Torsion bigravity: a purely geometric modified theory of gravity

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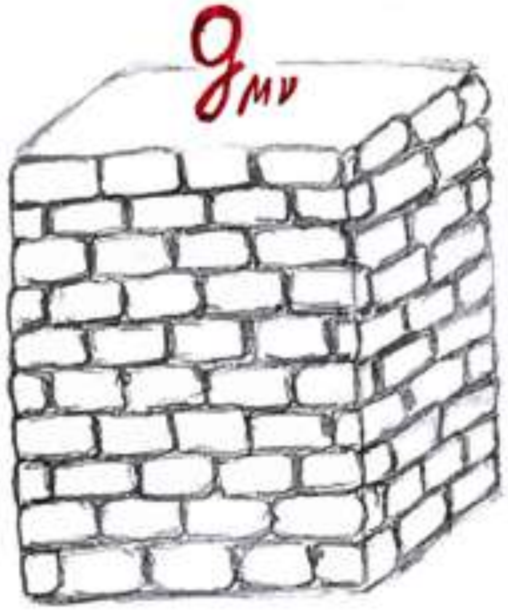
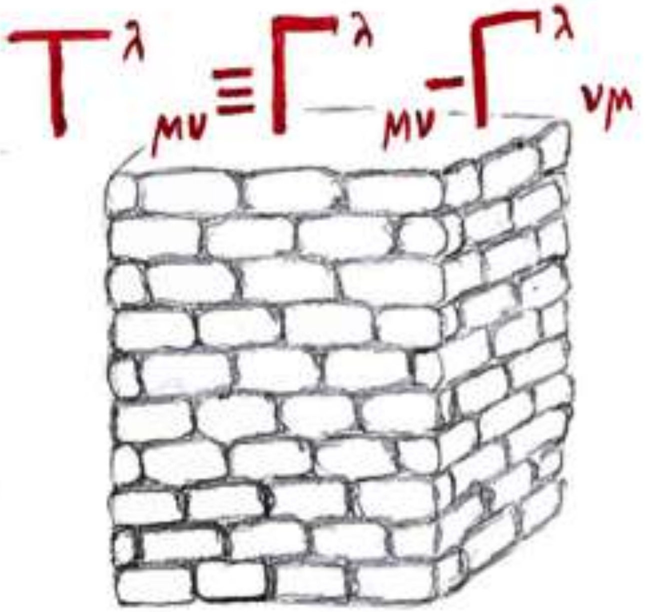
Laboratoire Charles Coulomb
Montpellier
24 September 2021

Metric and torsion

Elie Cartan

$(g_{\mu\nu}\ ,\ \Gamma^{\lambda}{}_{\mu\nu})$





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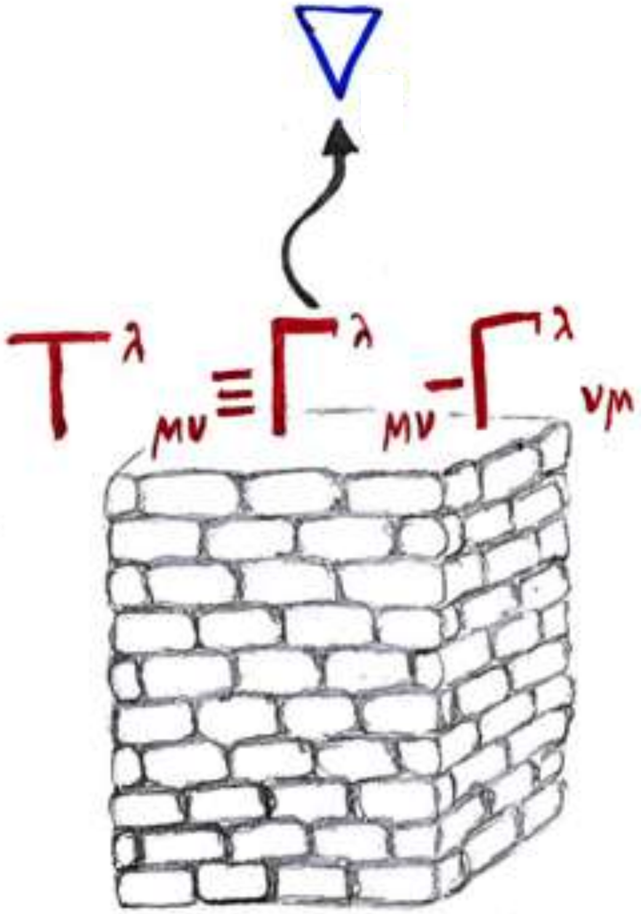

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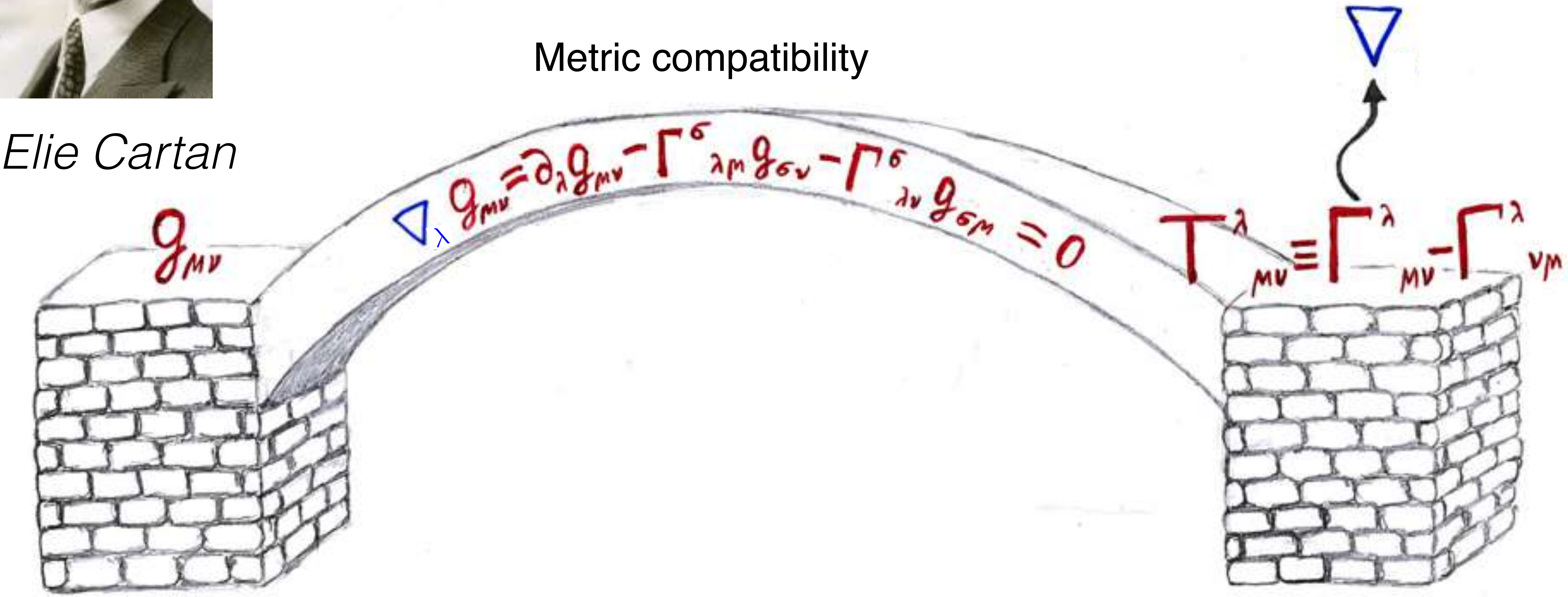
Metric compatibility



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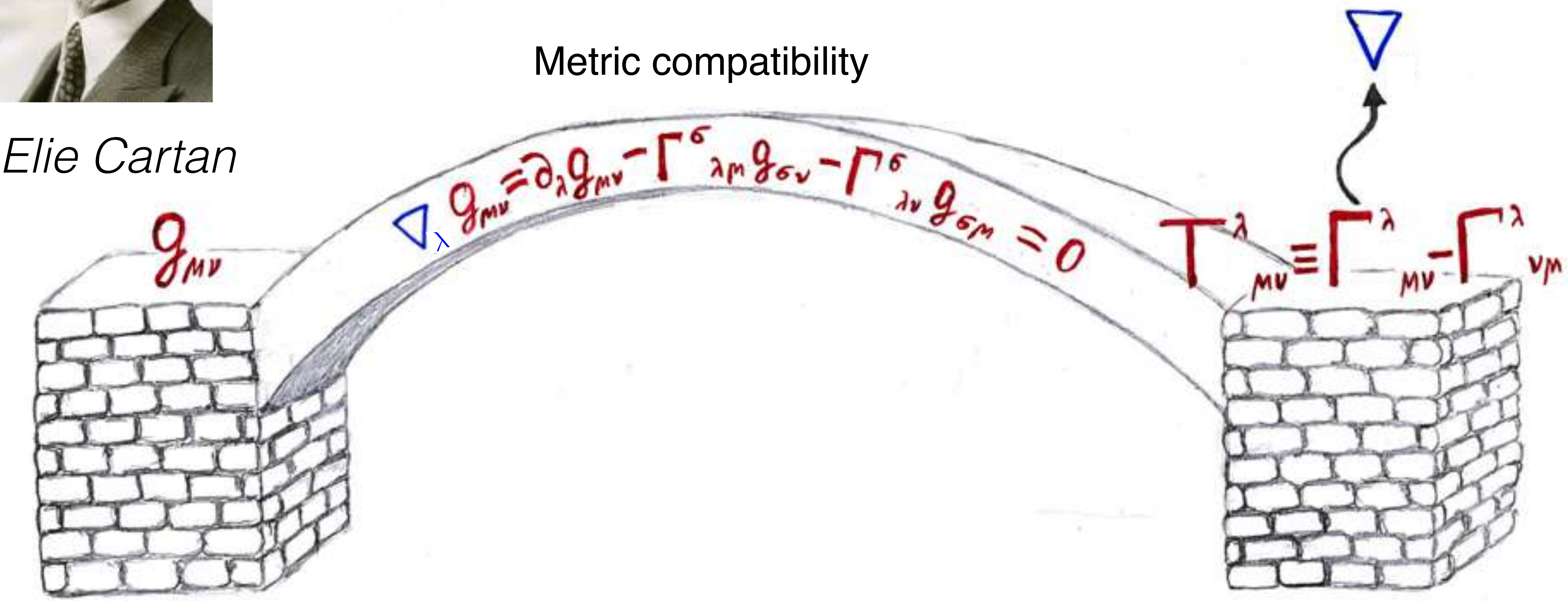


Moving frame 1-form

$$e^i = e^i{}_\mu dx^\mu$$

$$e_{i\mu} e^i{}_\nu = g_{\mu\nu}\ , \quad e_i{}^\mu e_{j\mu} = \eta_{ij}$$

Metric compatibility

$$A_{ij\mu} = -A_{ji\mu}$$

Connection 1-form

$$\mathcal{A}^i{}_j = A^i{}_{j\mu} dx^\mu$$

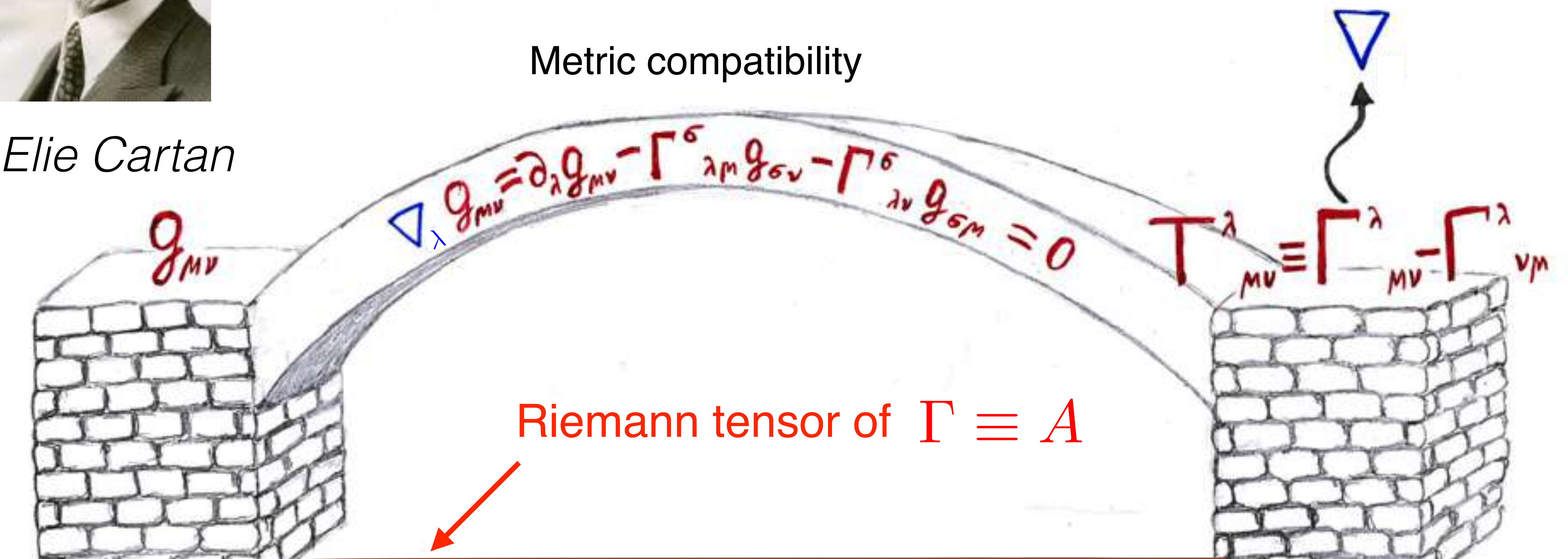
Torsion tensor of $\mathcal{A}^i{}_j$

$$de^i + \mathcal{A}^i{}_j e^j = -\frac{1}{2} T^i{}_{[jk]} e^j \wedge e^k$$

Metric and torsion

Elie Cartan

$$(g_{\mu\nu}\ ,\ \Gamma^{\lambda}{}_{\mu\nu})$$



Metric compatibility

$$g_{\mu\nu}$$

$$\nabla_\lambda g_{\mu\nu}=\partial_\lambda g_{\mu\nu}-\Gamma^\sigma{}_{\lambda\mu}g_{\sigma\nu}-\Gamma^\sigma{}_{\lambda\nu}g_{\sigma\mu}=0$$

$$T^\lambda{}_{\mu\nu}\equiv\Gamma^\lambda{}_{\mu\nu}-\Gamma^\lambda{}_{\nu\mu}$$

$$\nabla$$

Riemann tensor of $\Gamma \equiv A$

$$\boxed{\begin{aligned} &F^{\rho}{}_{\sigma\mu\nu}(\Gamma) = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma} \\ &\ \| \\ &F^{\rho}{}_{\sigma\mu\nu}(A) = e^{i\rho}e^{j}{}_{\sigma}\left(\partial_\mu A_{ij\nu} - \partial_\nu A_{ij\mu} + A_{ik\mu}A^{k}{}_{j\nu} - A_{ik\nu}A^{k}{}_{j\mu}\right) \end{aligned}}$$

Moving frame 1-form

Metric compatibility

$$A_{ij\mu} = -A_{ji\mu}$$

Connection 1-form

$$\mathcal{A}^{i}{}_{j} = A^{i}{}_{j\mu}dx^{\mu}$$

$$e_{i\mu}e^{i}{}_{\nu} = g_{\mu\nu}\ ,\quad e_{i}{}^{\mu}e_{j\mu} = \eta_{ij}$$

Torsion tensor of $\mathcal{A}^{i}{}_{j}$

$$de^{i} + \mathcal{A}^{i}{}_{j}e^{j} = -\frac{1}{2}T^{i}{}_{[jk]}e^{j}\wedge e^{k}$$

Einstein-Cartan theory

$$L_{\mathrm{Einstein-Cartan}} = \frac{1}{16\pi G_0} F(\Gamma) = \frac{1}{16\pi G_0} F(A)$$

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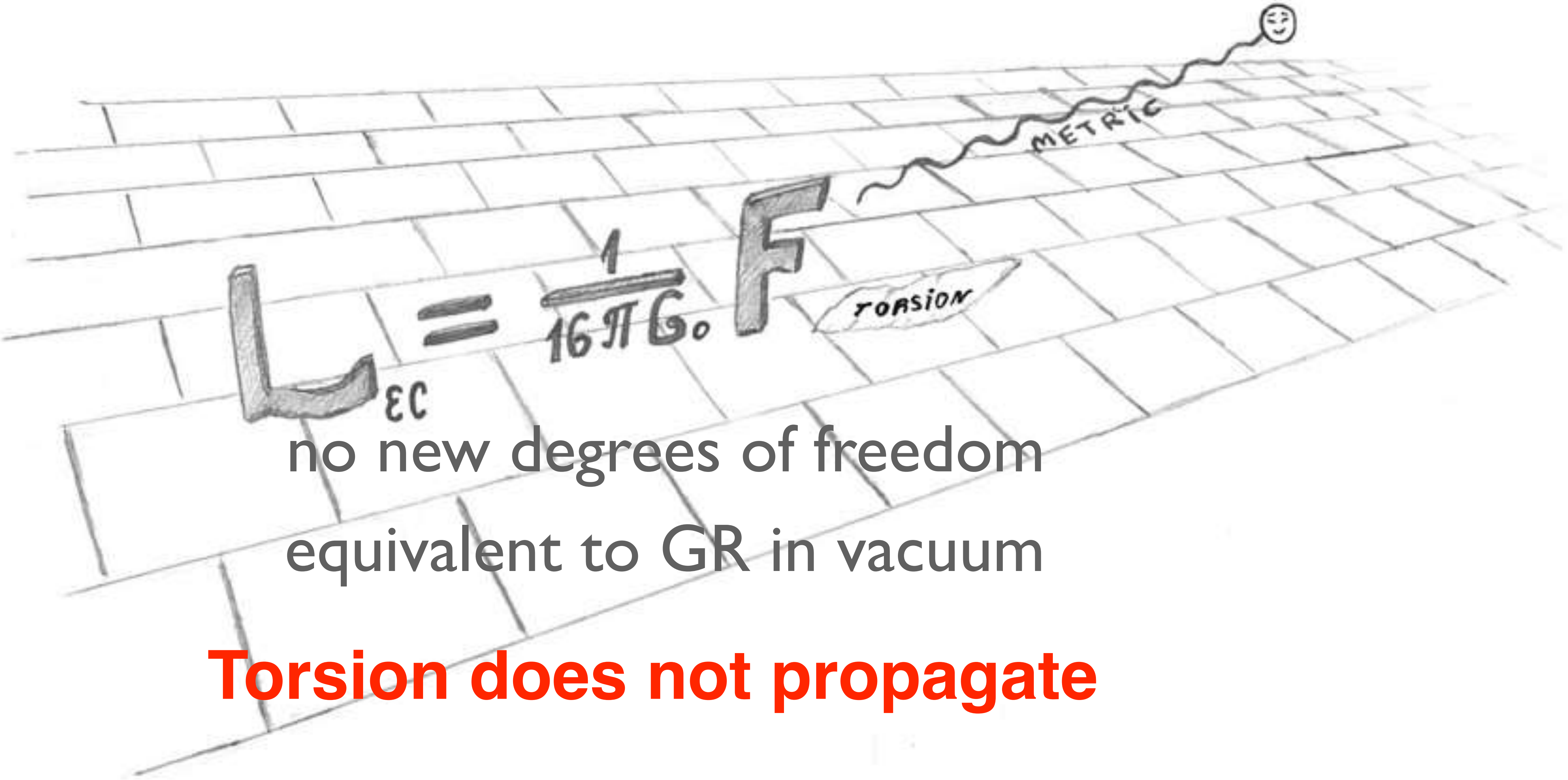


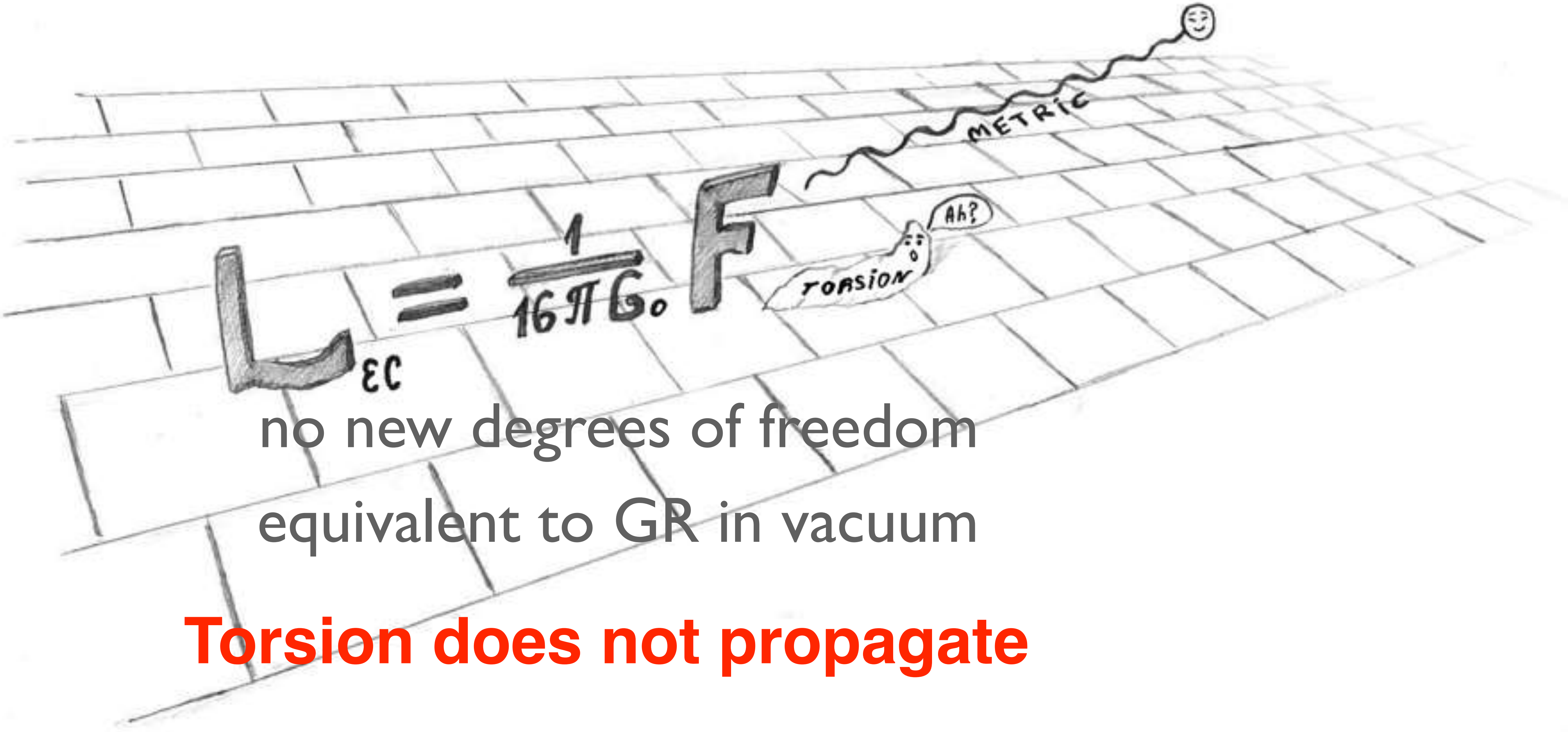
no new degrees of freedom

equivalent to GR in vacuum

Torsion does not propagate

Einstein-Cartan theory

$$L_{\mathrm{Einstein-Cartan}} = \frac{1}{16\pi G_0} F(\Gamma) = \frac{1}{16\pi G_0} F(A)$$



no new degrees of freedom

equivalent to GR in vacuum

Torsion does not propagate

Could there be a theory of gravity containing dynamical torsion ?

Torsion bigravity

Sezgin-van Nieuwenhuizen’80,
Hayashi-Shirafuji'81

→

Nair
Randjbar-Daemi
Rubakov'09

→

Nikiforova
Randjbar-Daemi
Rubakov’09

→

Damour
Nikiforova'19

Torsion bigravity

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Nikiforova'19

L_{GR}

$L_{\mathrm{Einstein-Cartan}}$

$(\text{Ricci of } A)^2$

Torsion bigravity

Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji'81 → Nair Randjbar-Daemi Rubakov'09 → Nikiforova Randjbar-Daemi Rubakov’09 → Damour Nikiforova'19

$$L_{\mathrm{GR}} \quad + \quad L_{\mathrm{Einstein-Cartan}} \qquad\qquad (\mathrm{Ricci\ of\ } A)^2$$

$$L = \frac{1}{16\pi G_0 (1+\eta)} R$$

Torsion bigravity

Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji'81 → Nair Randjbar-Daemi Rubakov'09 → Nikiforova Randjbar-Daemi Rubakov’09 → Damour Nikiforova'19

$$L_{\mathrm{GR}} \quad + \quad L_{\mathrm{Einstein-Cartan}} \qquad\qquad (\mathrm{Ricci\ of\ } A)^2$$

$$L = \frac{1}{16\pi G_0 (1+\eta)} R \quad + \quad \frac{\eta}{16\pi G_0 (1+\eta)} F(A)$$

Torsion bigravity

Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji'81

Nair Randjbar-Daemi Rubakov'09

Nikiforova Randjbar-Daemi Rubakov’09

Damour Nikiforova'19

$$L_{\rm GR} \qquad + \qquad L_{\rm Einstein-Cartan} \qquad + \qquad (\text{Ricci of } A)^2$$

$$L = \frac{1}{16\pi G_0 (1+\eta)} R \;+\; \frac{\eta}{16\pi G_0 (1+\eta)} F(A) \;+\; \frac{\eta}{16\pi G_0 \kappa^2}\left(F_{(ij)}F^{(ij)} - \frac{1}{3}F^2\right) + c_{34} F_{[ij]}F^{[ij]}$$

Torsion bigravity

Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji'81 → Nair Randjbar-Daemi Rubakov'09 → Nikiforova Randjbar-Daemi Rubakov’09 → Damour Nikiforova'19

$$L_{\rm GR} \quad + \quad L_{\rm Einstein-Cartan} \quad + \quad (\text{Ricci of } A)^2$$

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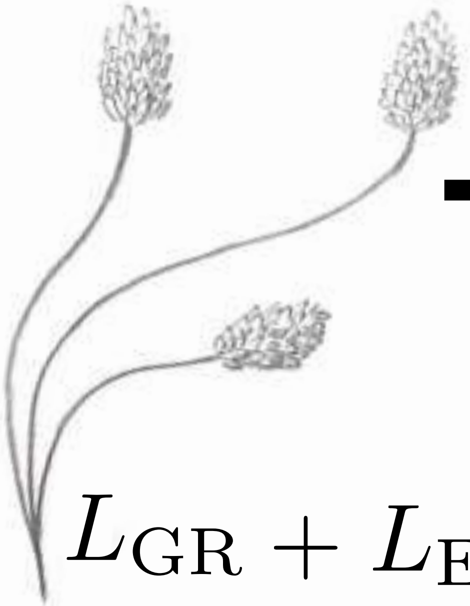
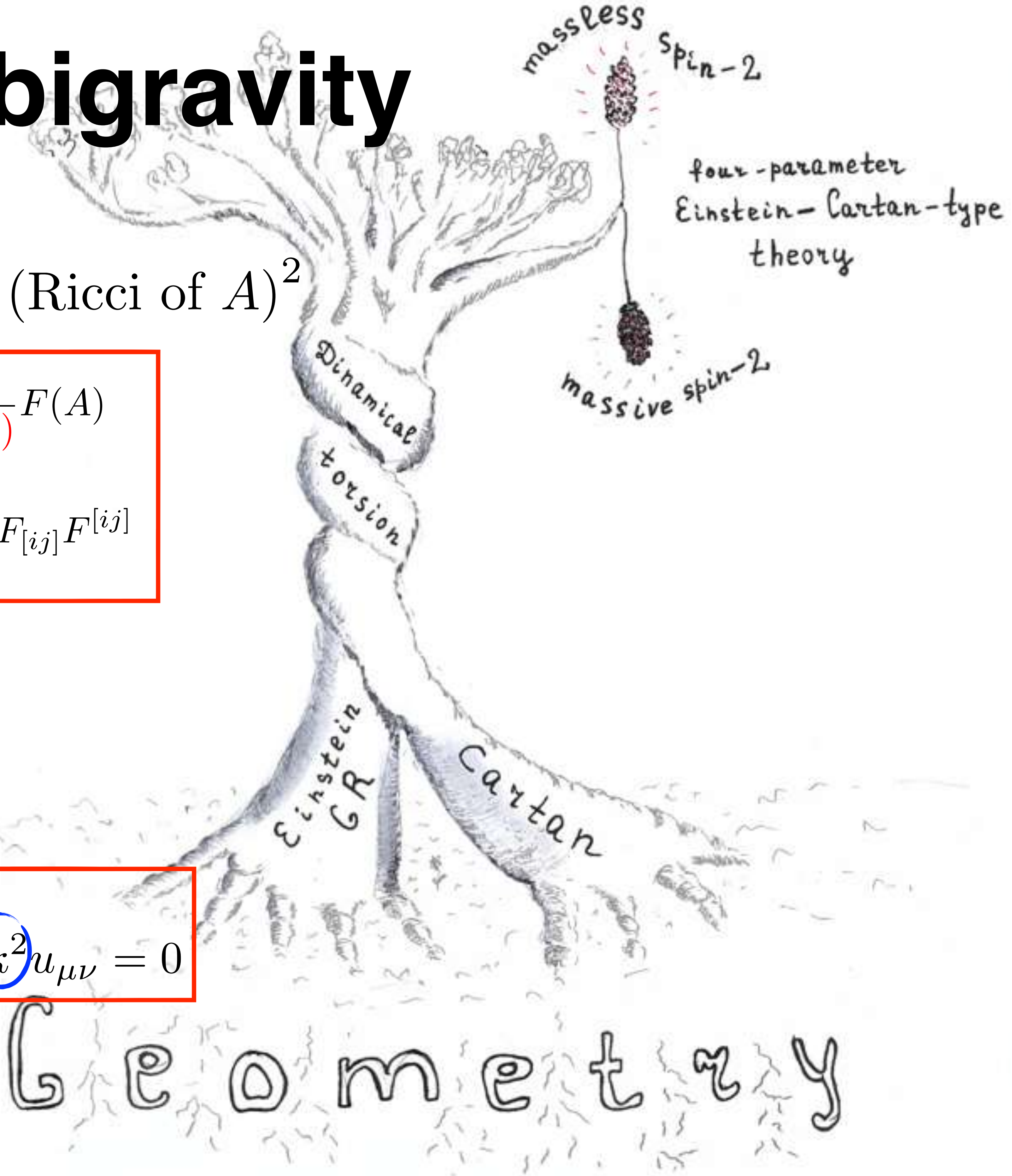
Reminder :

$$\boxed{F \text{ means curvature of } A}$$

Torsion bigravity





$$L_{\mathrm{GR}} + L_{\mathrm{Einstein-Cartan}} + (\mathrm{Ricci\ of\ } A)^2$$

$$L = \frac{1}{16\pi G_0 (1+\eta)} R + \frac{\eta}{16\pi G_0 (1+\eta)} F(A) + \frac{\eta}{16\pi G_0 \kappa^2} \left(F_{(ij)} F^{(ij)} - \frac{1}{3} F^2 \right) + c_{34} F_{[ij]} F^{[ij]}$$

Field content around flat space:

massless spin-2

$$\Box \bar{h}_{\mu\nu} + \partial_{\mu\nu} \bar{h} - \partial_{\mu\sigma} \bar{h}^{\sigma}_{\nu} - \partial_{\nu\sigma} \bar{h}^{\sigma}_{\mu} = 0$$

massive spin-2

$$\Box u_{\mu\nu} + \partial_{\mu\nu} u - \partial_{\mu\sigma} u^{\sigma}_{\nu} - \partial_{\nu\sigma} u^{\sigma}_{\mu} - \kappa^2 u_{\mu\nu} = 0$$

Explicit field equations

$$\frac{\eta}{(1+\eta)16\pi G_0}\left(F_{ij}-\frac{1}{2}\eta_{ij}F\right)+\frac{1}{(1+\eta)16\pi G_0}\left(R_{ij}-\frac{1}{2}\eta_{ij}R\right)$$

$$+\frac{\eta}{16\pi G_0\kappa^2}\left[F_{ki}F_{kj}+F_{kl}F_{kilj}-\frac{2}{3}F\,F_{ij}-\frac{1}{2}\eta_{ij}\left(F_{kl}F_{kl}-\frac{1}{3}F^2\right)\right]=T_{ij}$$

stress-energy tensor

2nd order in connection and metric !

$$\left[\eta_{ik}\left(D_mP_{jm}-\frac{2}{3}D_jP\right)-D_iP_{jk}\right]-\left[\eta_{jk}\left(D_mP_{im}-\frac{2}{3}D_iP\right)-D_jP_{ik}\right]$$

$$+\frac{\eta}{(1+\eta)16\pi G_0}\left(K_{ikj}-K_{jki}-K_{ill}\eta_{jk}+K_{jll}\eta_{ik}\right)$$

$$+\left(K_{mkn}-K_{nkm}-K_{mll}\eta_{nk}+K_{nll}\eta_{mk}\right)\left(\eta_{im}P_{jn}-\eta_{jm}P_{in}-\frac{2}{3}\eta_{im}\eta_{jn}P\right)=S_{ijk}$$

spin density

Theories with massive spin-2

bimetric gravity

$$g_{\mu\nu}\ , \quad f_{\mu\nu}$$

$$L \sim \sqrt{g}R(g) + \sqrt{f}R(f) - V\left(\sqrt{g^{-1}f}\right)$$

☺ $m = 0,\ s = 2 \quad \alpha\,\delta g_{\mu\nu} + \beta\,\delta f_{\mu\nu}$

☺ $m \neq 0,\ s = 2 \quad \bar{\alpha}\,\delta g_{\mu\nu} + \bar{\beta}\,\delta f_{\mu\nu}$

ghost-free (5 dof) around
generic backgrounds

torsion bigravity

$$g_{\mu\nu}\ , \quad T^{\lambda}{}_{\mu\nu}$$

$$L \sim \sqrt{g}R(g) + F + \left(F_{(ij)}F^{(ij)} - \frac{1}{3}F^2\right) + c_{34}F_{[ij]}F^{[ij]}$$

☺ $m = 0,\ s = 2 \quad \delta g_{\mu\nu} + \frac{1}{\kappa^2}\,\delta F_{\mu\nu}$

☺ $m \neq 0,\ s = 2 \quad \delta F_{\mu\nu} - \frac{1}{6}g_{\mu\nu}\,\delta F$

ghost-free (5 dof) around:
flat space,
Einstein spaces,
static spher.symm. solutions

A 5-parameter class of dynamical torsion theories revived with cosmological motivation

Nair—Randjbar-Daemi—Rubakov (2009)
V. Nikiforova, S. Randjbar-Daemi, V. Rubakov (2009)
Deffayet—Randjbar-Daemi (2011)
V. Nikiforova, S. Randjbar-Daemi, V. Rubakov (2016)
V. Nikiforova (2017)
V. Nikiforova, T. Damour (2018)

self-accelerating solution
where torsion accelerates the Universe

but
instabilities found.

Torsion bigravity world

OK let us model a world
on a base of torsion bigravity.

And then will see
whether it looks like our real world or not.

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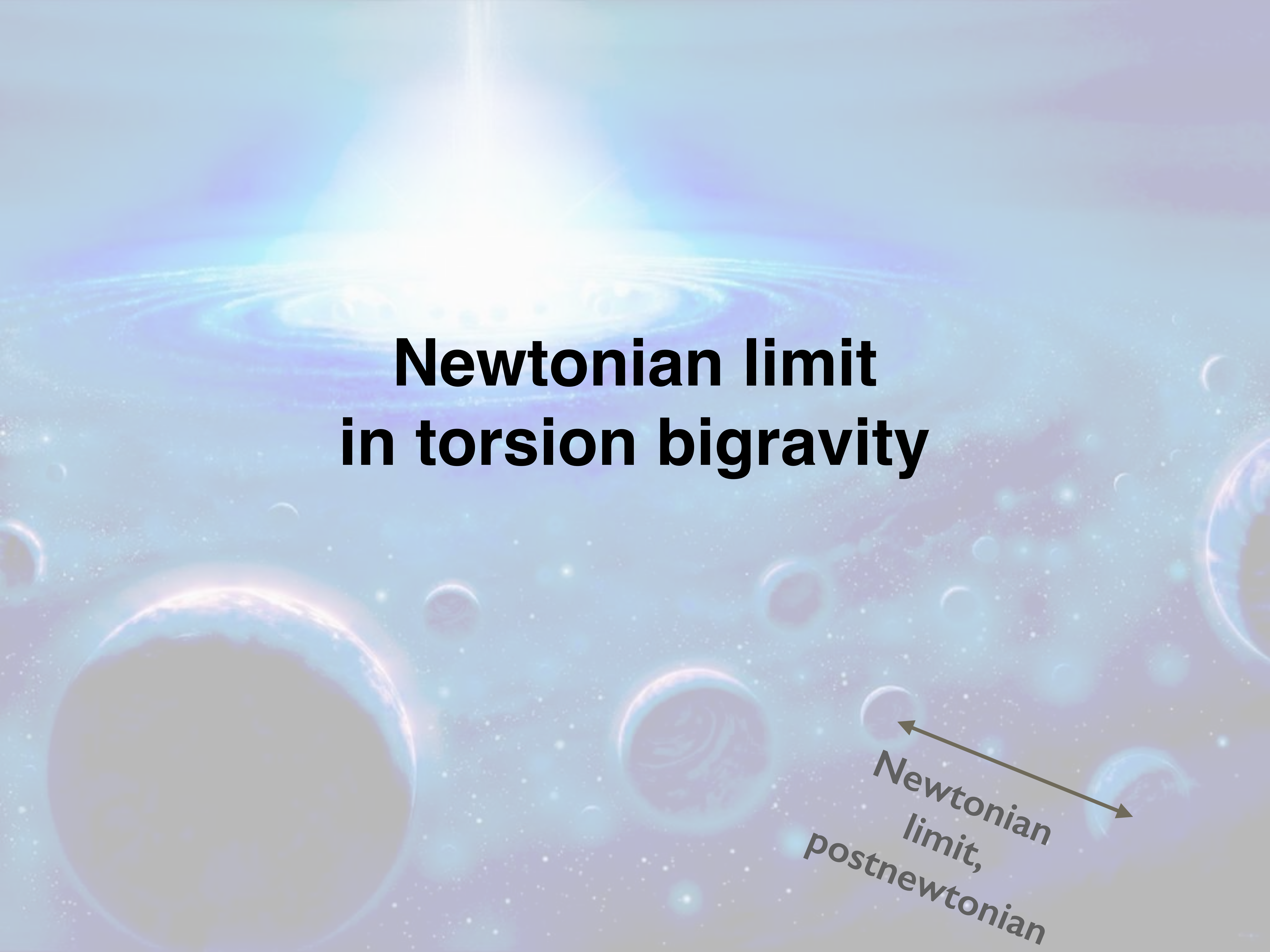
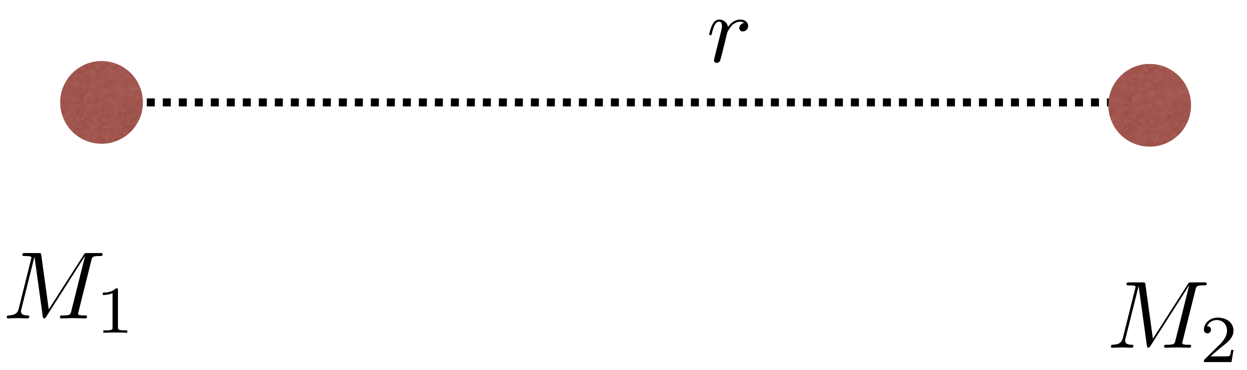
Newtonian
limit,
postnewtonian

Newtonian limit in torsion bigravity

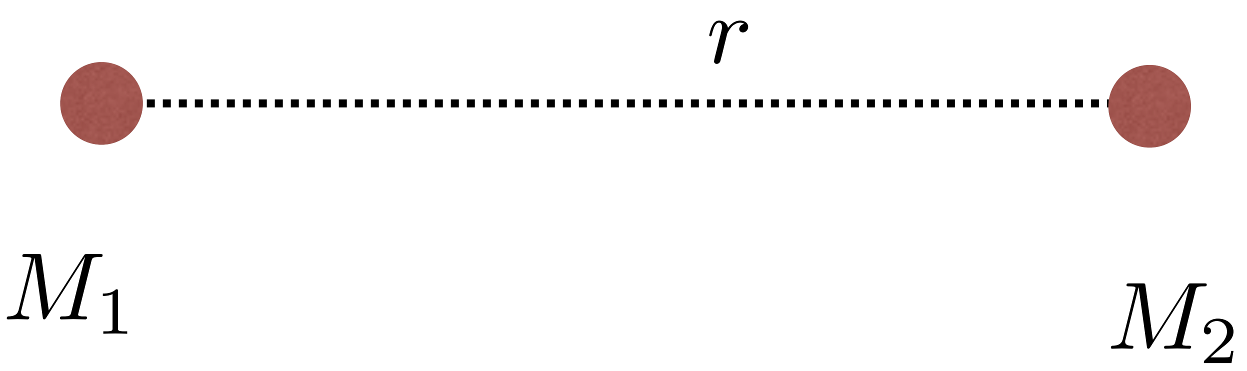


Newtonian limit, postnewtonian

Newtonian limit



Newtonian limit

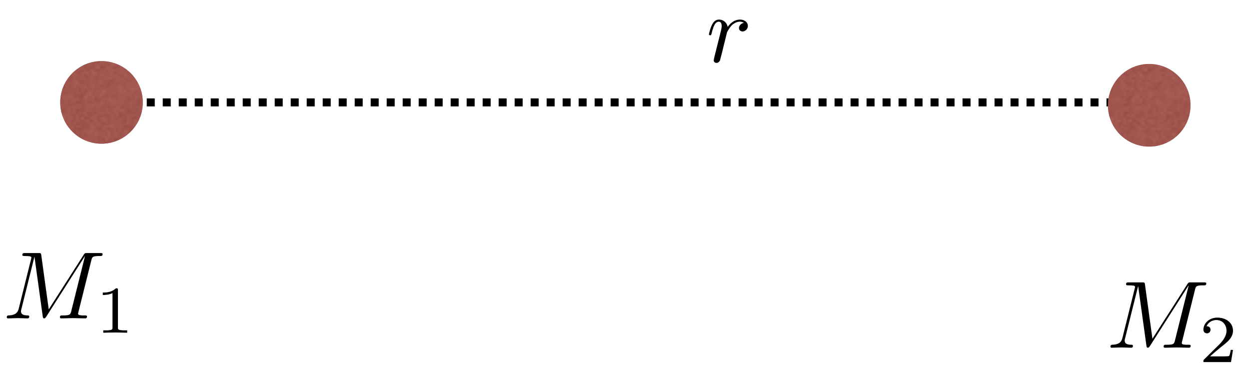


$$\boxed{V_{\text{int}}^{\text{Newtonian}} = -\underbrace{G_0\frac{M_1M_2}{r}}_{\substack{\text{massless}\\\text{spin-2}}} - \underbrace{G_m\frac{M_1M_2}{r}e^{-\kappa r}}_{\substack{\text{massive}\\\text{spin-2}}}}$$

Coupling constant :

$$\boxed{\frac{G_m}{G_0} = \frac{4}{3}\eta}$$

Newtonian limit



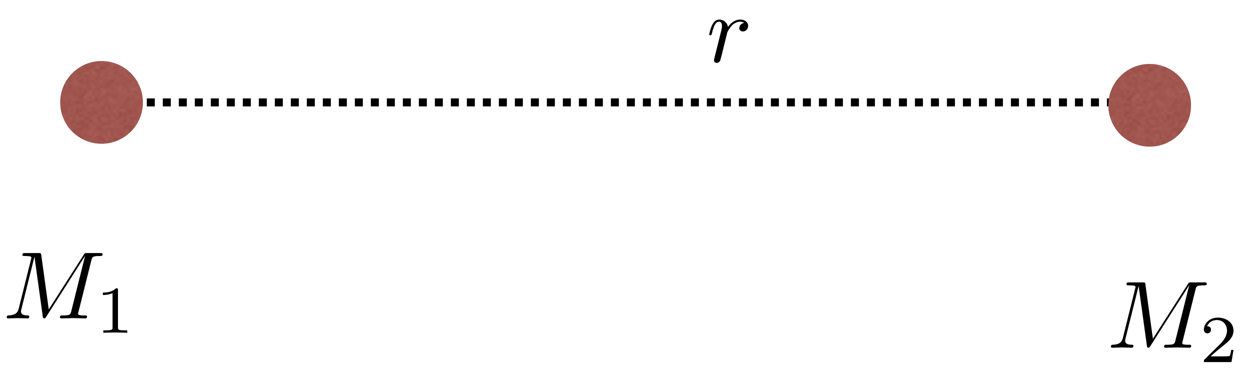
$$\boxed{V_{\rm int}^{\rm Newtonian} = -\underbrace{G_0\frac{M_1M_2}{r}}_{\text{massless spin-2}} - \underbrace{G_m\frac{M_1M_2}{r}e^{-\kappa r}}_{\text{massive spin-2 of mass } \kappa}}$$

Coupling constant :

$$\boxed{\frac{G_m}{G_0} = \frac{4}{3}\eta}$$

$$\boxed{\begin{aligned} L = & \frac{1}{16\pi G_0(1+\eta)}R + \frac{\eta}{16\pi G_0(1+\eta)}F(A) \\ & + \frac{\eta}{16\pi G_0\kappa^2}\left(F_{(ij)}F^{(ij)} - \frac{1}{3}F^2\right) + c_{34}F_{[ij]}F^{[ij]} \end{aligned}}$$

Newtonian limit



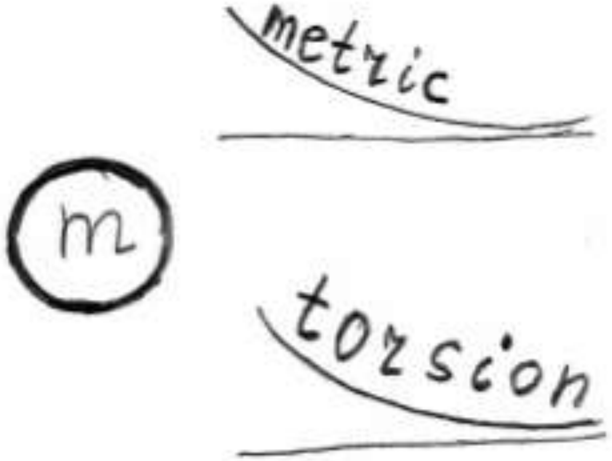
$$\boxed{V_{\text{int}}^{\text{Newtonian}} = -G_0\frac{M_1 M_2}{r} - G_m\frac{M_1 M_2}{r}e^{-\kappa r}}$$

massless spin-2

massive spin-2

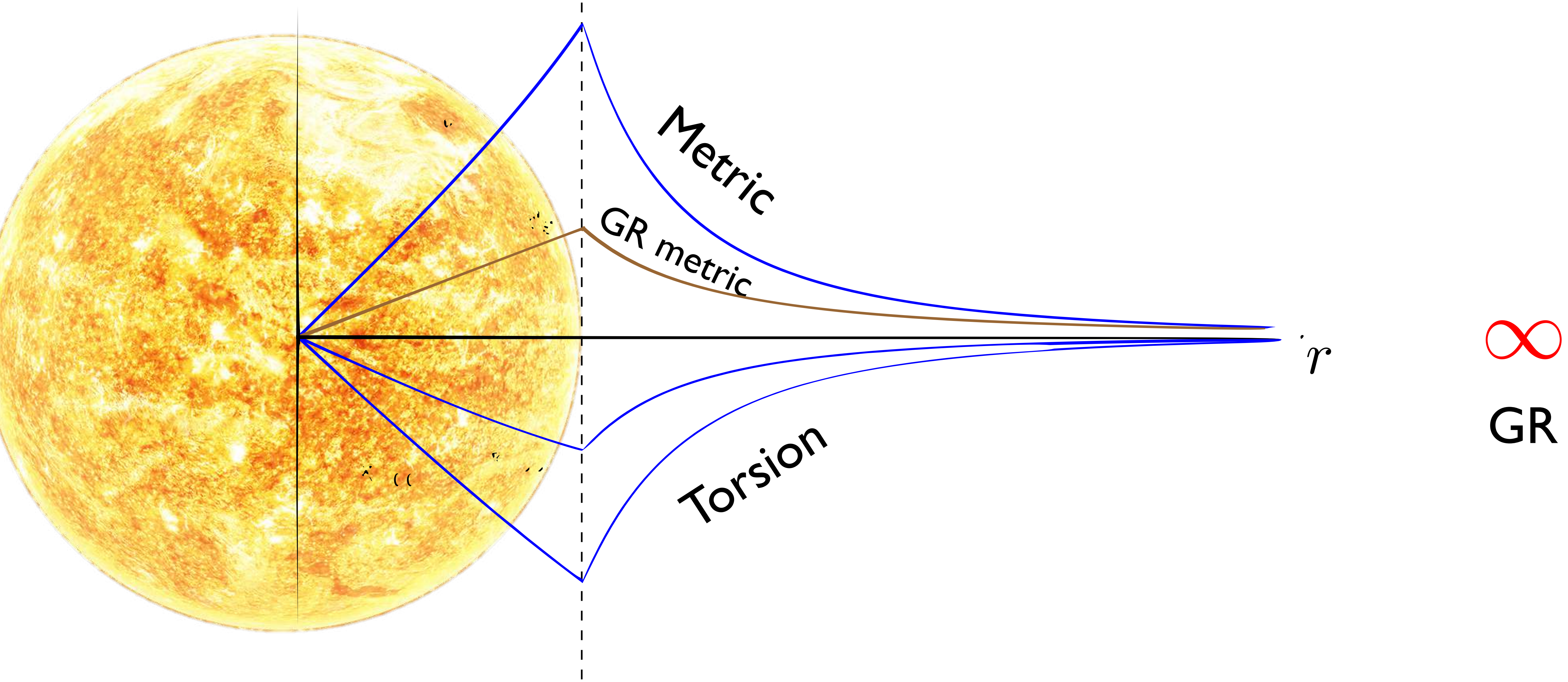
$$\boxed{\frac{G_m}{G_0} = \frac{4}{3}\eta}$$

$$L_{\text{int}} = 2G_0 T^{\mu\nu}\left(\frac{-4\pi}{\Box}\right)\left(T_{\mu\nu}-\frac{1}{2}T\eta_{\mu\nu}\right) + \frac{3}{2}G_m T^{\mu\nu}\left(\frac{-4\pi}{\Box-\kappa^2}\right)\left(T_{\mu\nu}-\frac{1}{3}T\eta_{\mu\nu}\right)$$



Stars in torsion bigravity

Torsion bigravity star

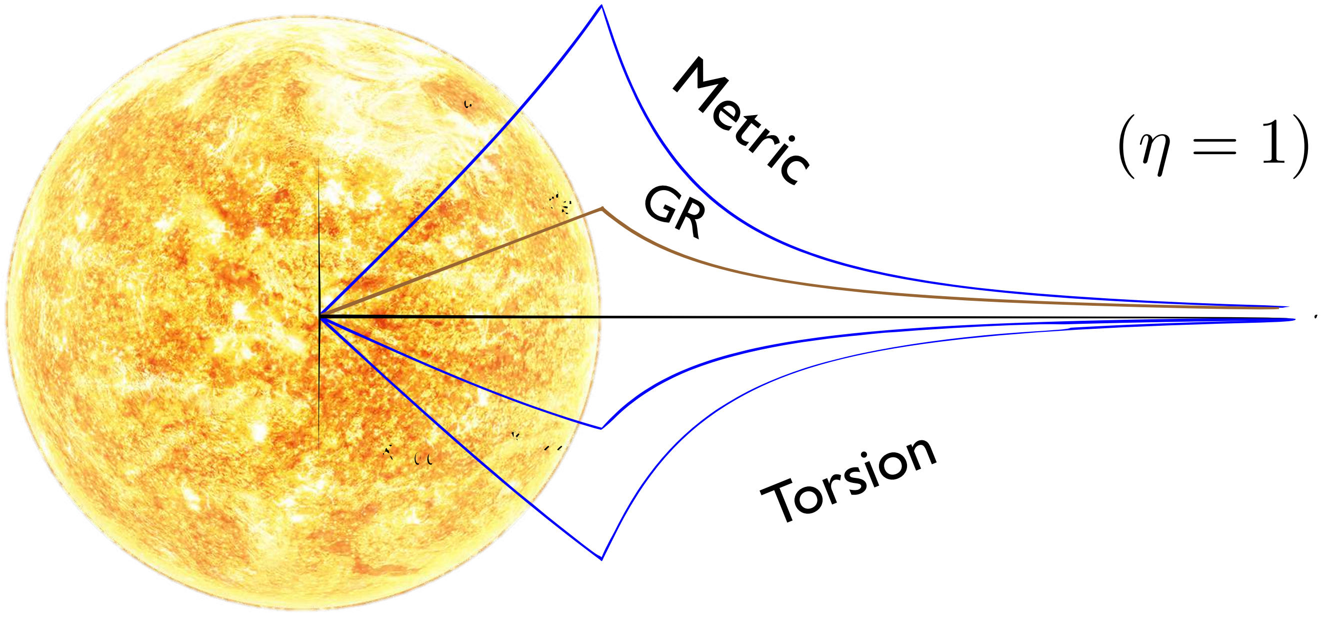


Spherically symmetric star solutions

Damour, Nikiforova'19

- dynamical torsion is generated by $T_{\mu\nu}$ of matter !

metric is different from the one predicted in GR
(compactness is different)



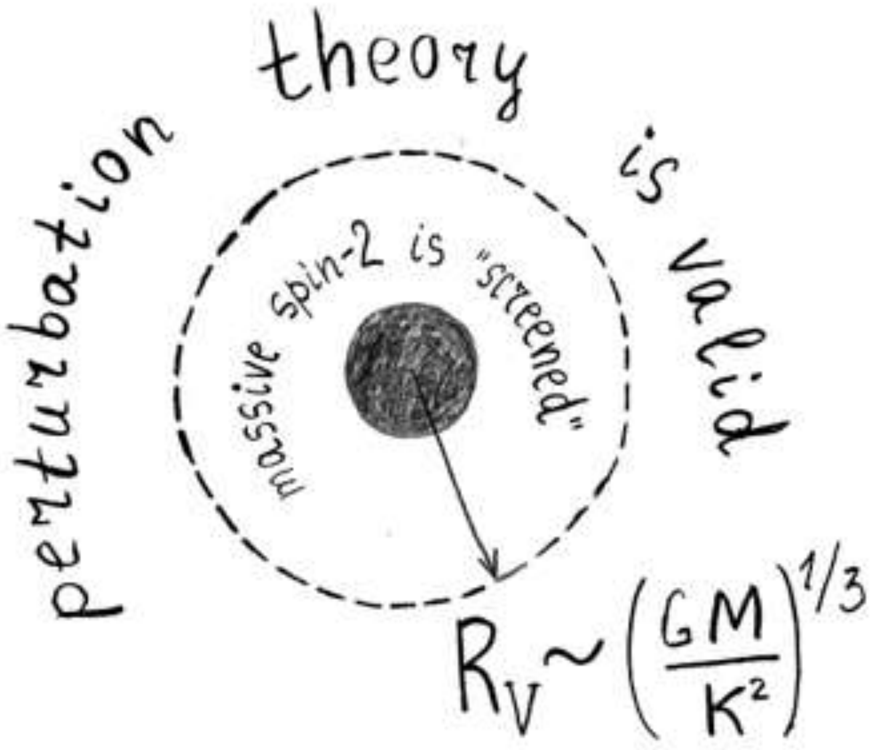
- the same number of dof (Babichev-Deffayet-Ziour) as in S.S. ghost-free bimetric gravity theories (DeRham-Gabadadze-Tolley, Hassan-Rosen)

Theories with massive spin-2 :

Vainshtein mechanism ?

bimetric gravity	torsion bigravity

Vainshtein screening



no $\frac{1}{\kappa^2}$ denominators

$\Downarrow$

absence of Vainshtein radius

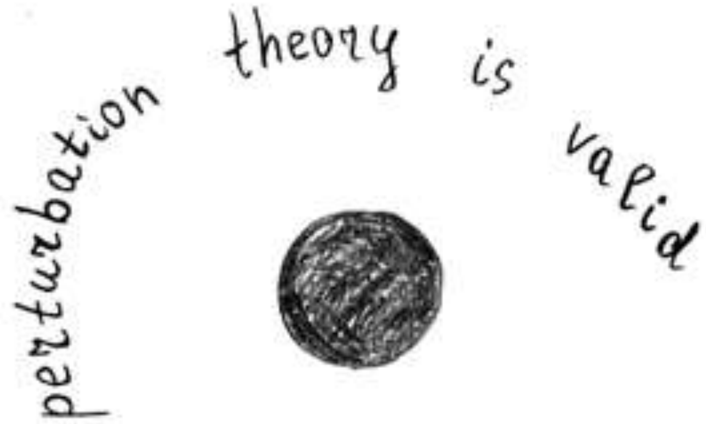
(Nikiforova'20)

$\Downarrow$

no Vainshtein screening

Solar system tests

$\boxed{\eta \ll 1}$

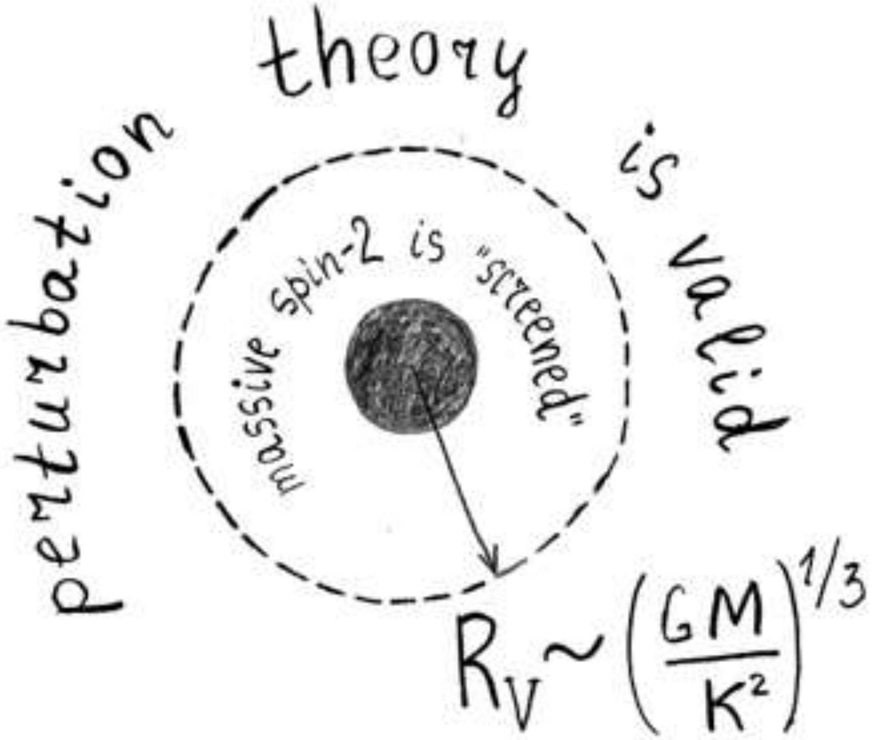


Theories with massive spin-2 :

Vainshtein mechanism ?

bimetric gravity	torsion bigravity

Vainshtein screening



no $\frac{1}{\kappa^2}$ denominators

$\Downarrow$

absence of Vainshtein radius

(Nikiforova'20)

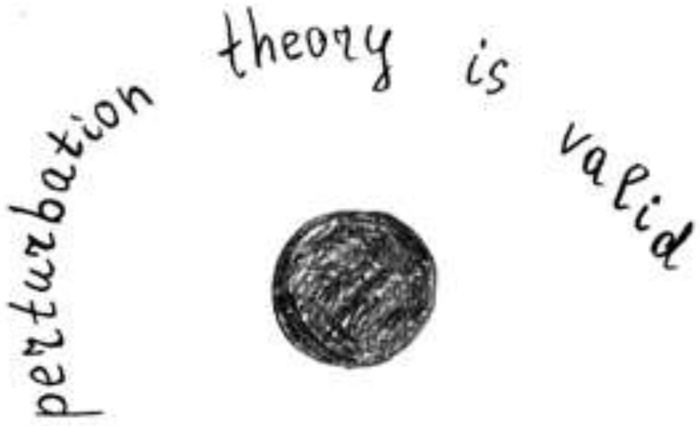
$\Downarrow$

no Vainshtein screening

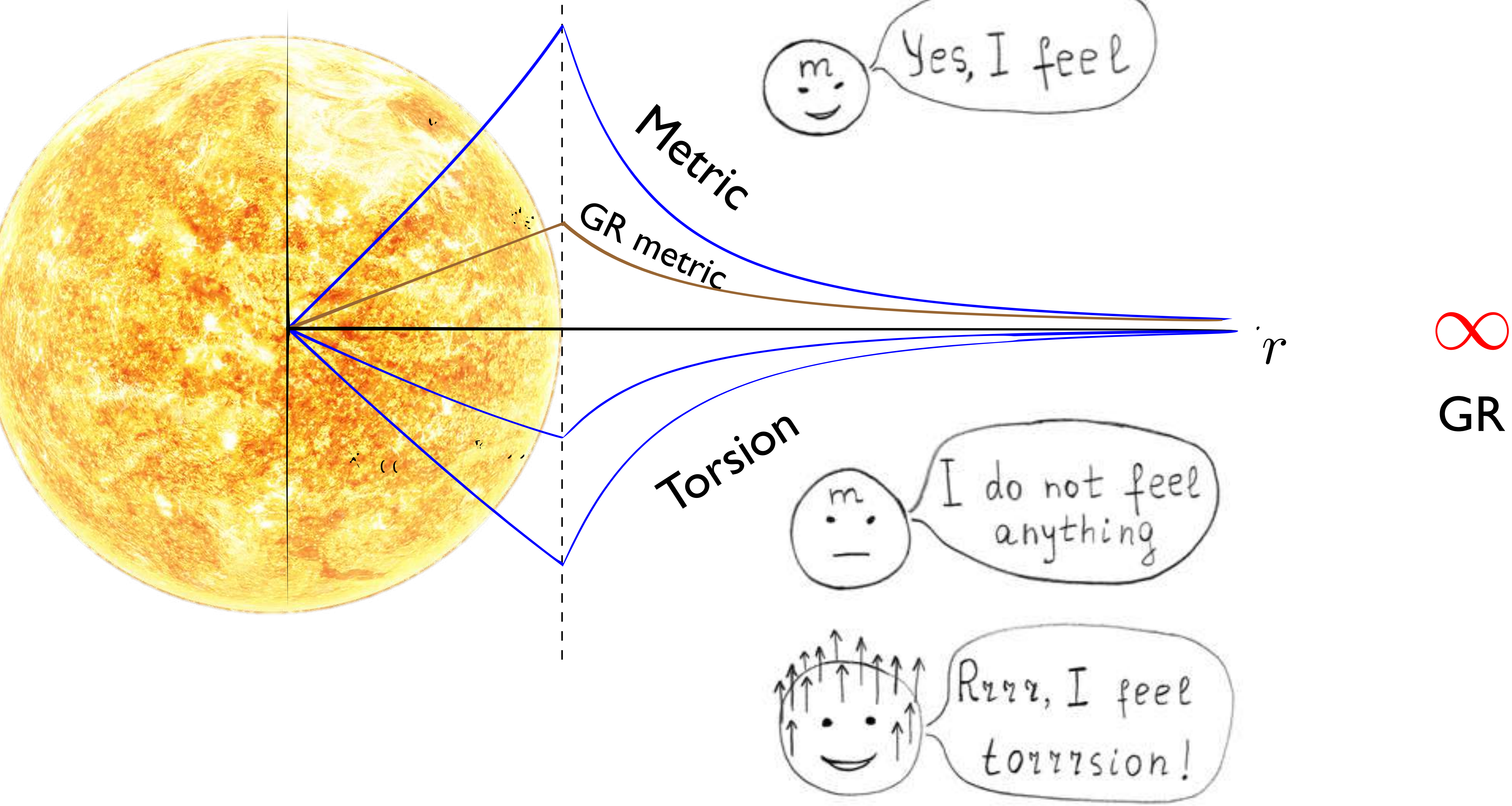
Solar system tests

$\boxed{\eta \ll 1}$

when $\eta \ll 1$ torsion remains not small !



Torsion bigravity star



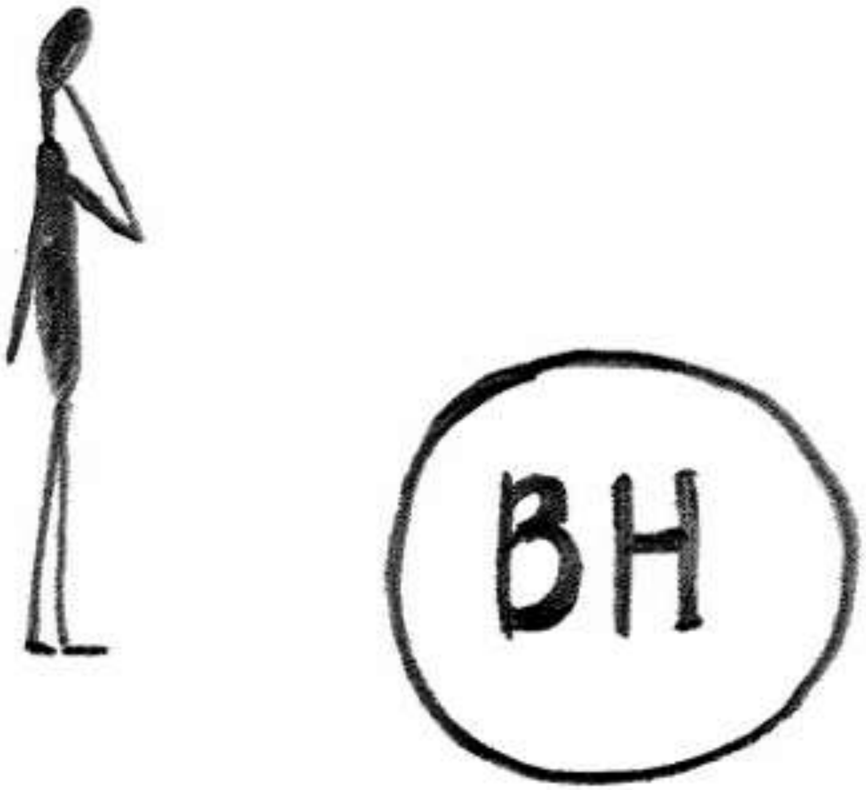
Black Holes in torsion bigravity

Theories with massive spin-2 :
Black hole ZOO

bimetric gravity	torsion bigravity
Einstein BHs (incl. Kerr) are exact solutions	Einstein BHs (incl. Kerr) are exact solutions (with zero torsion)
	
∃ Schwarzschild BHs with massive graviton hair (Brito-Cardoso-Pani'13)	∄ Schwarzschild BHs with linearized torsion hair (Nikiforova-Damour'20)
	

Black hole perturbations

Nikiforova'21 + work in progress



Black hole perturbations

Nikiforova'21 + work in progress

Black hole perturbations

Nikiforova'21 + work in progress

- <u>stability ?</u>
- <u>quasi-bound states ?</u>
- quasi-normal modes ?

Black hole perturbations

Nikiforova'21 + work in progress

- stability ?
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Black hole perturbations

Nikiforova'21 + work in progress

- stability ?
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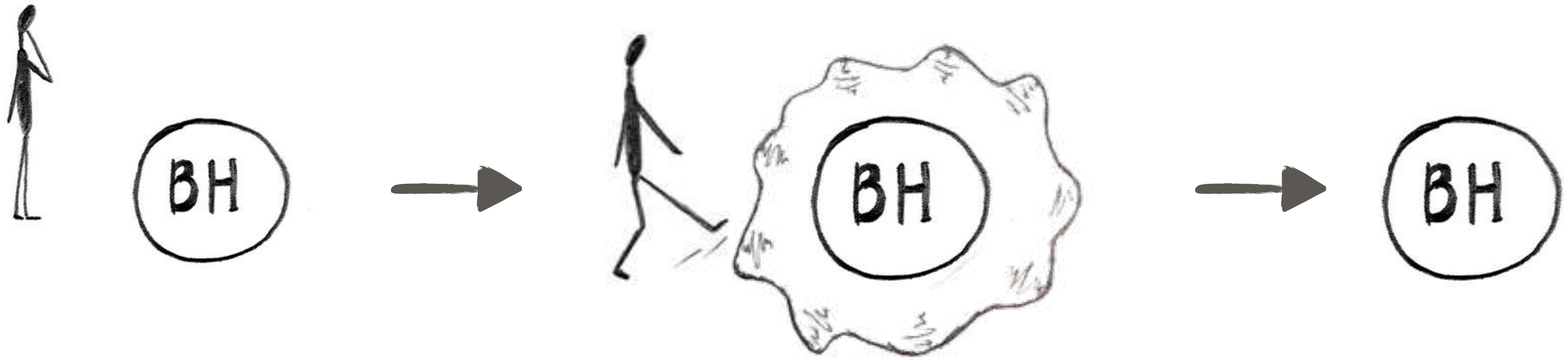

Black hole perturbations

Nikiforova'21 + work in progress

- stability ?
- quasi-bound states ?
- quasi-normal modes ?

Theories with massive spin-2 :

spherically symmetric perturbations of Schwarzschild black hole

bimetric gravity

torsion bigravity

if mass of massive spin-2 < 0.86
Schw. BH is unstable under
spher.symm. perts.
(Babichev-Fabbri'13,
Brito-Cardoso-Pani'13)

need large mass of massive spin-2
to avoid singularities

$$\kappa r_h > \sqrt{1+\eta}$$

Schw. BH is stable under spher.symm. perts.
(Nikiforova'21)

Stability proven by studying
the zero-energy wave-function:
absence of nodes
(Nikiforova'21)

Black hole perturbations

Nikiforova'21 + work in progress

non sph.-symm. perturbations :

- Reggie-Wheeler decomposition : even, odd sectors

$$h_{\mu\nu}^{\mathrm{axial},lm}(\omega, r, \theta, \phi) = \begin{pmatrix} 0 & 0 & h_0^{lm}(\omega, r) \csc\theta \partial_\phi Y_{lm}(\theta, \phi) & -h_0^{lm}(\omega, r) \sin\theta \partial_\theta Y_{lm}(\theta, \phi) \\ * & 0 & h_1^{lm}(\omega, r) \csc\theta \partial_\phi Y_{lm}(\theta, \phi) & -h_1^{lm}(\omega, r) \sin\theta \partial_\theta Y_{lm}(\theta, \phi) \\ * & * & -h_2^{lm}(\omega, r) \frac{X_{lm}(\theta, \phi)}{\sin\theta} & h_2^{lm}(\omega, r) \sin\theta W_{lm}(\theta, \phi) \\ * & * & * & h_2^{lm}(\omega, r) \sin\theta X_{lm}(\theta, \phi) \end{pmatrix},$$

$$h_{\mu\nu}^{\mathrm{polar},lm}(\omega, r, \theta, \phi)$$
$$= \begin{pmatrix} f(r) H_0^{lm}(\omega, r) Y_{lm} & H_1^{lm}(\omega, r) Y_{lm} & \eta_0^{lm}(\omega, r) \partial_\theta Y_{lm} & \eta_0^{lm}(\omega, r) \partial_\phi Y_{lm} \\ * & f(r)^{-1} H_2^{lm}(\omega, r) Y_{lm} & \eta_1^{lm}(\omega, r) \partial_\theta Y_{lm} & \eta_1^{lm}(\omega, r) \partial_\phi Y_{lm} \\ * & * & r^2 [K^{lm}(\omega, r) Y_{lm} + G^{lm}(\omega, r) W_{lm}] & r^2 G^{lm}(\omega, r) X_{lm} \\ * & * & * & r^2 \sin^2\theta [K^{lm}(\omega, r) Y_{lm} - G^{lm}(\omega, r) W_{lm}] \end{pmatrix}$$

- 5 dof for massive spin-2 perturbations (5 II order dif. eqs)
- no singularities if
- evidences for stability (work in progress)
- some quasi-bound states found (work in progress)