SORBONNE UNIVERSITÉ
CRÉATEURS DE FUTURS DEPUIS 1257

cnrs

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Institut d'Astrophysique de Paris

Charge–velocity–dependent one–scale model for cosmic string network evolution

Chacun a son défaut, où toujours il revient
J. de la Fontaine, Fables, 1668

Based on

Generalized velocity-dependent one-scale model for current-carrying strings
C. J. A. P. Martins, PP, I. Yu. Rybak and E. P. S. Shellard,
Phys. Rev. **D103**, 043538 (2021) [astro-ph 2011.09700]

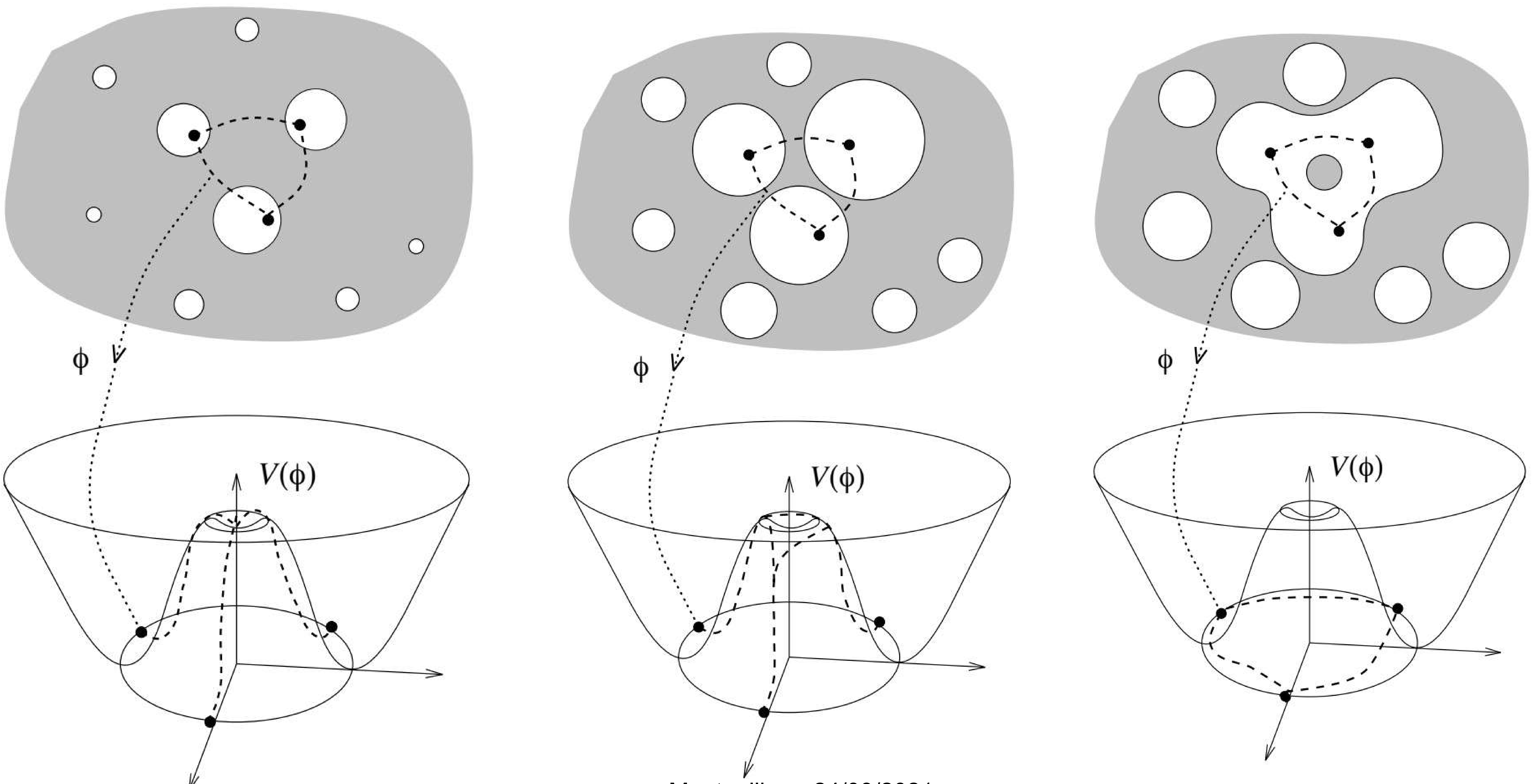
Charge-velocity-dependent one-scale linear model
C. J. A. P. Martins, PP, I. Yu. Rybak and E. P. S. Shellard,
Phys. Rev. **D** to appear (2021) [astro-ph 2108.03147]

Montpellier – 24/09/2021

Model: cosmic string

T.W.B. Kibble, *J. Phys.* **A9**, 1387 (1976)
M.B. Hindmarsh and T.W.B. Kibble, *Rep. Prog. Phys.* **58**, 477 (1995)

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} D_\mu \Phi \left(D^\mu \Phi\right)^* - V(\Phi)$$



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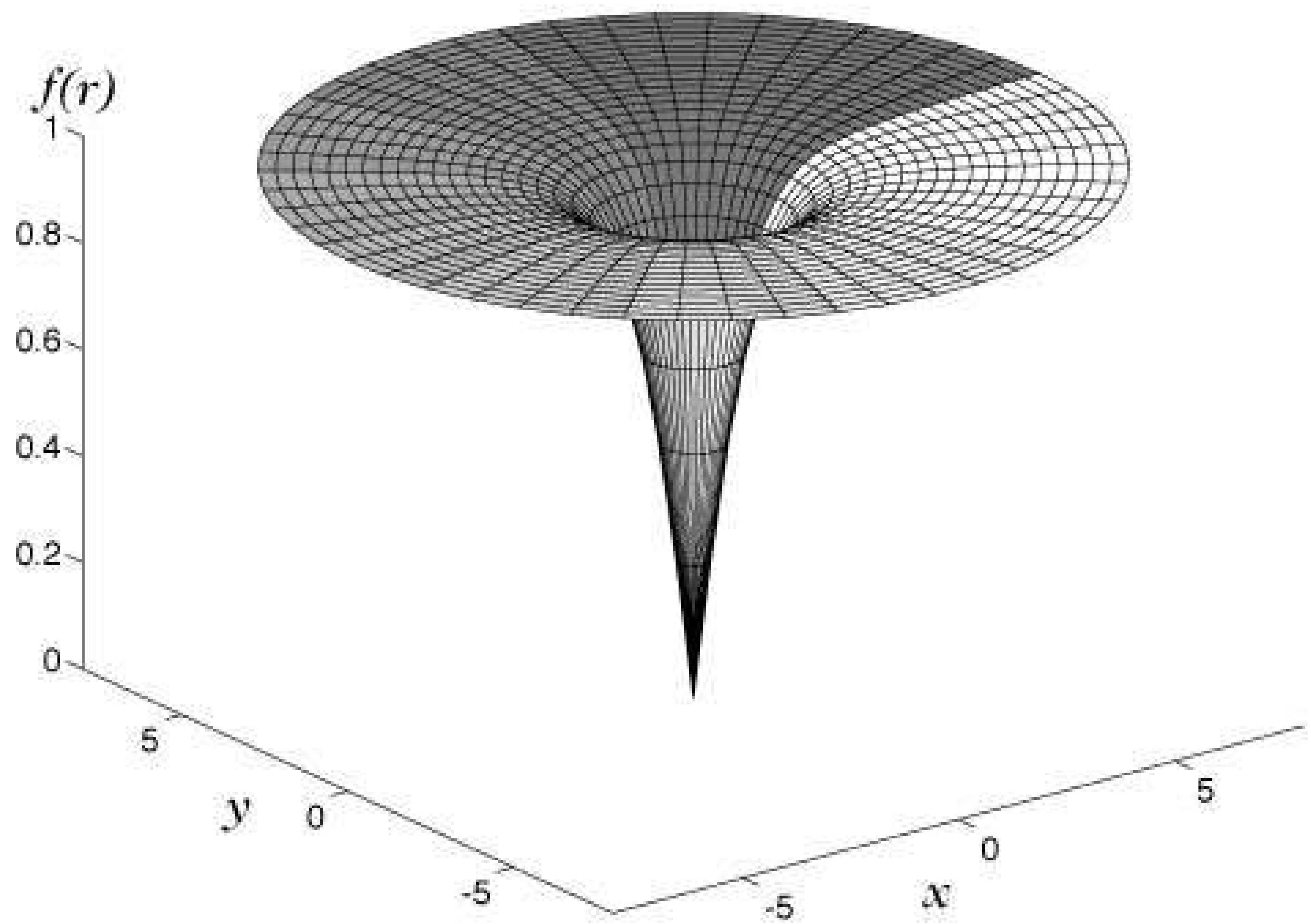
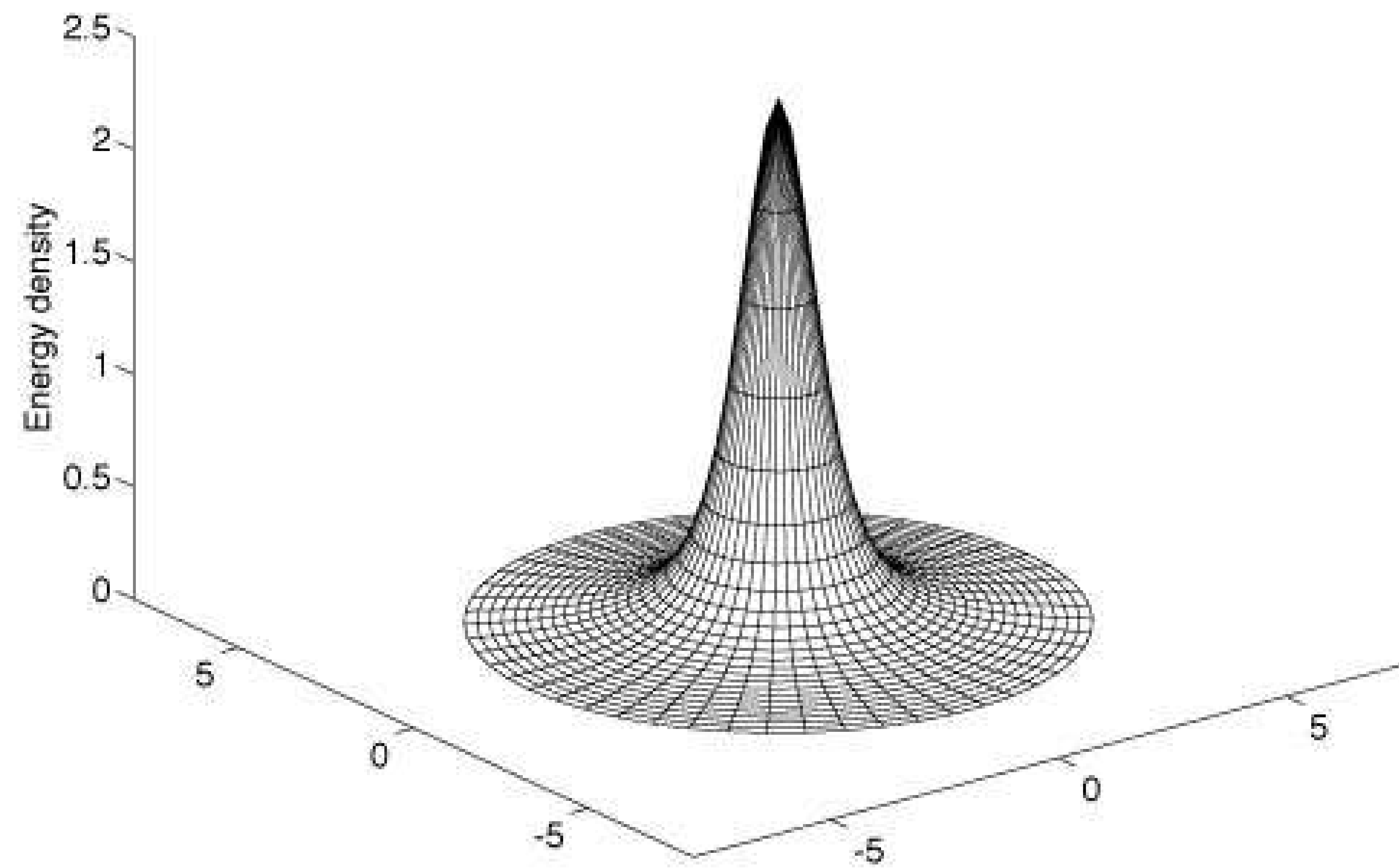
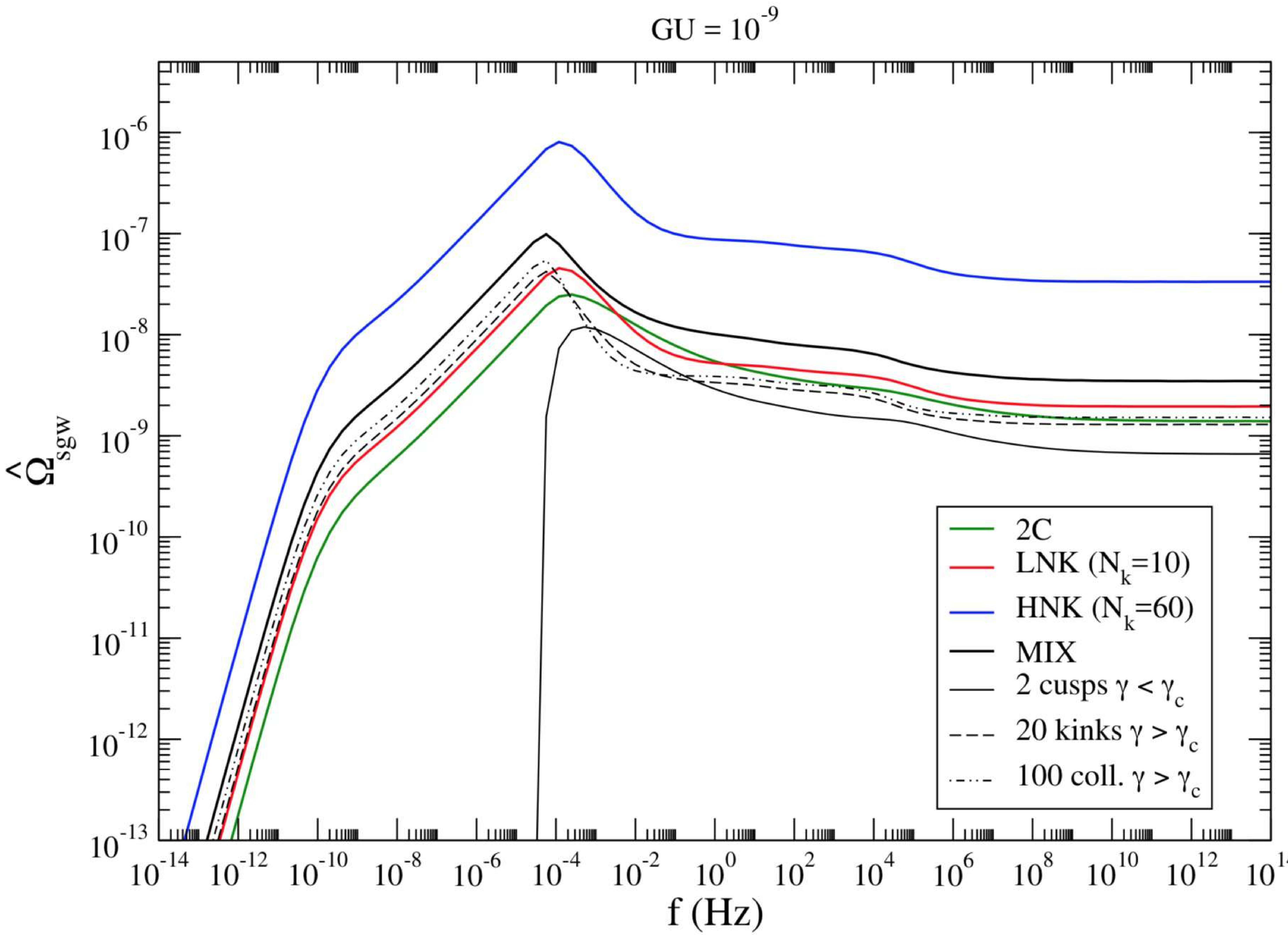




https://commons.wikimedia.org/wiki/File:Cosmic_strings_evolution_during_the_expansion_of_the_Universe.webm

GW stochastic background

C. Ringeval & T. Suyama, *JCAP* **17,** 12 (2017)
J. J. Blanco-Pillado & K. D. Olum, *Phys. Rev.* **D96**, 104046 (2017)
J. J. Blanco-Pillado K. D. Olum & X. Siemens, *Phys. Lett.* **B778,** 392 (2018)



Model: Witten superconducting cosmic string

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}D_\mu\Phi\left(D^\mu\Phi\right)^* + \frac{1}{2}D_\mu\Sigma\left(D^\mu\Sigma\right)^* - V(\Phi,\Sigma)$$

Field strength tensors

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Covariant derivatives

$$D_\mu\Phi = \left(\partial_\mu - \mathrm{i}\, e_\phi B_\mu\right)\Phi$$

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Potential

$$V(\Phi,\Sigma) = \frac{\lambda_\phi}{4}(|\Phi|^2 - \eta^2)^2 + f(|\Phi|^2 - \eta^2)|\Sigma|^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4 + \frac{m_\sigma^2}{2}|\Sigma|^2$$

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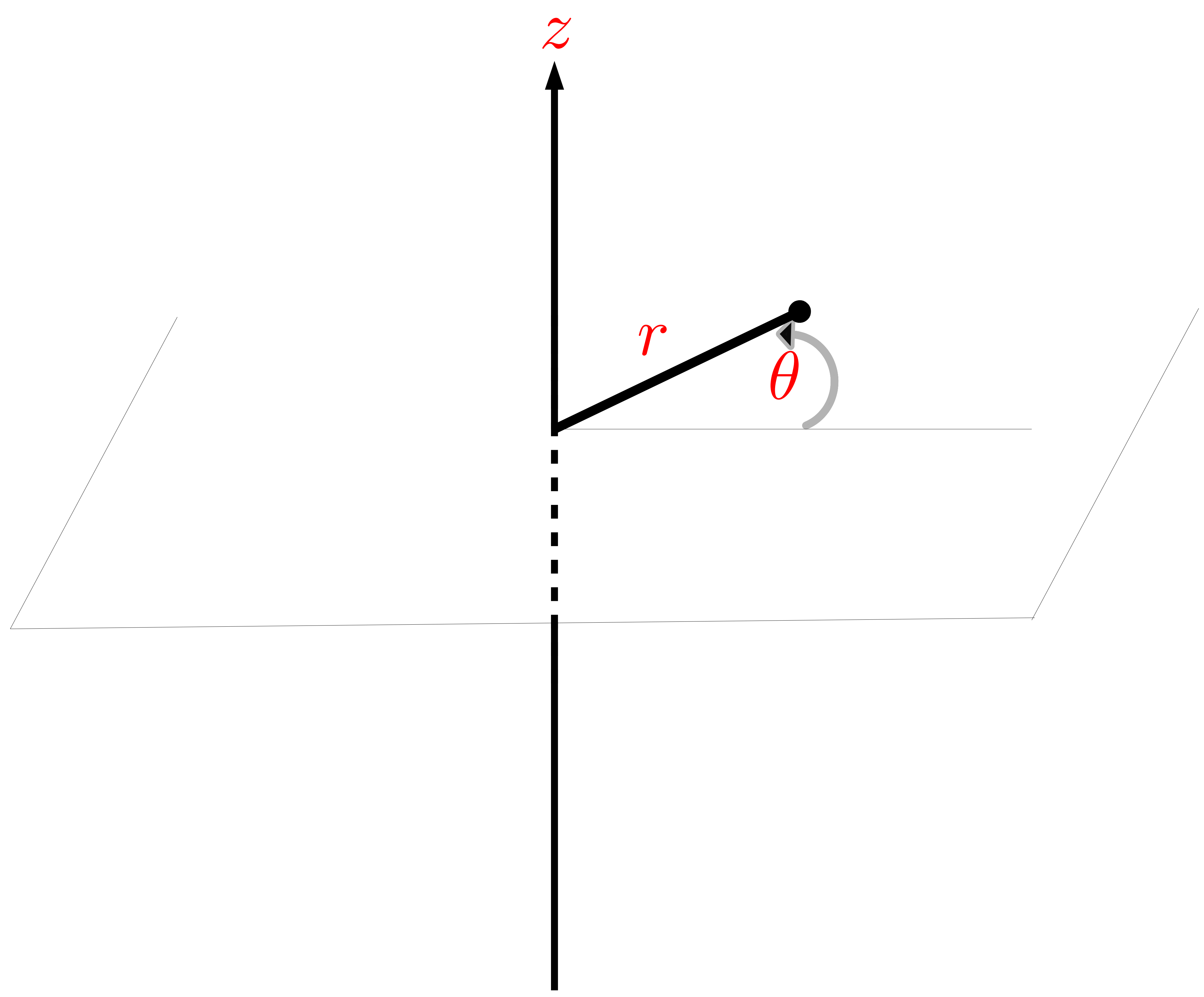
Cylindrical coordinates (t, r, θ, z)



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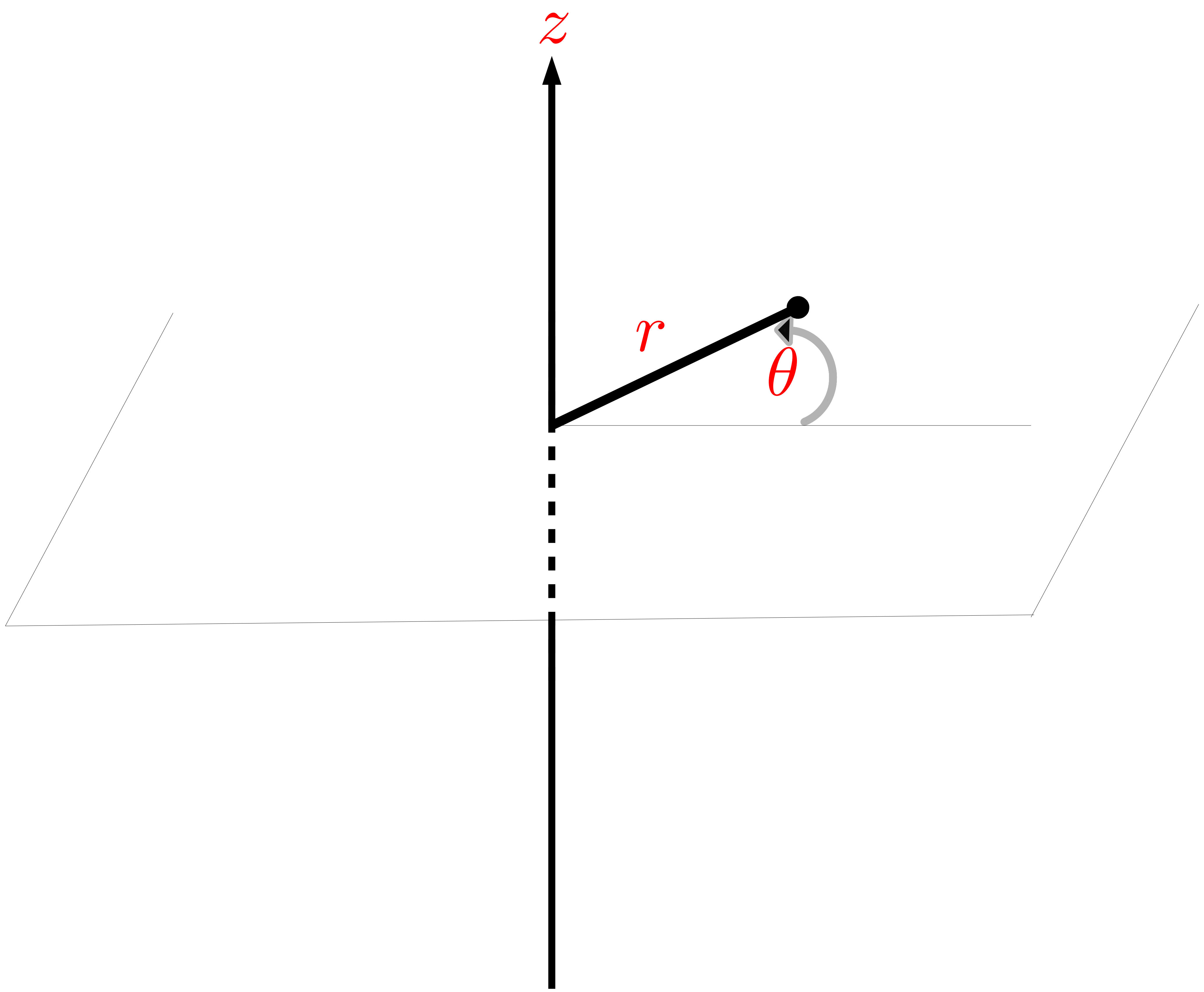

Ansatz

$$B_\mu \mathrm{d}x^\mu = \frac{1}{e_\phi}\left[n - P(r)\right]\mathrm{d}\theta$$

$$\Phi(r,\theta) = \eta\phi(r)\mathrm{e}^{\mathrm{i}n\theta}$$

$$A_\mu \mathrm{d}x^\mu = A_z(r)\mathrm{d}z + A_t(r)\mathrm{d}t$$

$$\Sigma(t,r,z) = \eta\sigma(r)\mathrm{e}^{\mathrm{i}(\omega t - kz)}$$

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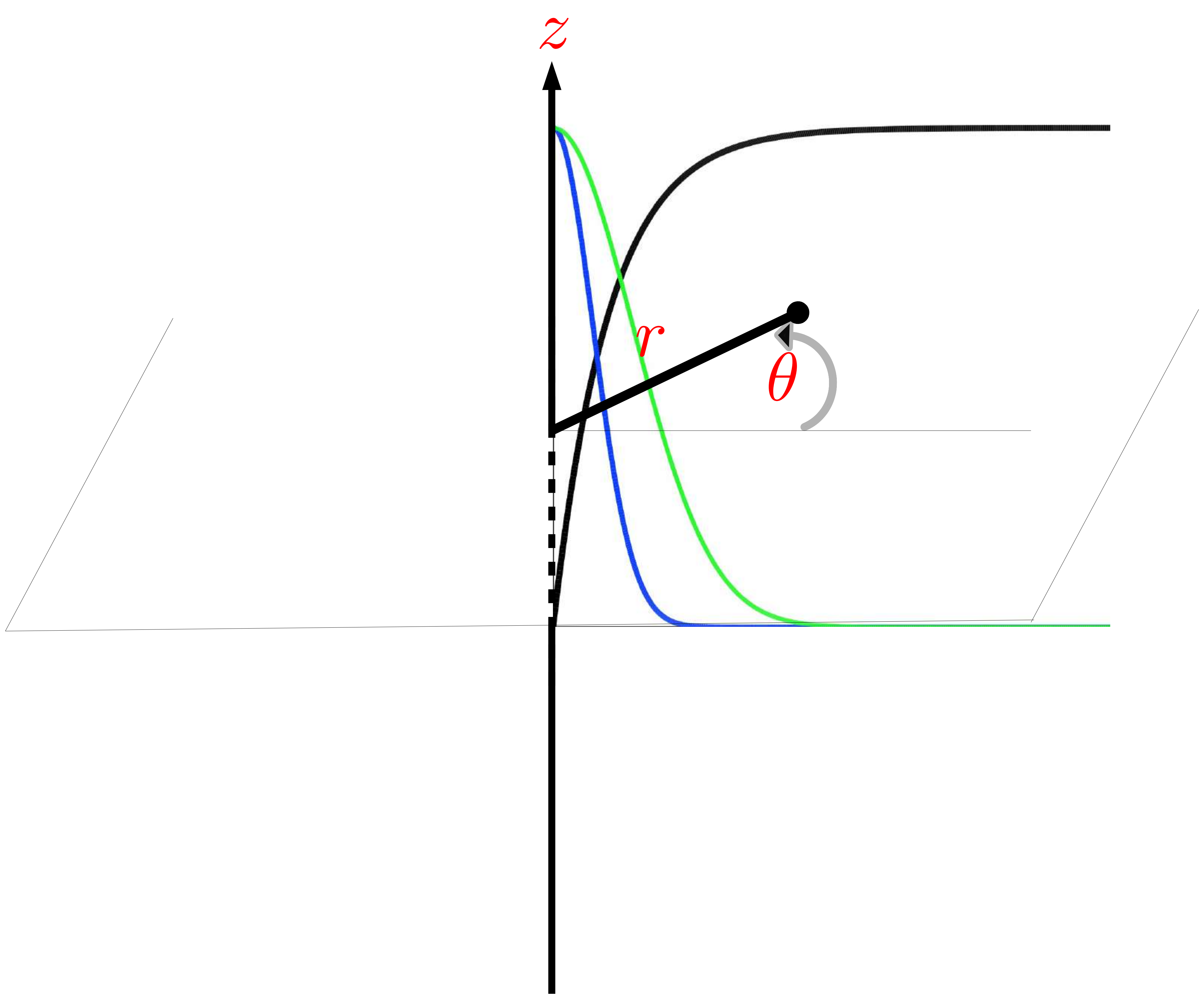

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Cylindrical coordinates (t, r, θ, z)

Background and condensate

$$\Phi(r,\theta) = \eta\phi(r)\mathrm{e}^{\mathrm{i}n\theta}$$

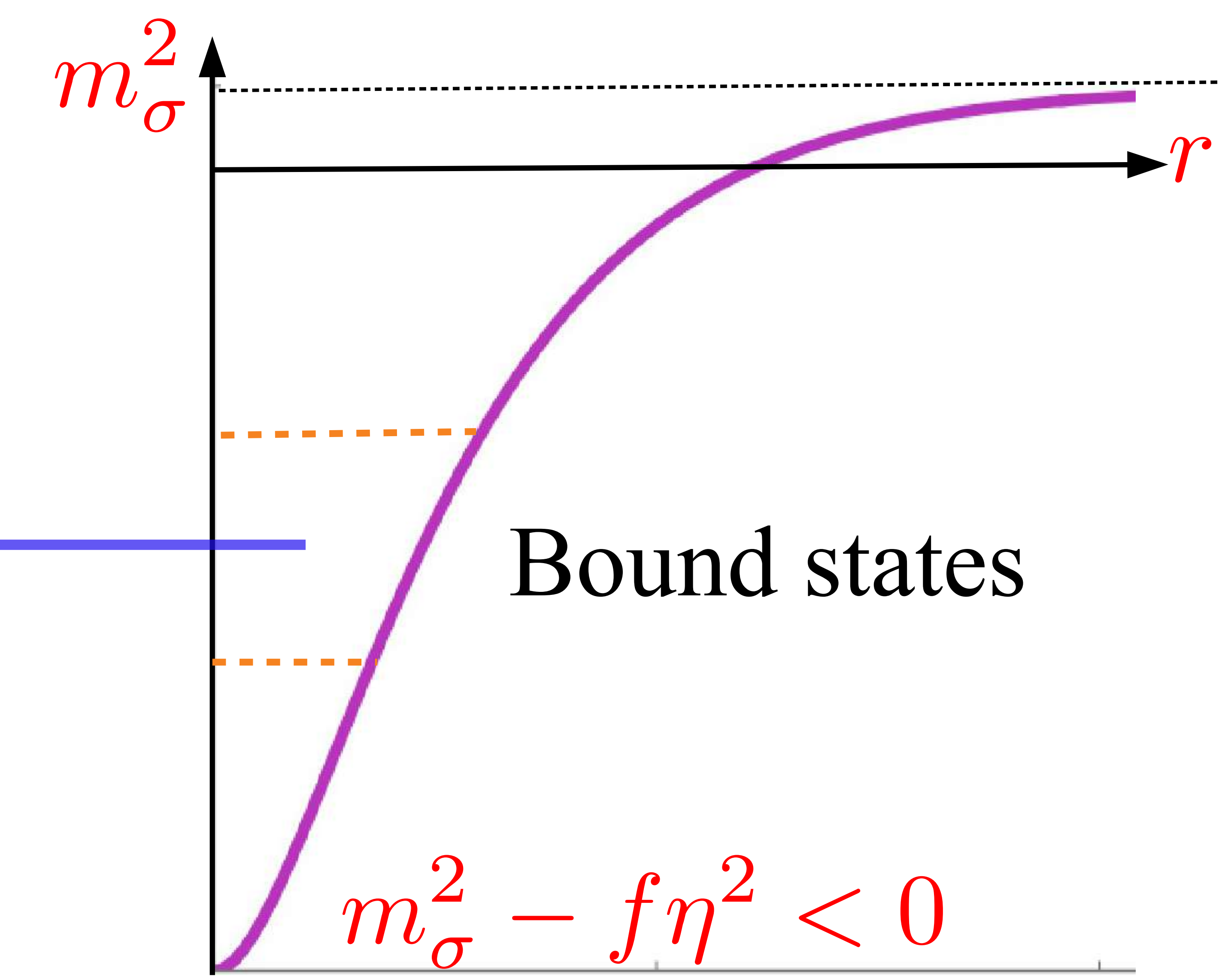
$$\Sigma(t,r,z) = \eta\sigma(r)\mathrm{e}^{\mathrm{i}Et}$$

$\Longrightarrow$

2D Schrödinger equation

$$\left[-\Delta + V(r)\right]\sigma = E^2\sigma$$

$$V(r) = f\left[\phi^2(r) - \eta^2\right] + m_\sigma^2$$



Negative energy solutions
$\Longrightarrow$ instabilities

E. Witten, *Nucl. Phys.* **B249**, 557 (1985)

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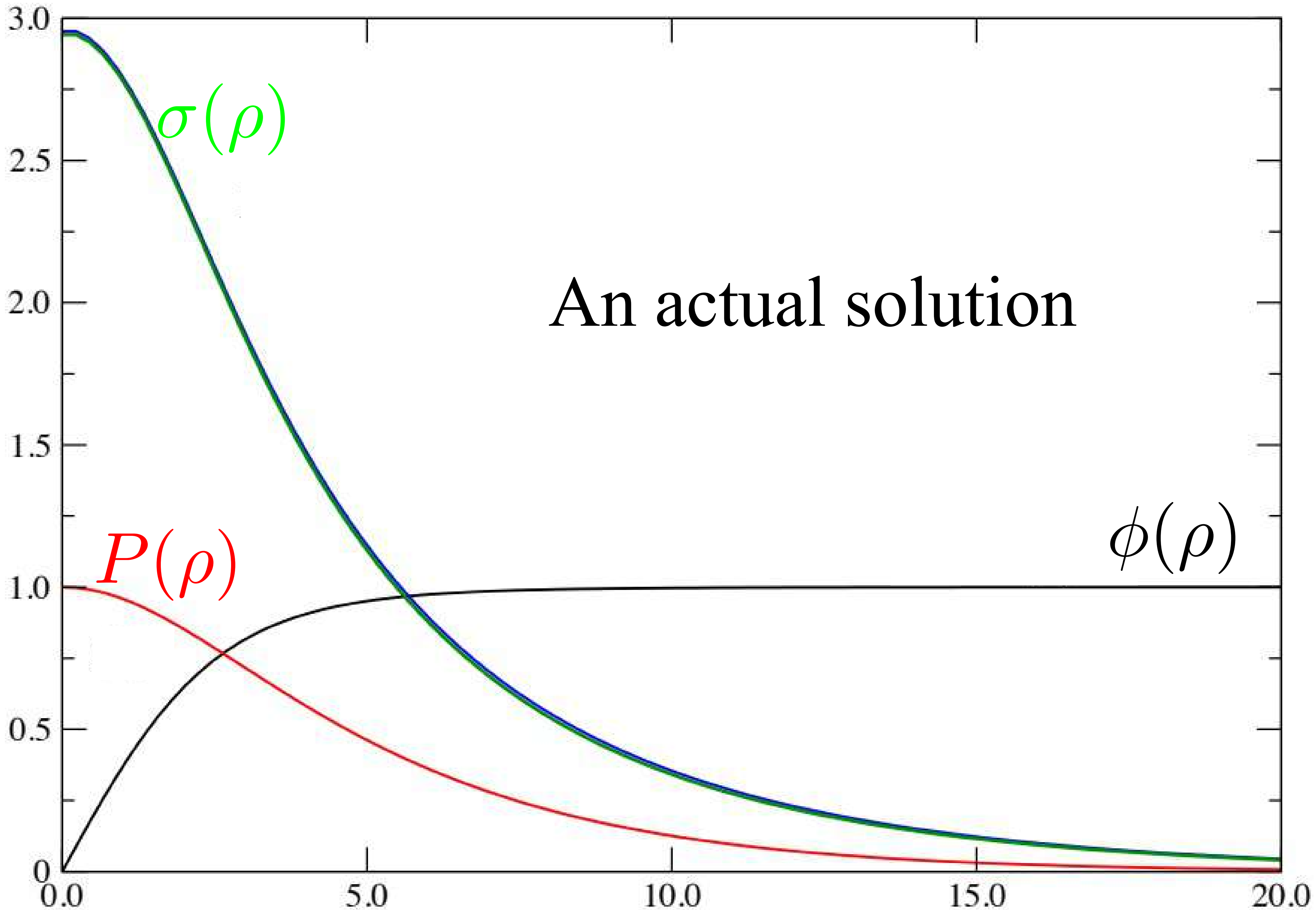

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PP, *Phys. Rev.* **D45**, 1091 (1992)

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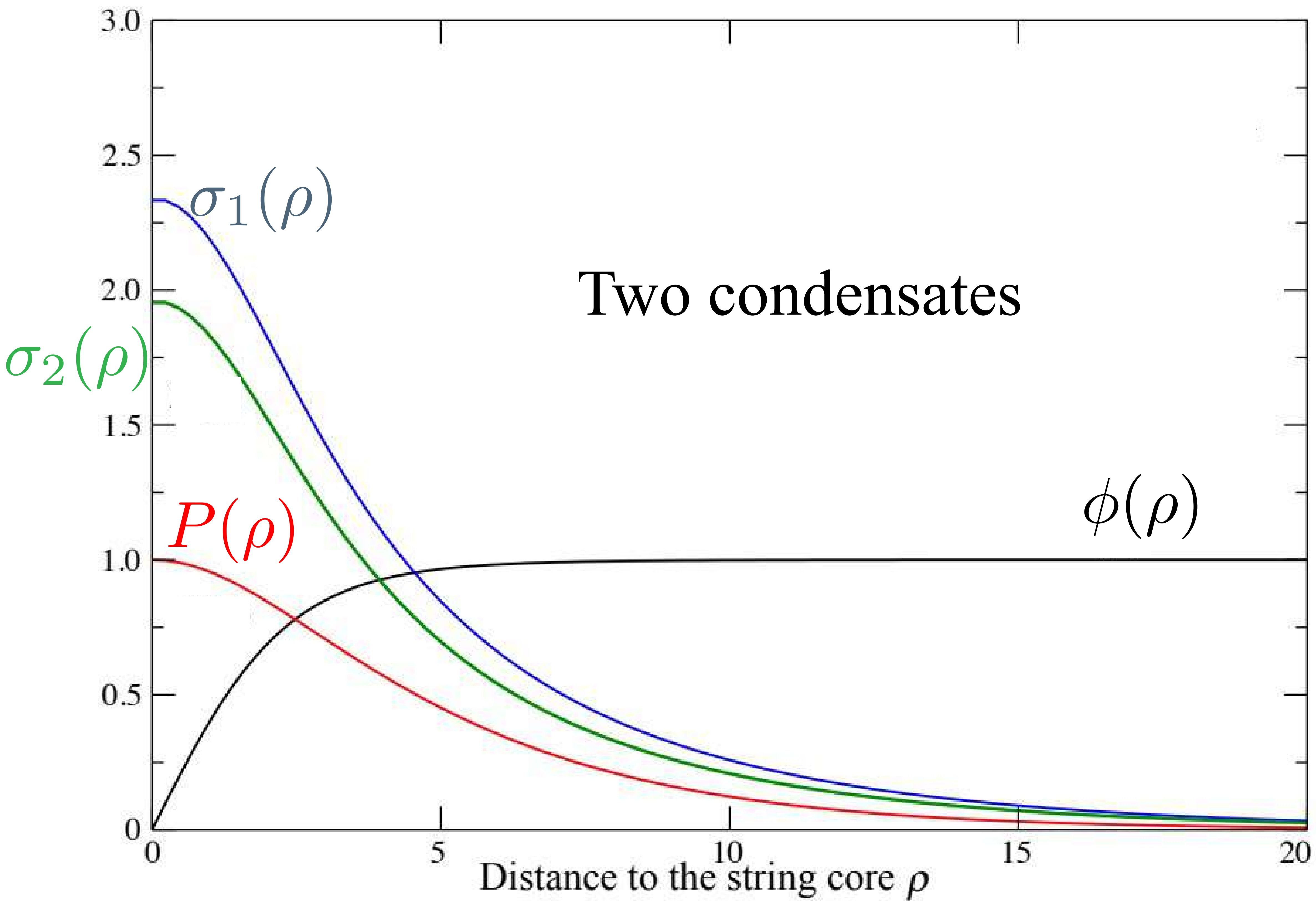
Cylindrical coordinates (t, r, θ, z)



M. Lilley, X. Martin & PP, *Phys. Rev.* **D79**, 103514 (2009)

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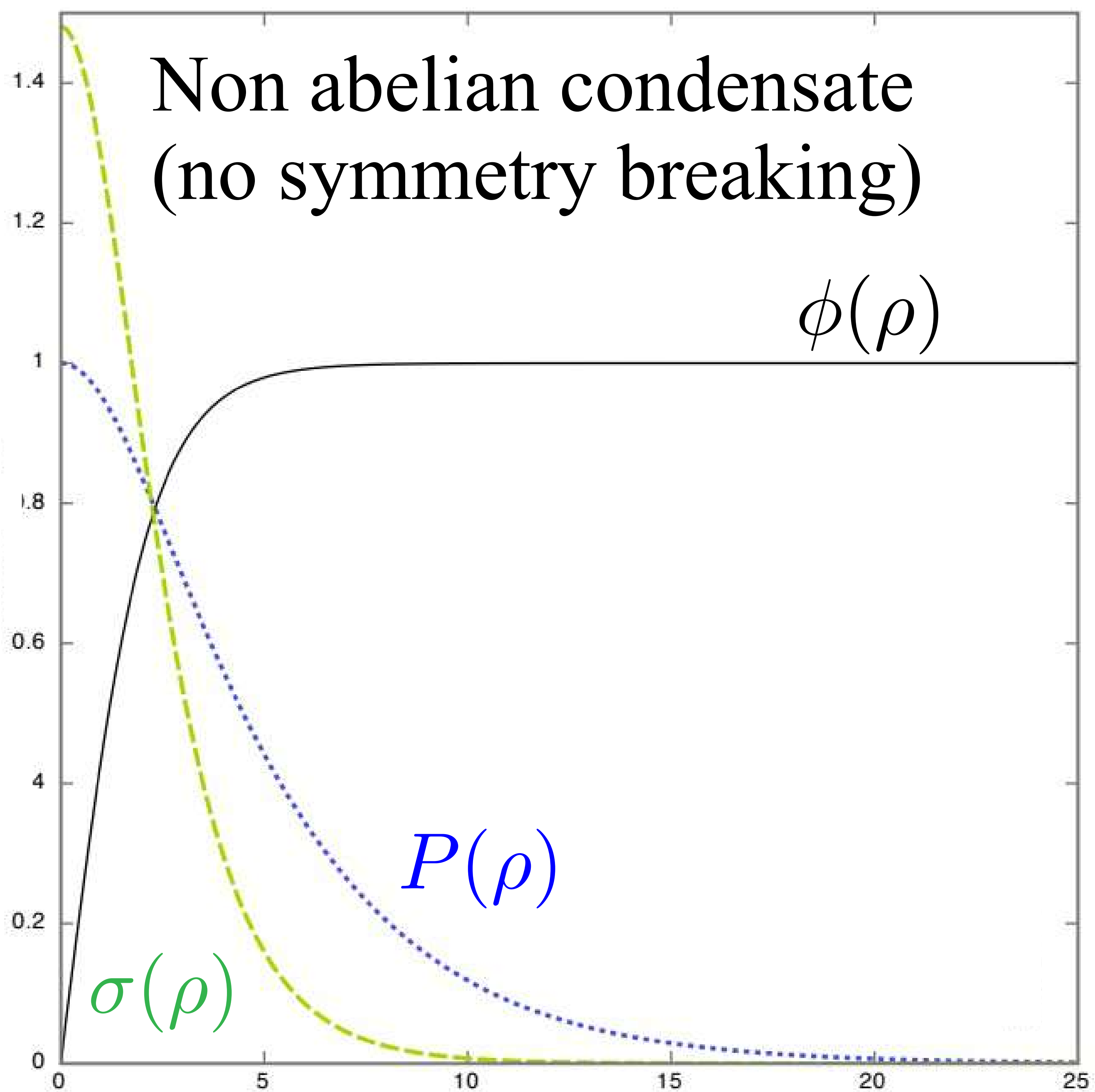

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M. Lilley, F. Di Marco, J. Martin & PP, *Phys. Rev.* **D82**, 023510 (2010)

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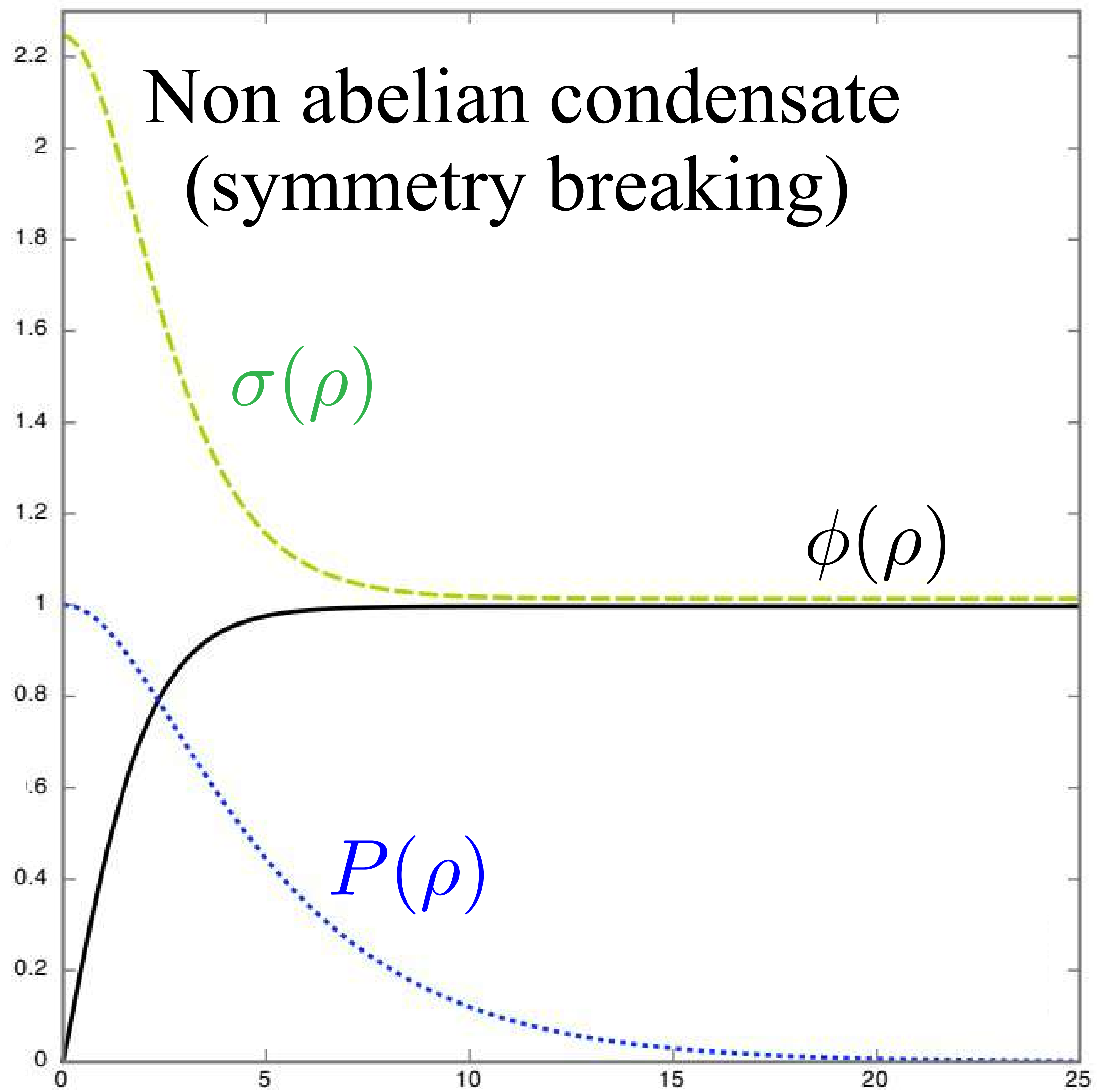

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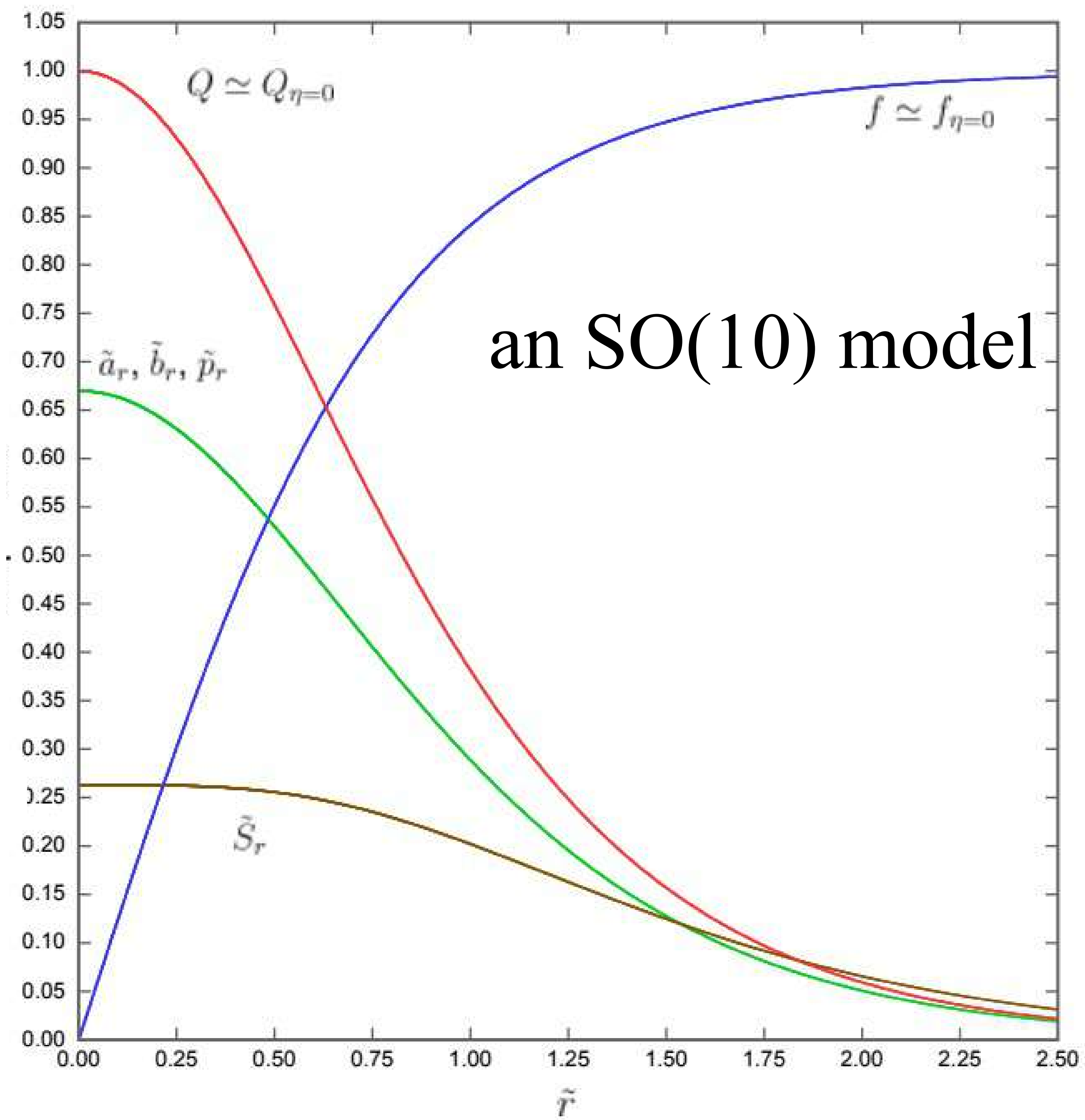

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A.-C. Davis & S. C. Davis, *Phys. Rev.* **D55**, 1879 (1997)
E. Allys, *Phys. Rev.* **D93**, 105021 (2016)

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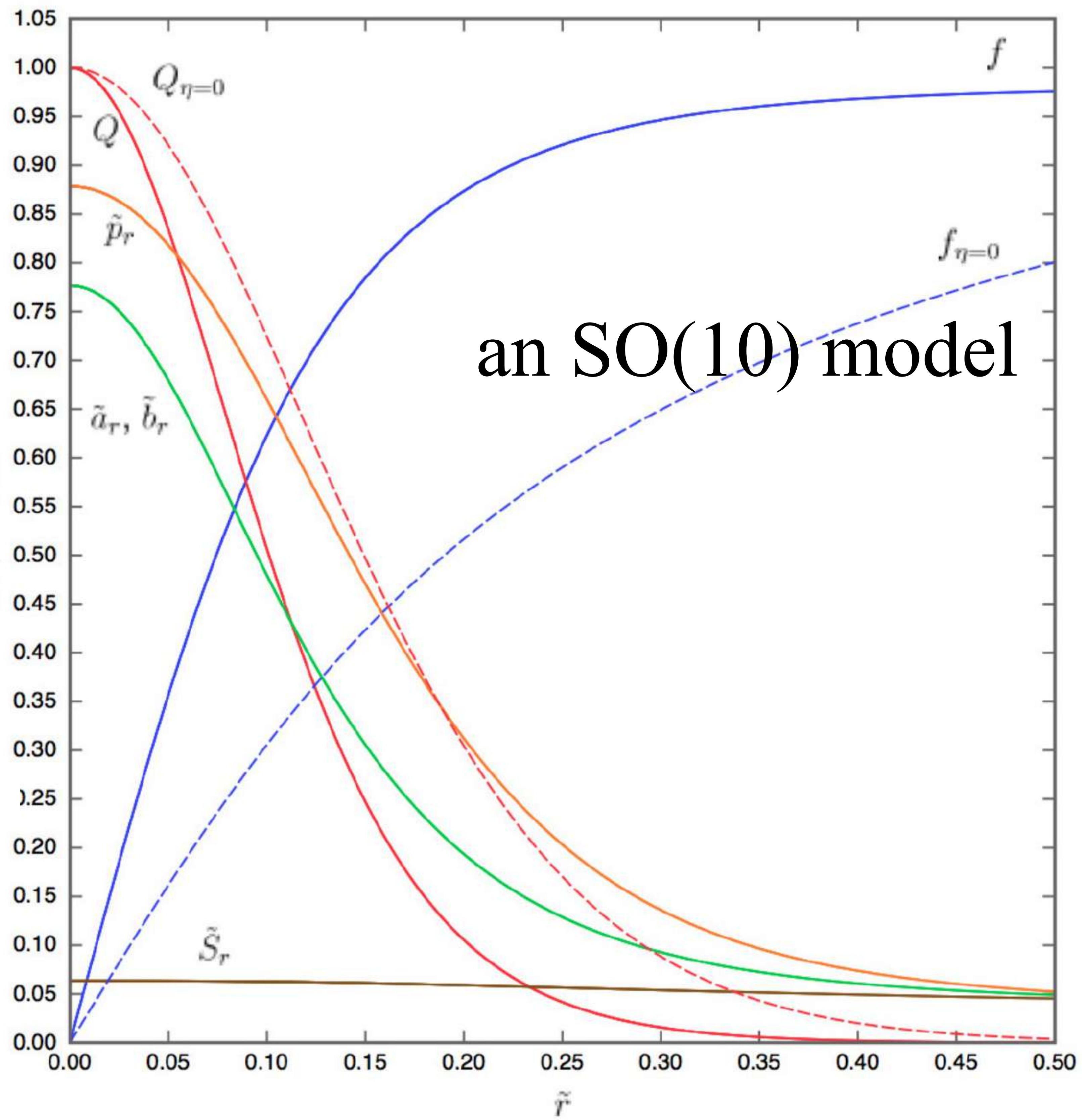

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Neutral limit: (avoid complications due to long-range interaction...) PP, *Phys. Rev.* **D46**, 3335 (1992) *Phys. Rev.* **D47**, 3169 (1993)

$e_\sigma \to 0$

$A_\mu \to 0$

$$\Sigma(x^\alpha) = \sigma(x^\perp)\mathrm{e}^{\mathrm{i}\psi(\xi_a)}$$

Conserved current $\mathcal{J}_\mu = \sigma^2(r)\partial_\mu\psi$

$$\omega t_\mu - k z_\mu = \omega \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - k \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Integrated (macroscopic) current $c_\mu \equiv \int \mathrm{d}^2 x^\perp \mathcal{J}_\mu = 2\pi\partial_\mu\psi \int \sigma^2(r) r\mathrm{d}r$

Scalar current $\mu = \sqrt{|c_\mu c^\mu|}$

Scalar state parameter $\kappa = g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi \to k^2 - \omega^2$

$$\mu = 2\pi\sqrt{|\kappa|}\int \sigma^2(r) r\mathrm{d}r$$

ν → k, $-\omega$

Stress energy tensor $T^{\mu\nu} = -2\frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} + g^{\mu\nu}\mathcal{L}$

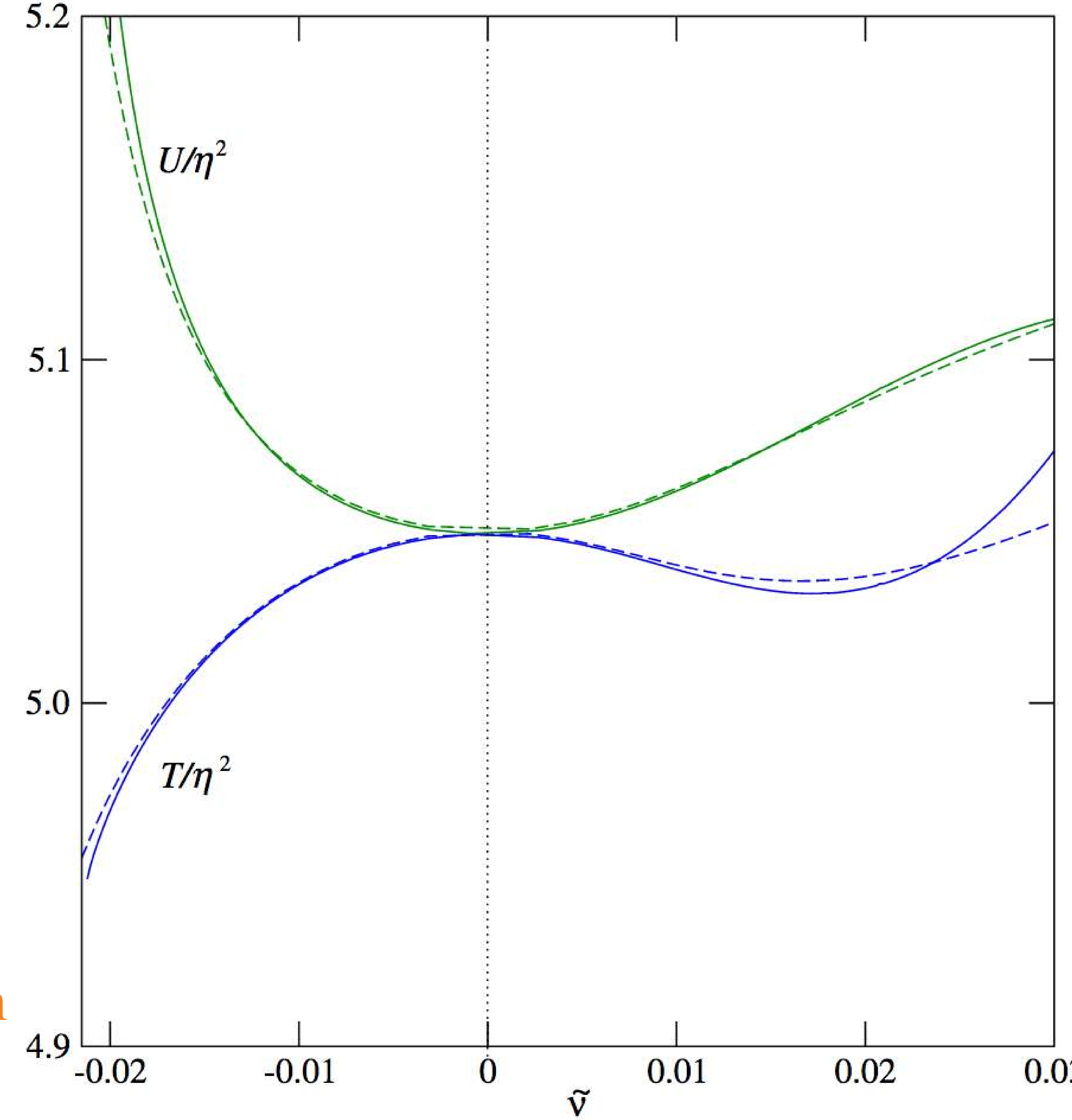
Integrated for macroscopic treatment

$$\mathcal{T}^{\mu\nu} \equiv \int T^{\mu\nu} \mathrm{d}^2 x^{\perp}$$
$$= U u^\mu u^\nu - T v^\mu v^\nu$$
$$= (U - T) u^\mu u^\nu - T \eta^{\mu\nu}$$

$U(\nu)$ $T(\nu)$

$$U - T = \nu\mu$$

Legendre transform $\Longrightarrow$ dual formalism



$$U = T + \mu\nu$$

Legendre transform $\Longrightarrow$ dual formalism

B. Carter & PP, *Phys. Rev.* **D52**, R1744 (1995)
B. Carter, PP & A. Gangui, *Phys. Rev.* **D55**, 4647 (1997)

Macroscopic formalism

State parameter $\kappa \Longrightarrow$ worldsheet lagrangian $\mathcal{L}(\kappa)$ and $\kappa = \kappa_0 \gamma^{ab} \partial_a \varphi \partial_b \varphi$ → $\nu^2 = |\kappa|$

$$\mathcal{S}_\mathcal{L} = -m^2 \int \mathrm{d}^2\xi \sqrt{-\gamma} \mathcal{L}(\kappa)$$

Master function (dual to lagrangian) $\Lambda(\chi)$: set $\chi = \tilde{\kappa}_0 \gamma^{ab} \partial_a \psi \partial_b \psi$ → $\mu^2 = |\chi|$

→ 2 conserved (orthogonal:$\gamma_{ab} n^a z^a = 0$) currents

$$z^a = -\frac{\partial \mathcal{L}}{\partial \partial_a \varphi} \qquad n^a = -\frac{\partial \Lambda}{\partial \partial_a \psi}$$

Equivalent alternative dynamical description

$$\mathcal{S}_\mathcal{L} \Longleftrightarrow \mathcal{S}_\Lambda = -m^2 \int \mathrm{d}^2\xi \sqrt{-\gamma} \Lambda(\chi)$$

Current amplitudes $\mathcal{K} z^a = \kappa_0 \partial^a \varphi \quad \Longleftrightarrow \quad \tilde{\mathcal{K}} n^a = \tilde{\kappa}_0 \partial^a \psi$

$$\mathcal{K}^{-1} = -2 \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\kappa} \quad \Longleftrightarrow \quad \tilde{\mathcal{K}}^{-1} = -2 \frac{\mathrm{d}\Lambda}{\mathrm{d}\chi}$$

Equivalent formulations

$$\tilde{\mathcal{K}} = -\mathcal{K}^{-1} \quad \Longrightarrow \quad \kappa = \mathcal{K}^2 \chi$$

Legendre transformation $\Lambda = \mathcal{L} + \mathcal{K}\chi$

$$U = T + \mu\nu$$

$$\mu = \frac{\mathrm{d}U}{\mathrm{d}\nu}$$

$$\nu = -\frac{\mathrm{d}T}{\mathrm{d}\mu}$$

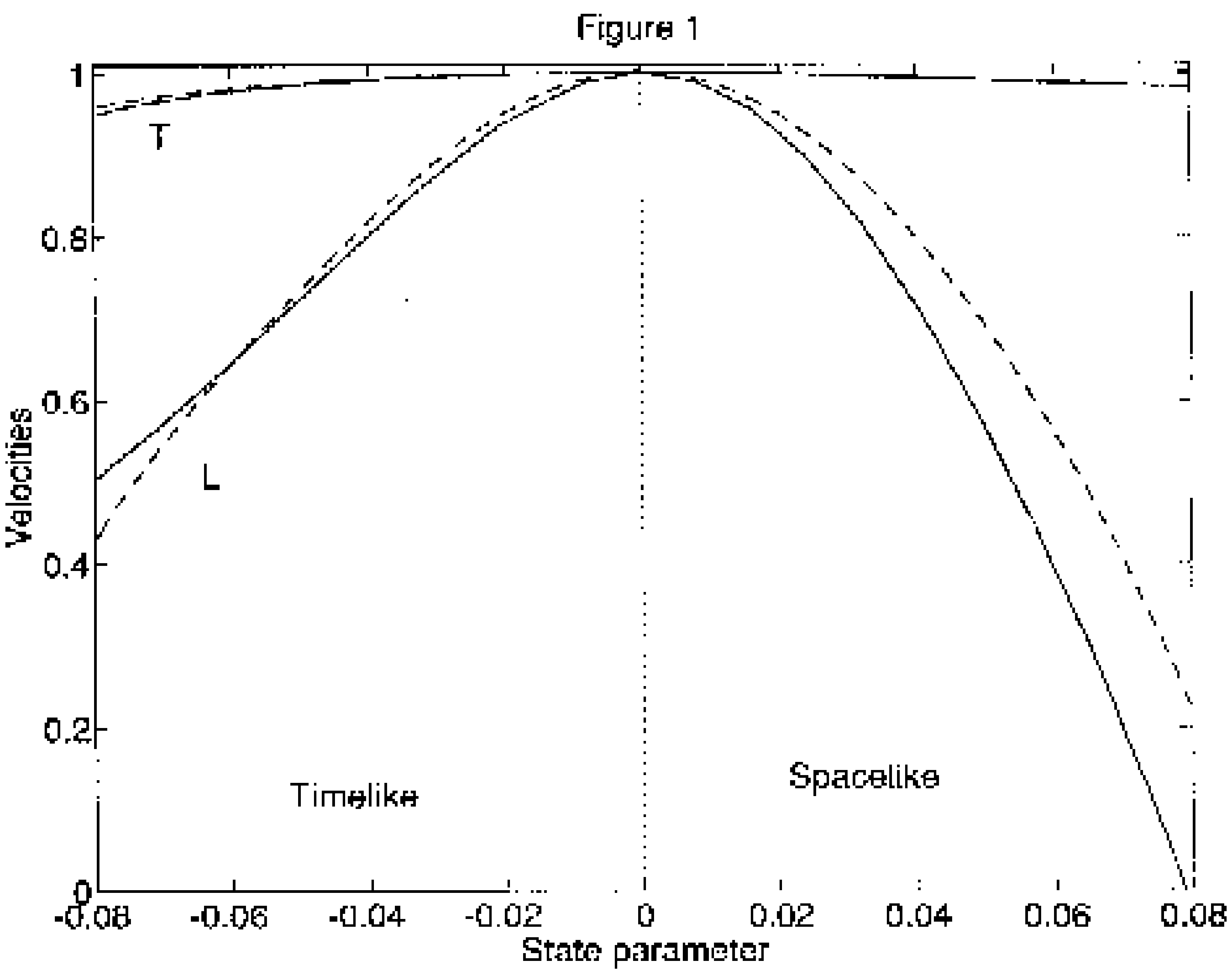
Identification for the macroscopic description

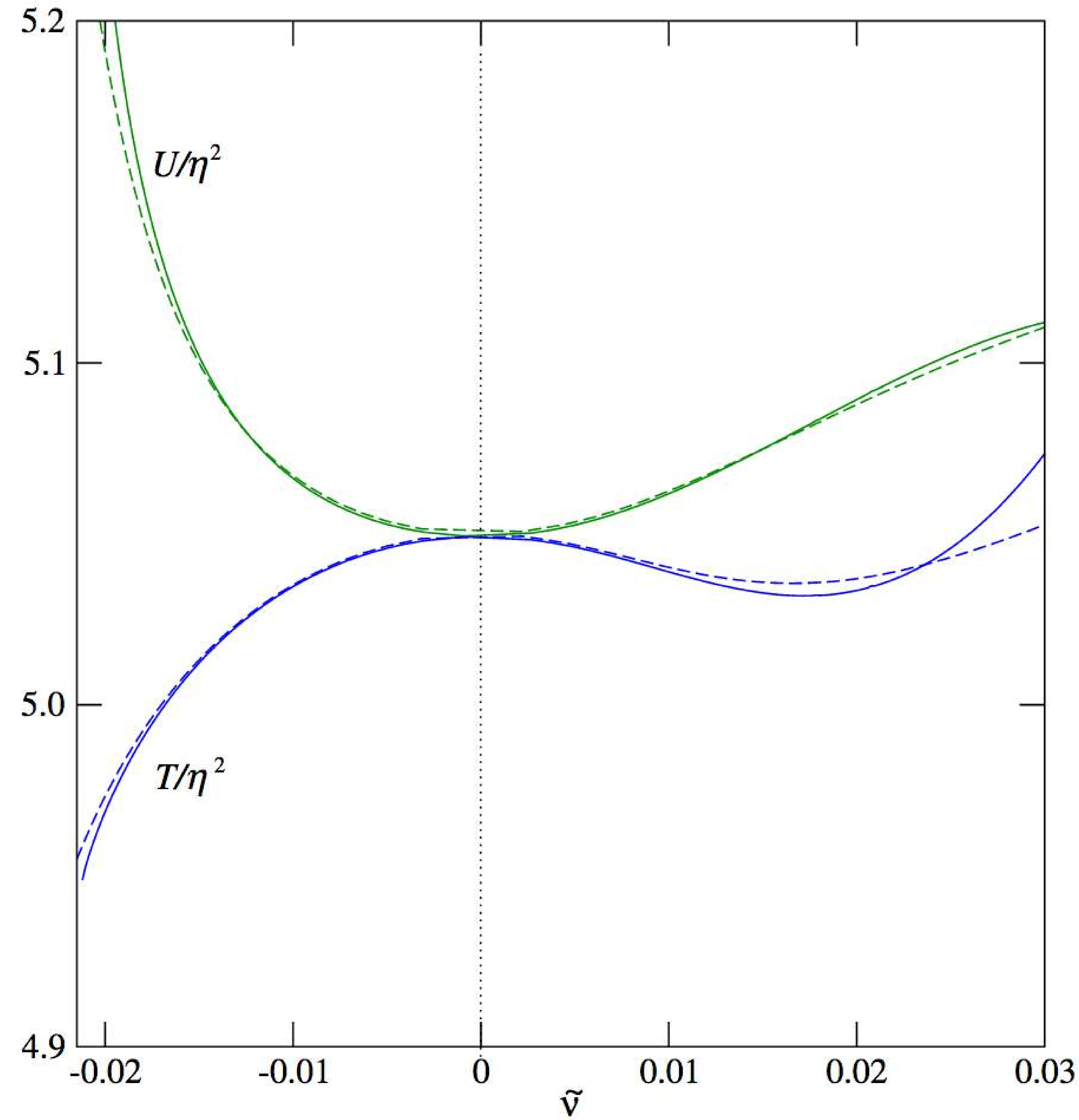
Regime	U	T	κ and χ	current
Electric	$-\Lambda$	$-\mathcal{L}$	≤ 0	timelike
Magnetic	$-\mathcal{L}$	$-\Lambda$	≥ 0	spacelike

Macroscopic perturbations (stability constraint)

$$c_{\mathrm{T}}^2 = \frac{T}{U} > 0 \qquad c_{\mathrm{L}}^2 = -\frac{\mathrm{d}T}{\mathrm{d}U} = \frac{\nu}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}\nu} > 0$$

(no spring)





Specific models

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Subsonic equation of state $U + T = 2m^2 \quad \Longrightarrow \quad c_{\mathrm{T}} < c_{\mathrm{L}} = 1$

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- Kaluza-Klein $\mathcal{L} = -m\sqrt{m^2 - \kappa} \quad \Longrightarrow \quad \Lambda = -m\sqrt{m^2 - \chi}$ self-dual

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- Small field expansion (higher order) $\mathcal{L} = -m^2 + \frac{\kappa}{2} + \frac{\kappa^2}{4m_*^2} + \mathcal{O}\left(\frac{\kappa^3}{m_*^4}\right)$

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- Witten magnetic model $\mathcal{L} = -m^2 + \frac{\kappa}{2}\left(1 - \frac{\kappa}{m_*^2}\right)^{-1}$

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  Subsonic equation of state $U + T = 2m^2 \implies c_{\rm T} < c_{\rm L} = 1$

- Kaluza-Klein $\mathcal{L} = -m\sqrt{m^2 - \kappa} \implies \Lambda = -m\sqrt{m^2 - \chi}$ self-dual

  Transonic equation of state $UT = m^4 \implies c_{\rm T} = c_{\rm L} \leq 1$

- Small field expansion (higher order) $\mathcal{L} = -m^2 + \frac{\kappa}{2} + \frac{\kappa^2}{4m_*^2} + \mathcal{O}(\kappa^3)$
- Witten magnetic model $\mathcal{L} = -m^2 + \frac{\kappa}{2}\left(1 - \frac{\kappa}{m_*^2}\right)^{-1}$
- Witten electric model $\mathcal{L} = -m^2 - \frac{m_*^2}{2}\ln\left(1 - \frac{\kappa}{m_*^2}\right)$

Specific models

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Supersonic equation of state $c_{\mathrm{L}} \leq c_{\mathrm{T}} \leq 1$ (Witten magnetic and electric models)

Specific models

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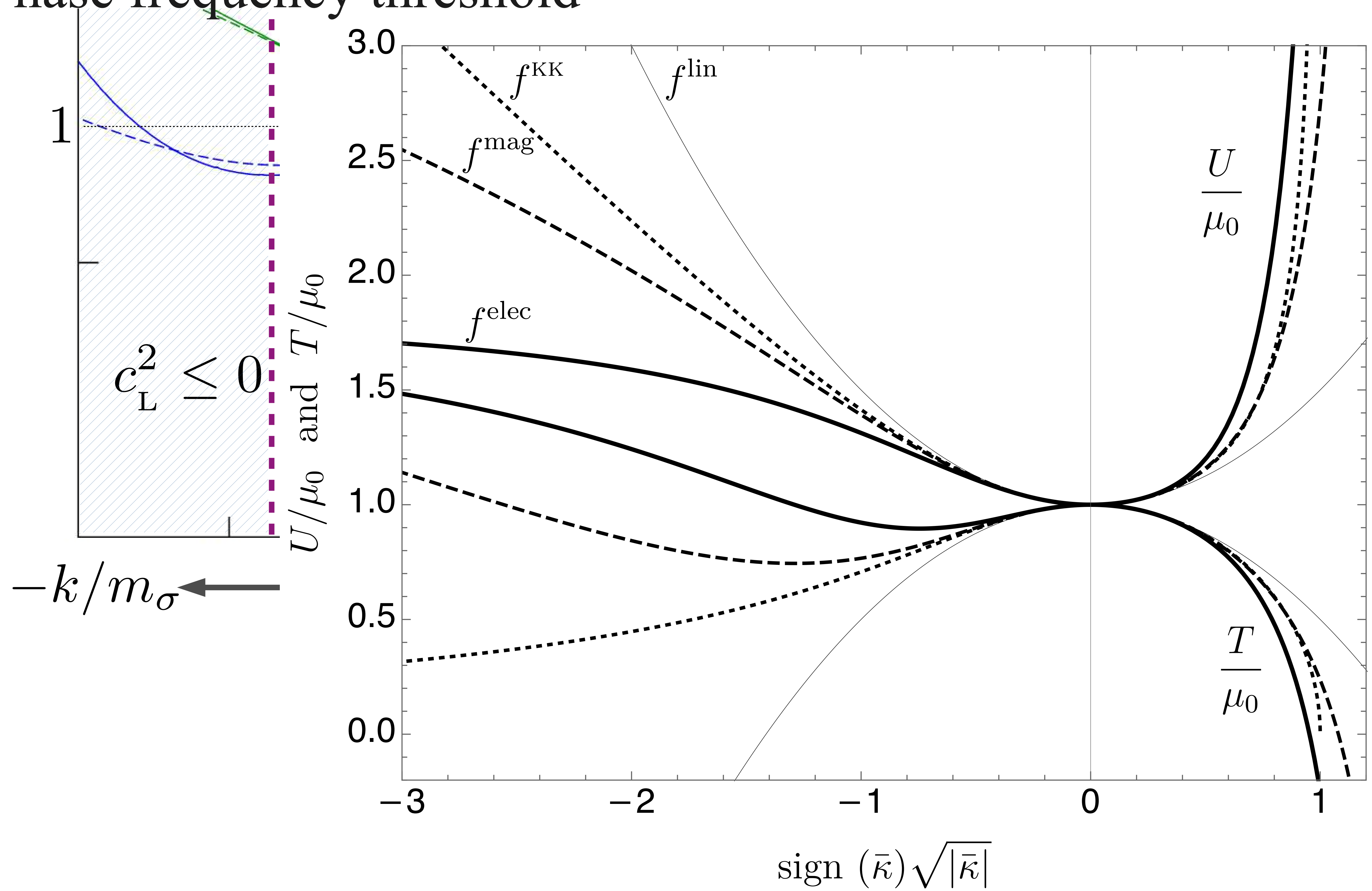
} Supersonic equation of state $c_{\text{L}} \leq c_{\text{T}} \leq 1$

Phase frequency threshold $\lim_{\nu \to -m_*} \mu = \infty$

Phase frequency threshold



- Witten magnetic model $\mathcal{L} = -m^2 + \frac{\kappa}{2}\left(1 - \frac{\kappa}{m_*^2}\right)^{-1}$
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Dynamics of current-carrying cosmic strings in expanding universes

Effective action $S = \int \mathcal{L}(\kappa)\sqrt{-\gamma}\,\mathrm{d}^2\sigma = -\mu_0 \int f(\kappa)\sqrt{-\gamma}\,\mathrm{d}\sigma^0\mathrm{d}\sigma^1$

Induced metric $\gamma_{ab} \equiv g_{\mu\nu}\frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b} = g_{\mu\nu}X^\mu_{,a}X^\nu_{,b}$

Worldsheet $X^\mu(\sigma^a) = \left\{X^0(\sigma^0,\sigma^1), \boldsymbol{X}(\sigma^0,\sigma^1)\right\}$ Gauge choice 1: $X^0 = \tau$

Background metric $\mathrm{d}s^2_{\mathrm{FLRW}} = a^2(\tau)\left(\mathrm{d}\tau^2 - \mathrm{d}\boldsymbol{x}^2\right) = \mathrm{d}t^2 - a^2(t)\mathrm{d}\boldsymbol{x}^2$

Gauge choice 2: $\frac{\partial \boldsymbol{X}}{\partial \tau}\cdot\frac{\partial \boldsymbol{X}}{\partial \sigma} \equiv \dot{\boldsymbol{X}}\cdot\boldsymbol{X}' = 0 \implies \gamma_{ab} = \mathrm{diag}\left[a^2\left(1-\dot{\boldsymbol{X}}^2\right), -a^2\boldsymbol{X}'^2\right]$

$\implies$ State parameter $\kappa = \frac{\dot{\varphi}^2}{a^2\left(1-\dot{\boldsymbol{X}}^2\right)} - \frac{\varphi'^2}{a^2\boldsymbol{X}'^2} \equiv q^2 - j^2$

Equations of motion

$$\partial_\tau\left(\epsilon\bar{U}\right)+\frac{\dot{a}}{a}\epsilon\left[\left(\bar{U}+\bar{T}\right)\dot{\boldsymbol{X}}^2+\bar{U}-\bar{T}\right]=\partial_\sigma\Phi$$

$$\ddot{\boldsymbol{X}}\epsilon\bar{U}+\frac{\dot{a}}{a}\epsilon\left(\bar{U}+\bar{T}\right)\left(1-\dot{\boldsymbol{X}}^2\right)\dot{\boldsymbol{X}}=\partial_\sigma\left(\frac{\bar{T}}{\epsilon}\boldsymbol{X}'\right)+2\Phi\dot{\boldsymbol{X}}'+\boldsymbol{X}'\left(\dot{\Phi}+2\frac{\dot{a}}{a}\Phi\right)$$

$$\partial_\tau\left(f_\kappa a\sqrt{q^2\boldsymbol{X}'^2}\right)=\partial_\sigma\left[f_\kappa a\sqrt{j^2(1-\dot{\boldsymbol{X}}^2)}\right]$$

$$\boxed{\epsilon^2=\frac{\boldsymbol{X}'^2}{1-\dot{\boldsymbol{X}}^2}}$$

Averaging process

$$E = a\mu_0 \int \bar{U} \epsilon \, \mathrm{d}\sigma$$ Energy

$$E_0 = a\mu_0 \int \epsilon \, \mathrm{d}\sigma$$ Bare energy

$$\langle \mathcal{O} \rangle \equiv \frac{\displaystyle\int \mathcal{O} \epsilon \, \mathrm{d}\sigma}{\displaystyle\int \epsilon \, \mathrm{d}\sigma}$$ General variable

Integrated state parameter

$$K = Q^2 - J^2$$

Total charge $Q^2 \equiv \langle q^2 \rangle$ and current $J^2 \equiv \langle j^2 \rangle$

RMS velocity $v \equiv \sqrt{\langle \dot{\boldsymbol{X}}^2 \rangle}$

General assumptions / hypothesis

Uncorrelated variables
$$\langle \mathcal{F}(\mathcal{O}) \rangle \approx \mathcal{F}(\langle \mathcal{O} \rangle)$$

Vanishing boundary terms
$$\int \partial_\sigma \{\mathcal{F}[\boldsymbol{X}(\sigma,\tau)]\} \, \mathrm{d}\sigma \;\to\; \oint \partial_\sigma \{\mathcal{F}[\boldsymbol{X}(\sigma,\tau)]\} \, \mathrm{d}\sigma \approx 0$$

+ Brownian string network $E = \dfrac{\mu_0 V}{L_\mathrm{c}^2 a^2} \iff E_0 = \dfrac{\mu_0 V}{\xi_\mathrm{c}^2 a^2}$

$$E = E_0 \langle f - 2q^2 f_\kappa \rangle \implies \frac{E}{E_0} = F - 2Q^2 F' \qquad F(K) \equiv \langle f(\kappa) \rangle \quad F' \equiv \langle f_\kappa \rangle \quad F'' \equiv \langle f_{\kappa\kappa} \rangle$$

Equations of motion

$$\frac{\mathrm{d}L_\mathrm{c}}{\mathrm{d}\tau} = \frac{\dot{a}}{a}\frac{L_\mathrm{c}}{F-2Q^2F'}\left\{v^2\left[F-\left(Q^2-J^2\right)F'\right]-\left(Q^2+J^2\right)F'\right\}$$

$$\frac{\mathrm{d}v}{\mathrm{d}\tau} = \frac{\left(1-v^2\right)}{F-2Q^2F'}\left\{\frac{k(v)}{L_\mathrm{c}\sqrt{F-2Q^2F'}}\left(F+2J^2F'\right)-2v\frac{\dot{a}}{a}\left[F-\left(Q^2-J^2\right)F'\right]\right\}$$

$$\frac{\mathrm{d}J^2}{\mathrm{d}\tau} = 2J^2\left[\frac{vk(v)}{L_\mathrm{c}\sqrt{F-2Q^2F'}}-\frac{\dot{a}}{a}\right]$$

$$\frac{\mathrm{d}Q^2}{\mathrm{d}\tau} = 2Q^2\frac{F'+2J^2F''}{F'+2Q^2F''}\left[\frac{vk(v)}{L_\mathrm{c}\sqrt{F-2Q^2F'}}-\frac{\dot{a}}{a}\right]$$

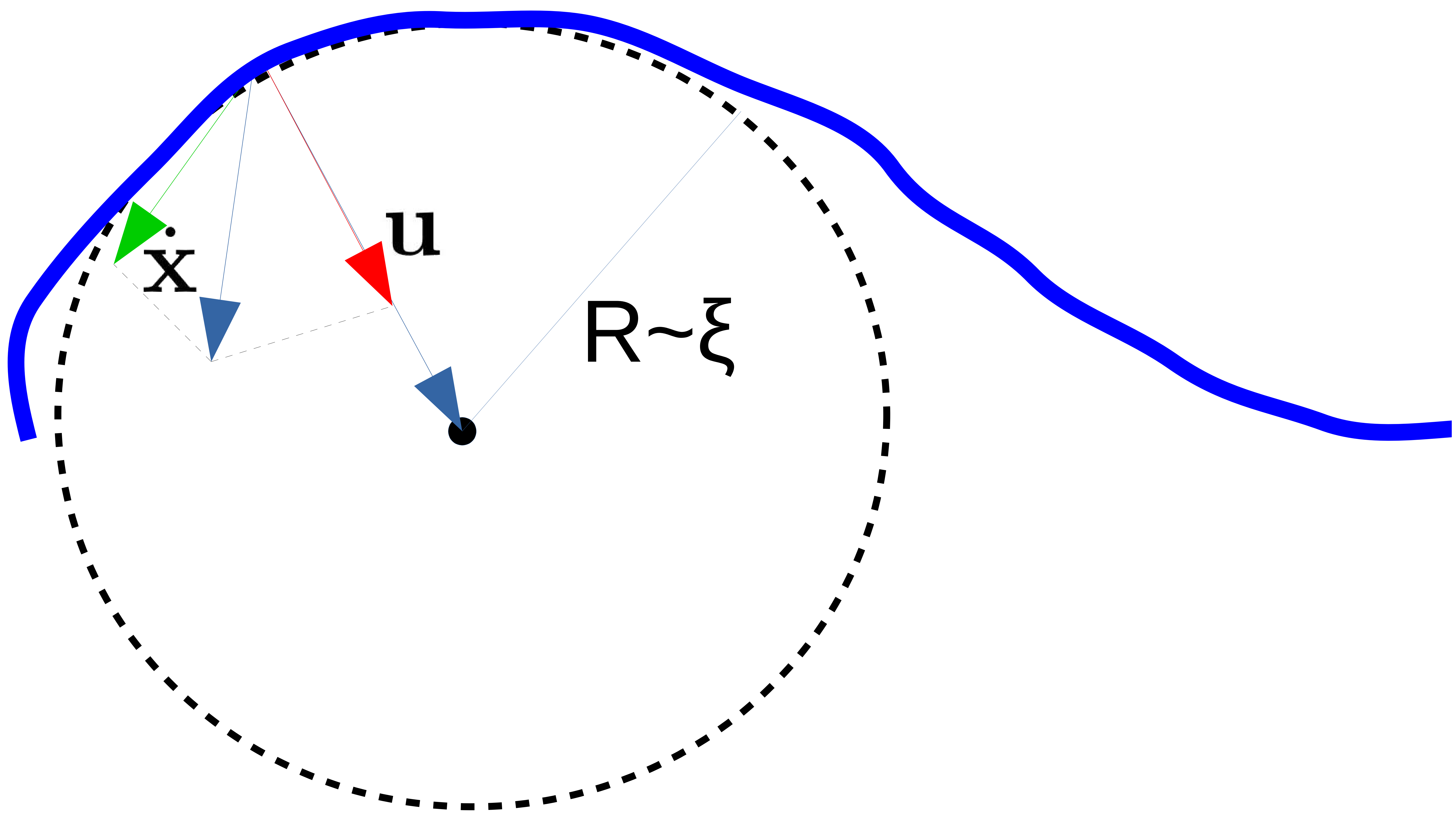
Chirality

$$K = Q^2 - J^2$$

Charge

$$Y = \tfrac{1}{2}(Q^2+J^2)$$

Momentum parameter: $k(v) \equiv \frac{\left\langle (1-\dot{\boldsymbol{X}}^2)(\dot{\boldsymbol{X}} \cdot \boldsymbol{u}) \right\rangle}{v(1-v^2)}$

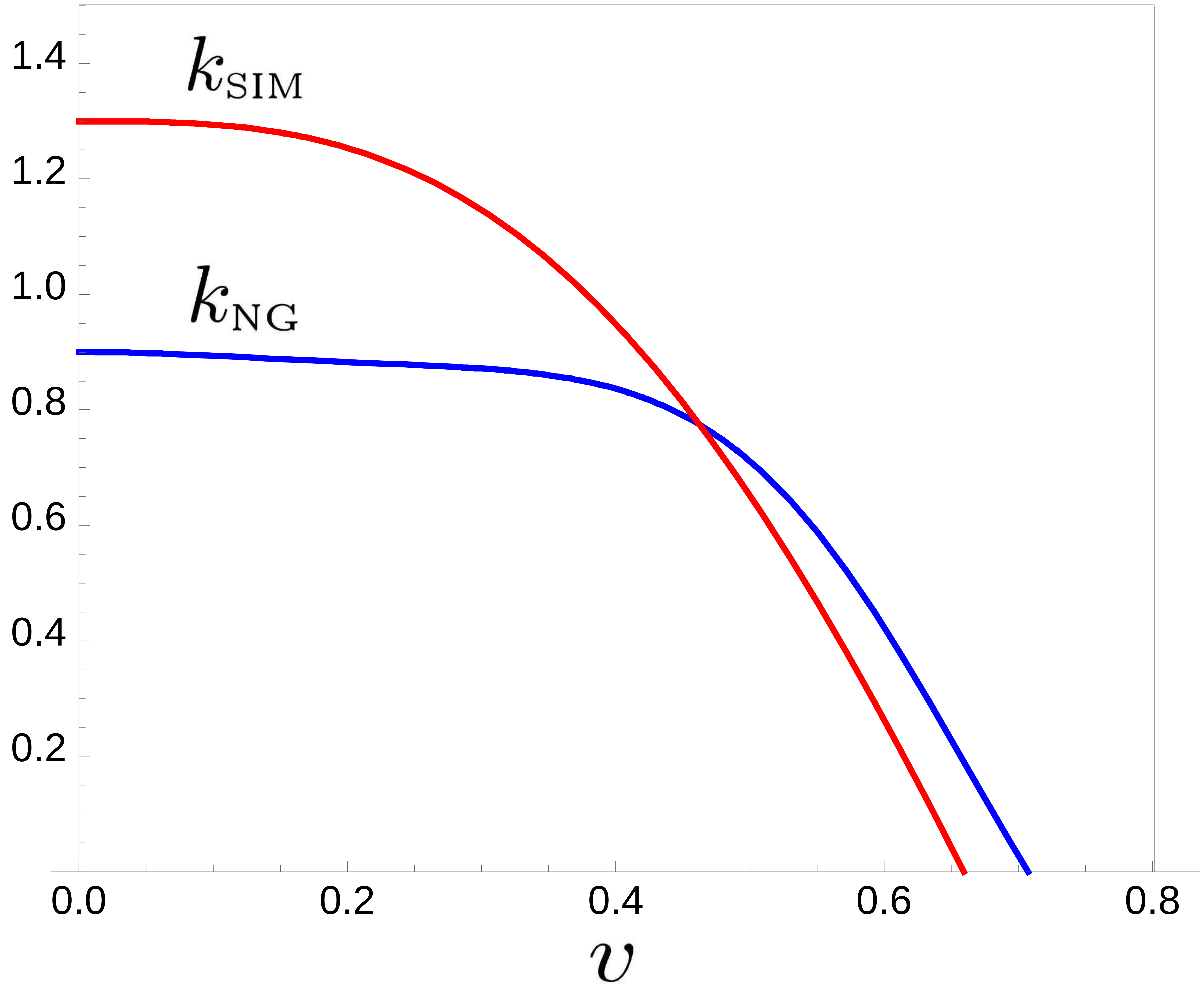


Momentum parameter: $$k(v) \equiv \frac{\left\langle (1-\dot{\boldsymbol{X}}^2)(\dot{\boldsymbol{X}}\cdot\boldsymbol{u}) \right\rangle}{v(1-v^2)}$$

From Nambu-Goto network simulations $$k_{\rm NG}(v) = \frac{2\sqrt{2}}{\pi}\frac{1-8v^6}{1+8v^6}\left(1-v^2\right)\left(1+2\sqrt{2}v^3\right)$$

From Abelian-Higgs simulations $$k_{\rm SIM}(v) = k_0\frac{1-(\alpha v^2)^\beta}{1+(\alpha v^2)^\beta}$$

Momentum parameter: $k(v) \equiv \frac{\left\langle (1 - \dot{\boldsymbol{X}}^2)(\dot{\boldsymbol{X}} \cdot \boldsymbol{u}) \right\rangle}{v(1 - v^2)}$



Phenomenological parameter:

- loop chopping efficiency $$\left.\frac{\mathrm{d}E_0}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -\, c\, v \frac{E_0}{\xi_{\mathrm{c}}}$$

($c \simeq 0.5$ in ordinary "VOS" model)

- current chopping efficiency $$\left.\frac{\mathrm{d}E}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -\, g\, cv \frac{E}{\xi_{\mathrm{c}}}$$

- charge leakage $$\left.\frac{\mathrm{d}Y}{\mathrm{d}\tau}\right|_{\mathrm{leakage}} = -\, A\, \frac{Y}{\xi_{\mathrm{c}}} = -\, A\, \frac{Y}{L_{\mathrm{c}}\sqrt{F - 2Q^2F'}} \to \frac{Y}{L_{\mathrm{c}}\sqrt{1+Y}}$$

Universe expansion: $$a(\tau) = a_{\mathrm{eq}}\left[2\left(\frac{\tau}{\tau_{\mathrm{eq}}}\right) + \left(\frac{\tau}{\tau_{\mathrm{eq}}}\right)^2\right]$$

Scaling solution $L_c = \zeta\tau$ with $\dot{\zeta} = 0$

$$\dot{v} = 0$$

$$\dot{Y} = \dot{K} = 0$$

Linear regime $F(K) = 1 - \frac{\kappa_0}{2} K$

$$\dot{\zeta}\tau = \frac{v^2 + Y}{1+Y}\frac{\dot{a}}{a}\zeta + \frac{gcv(1+Y) + AY}{2(1+Y)^{3/2}} - \zeta$$

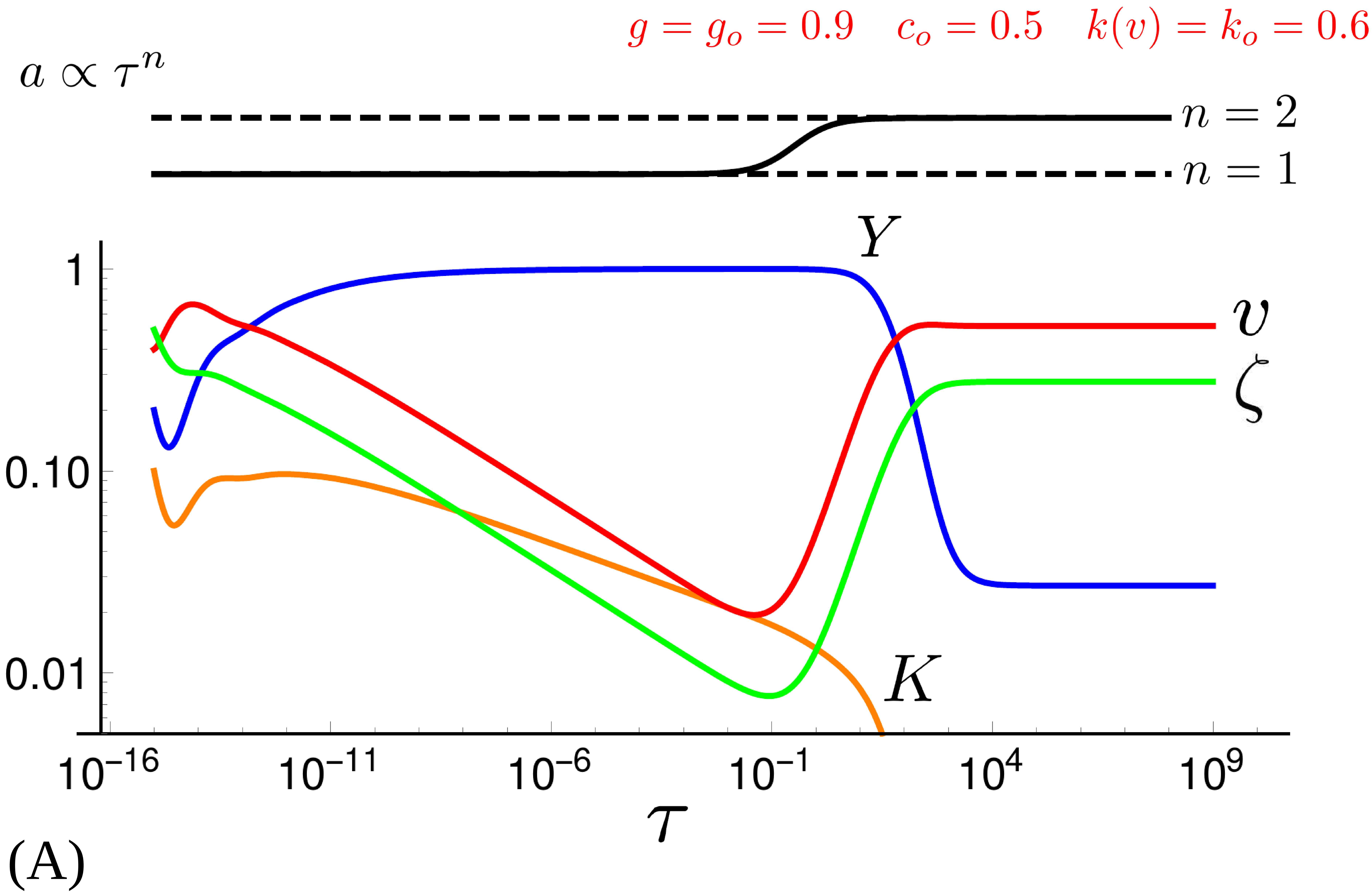
$$\dot{v}\tau = \frac{1-v^2}{1+Y}\left[\frac{k\,(1-Y)}{\zeta\sqrt{1+Y}} - 2v\frac{\dot{a}}{a}\right]$$

$$\dot{Y}\tau = 2Y\left(\frac{vk}{\zeta\sqrt{1+Y}} - \frac{\dot{a}}{a}\right) - \frac{vc(g-1)}{\zeta}\sqrt{1+Y} - \frac{AY}{\zeta\sqrt{1+Y}}$$

$$\dot{K} = 2K\left(\frac{vk}{L_c\sqrt{1+Y}} - \frac{\dot{a}}{a}\right) - \frac{2(1-2\rho_A)AY}{L_c\sqrt{1+Y}} - 2\frac{v}{L_c}c\,(g-1)\,(1-2\rho)\sqrt{1+Y}$$

Dynamical solutions

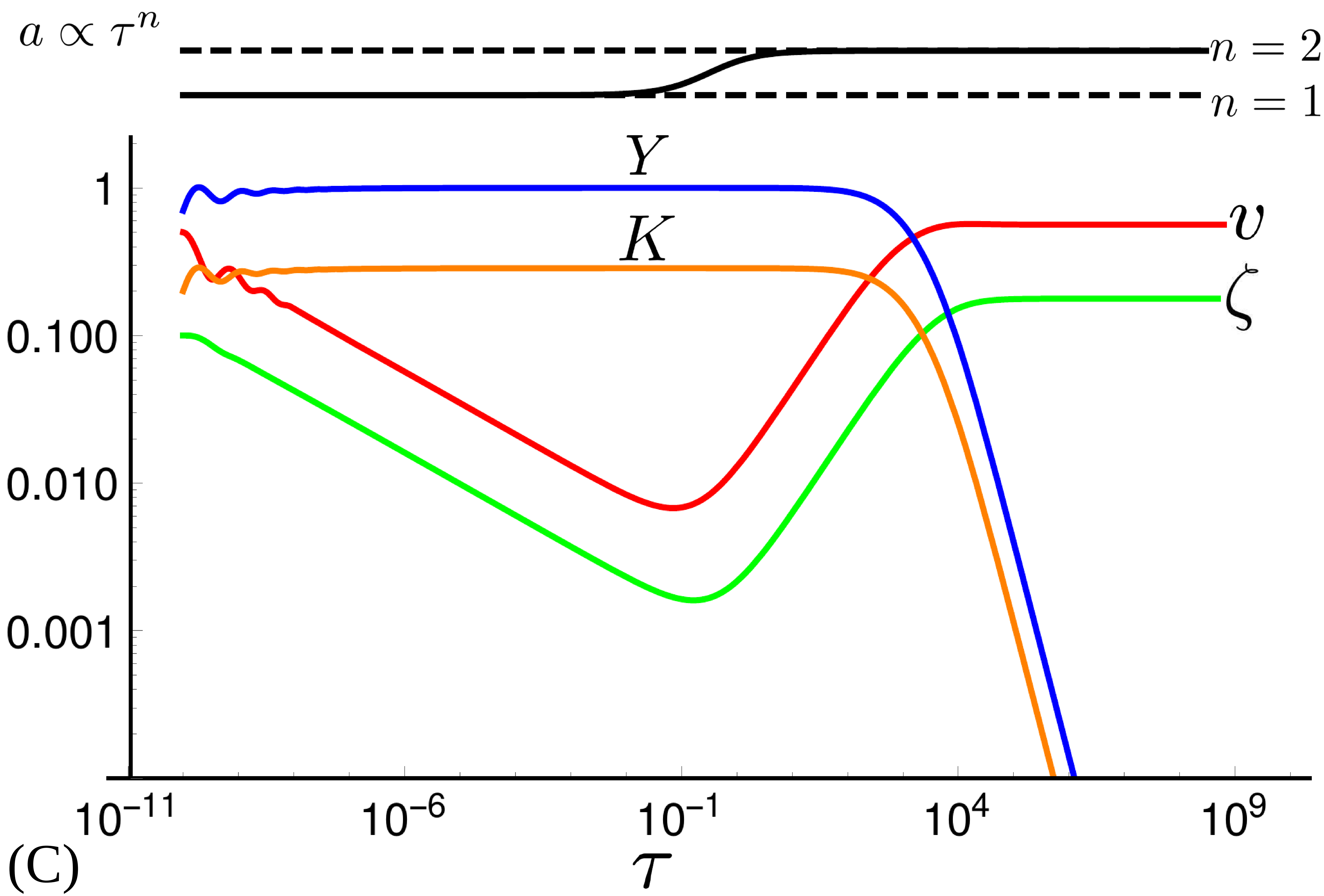
No leakage



Dynamical solutions

No leakage

$g = g_o = 1 \quad c_o = 0.23 \quad k(v) = k_o = 0.4$

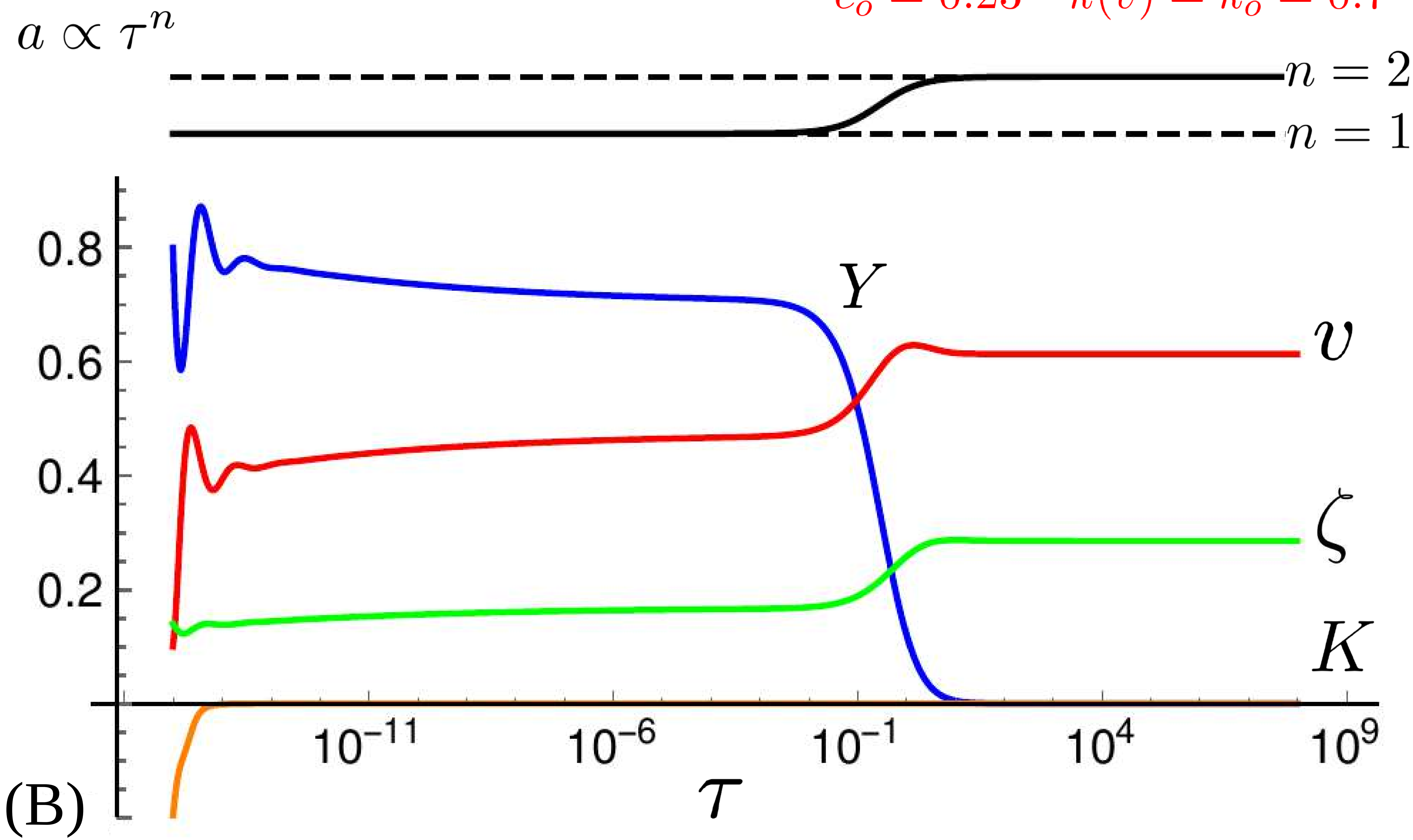


(C)

Dynamical solutions

No leakage $g = 1 + 2bY$

$c_o = 0.23 \quad k(v) = k_o = 0.7 \quad b = 0.6$

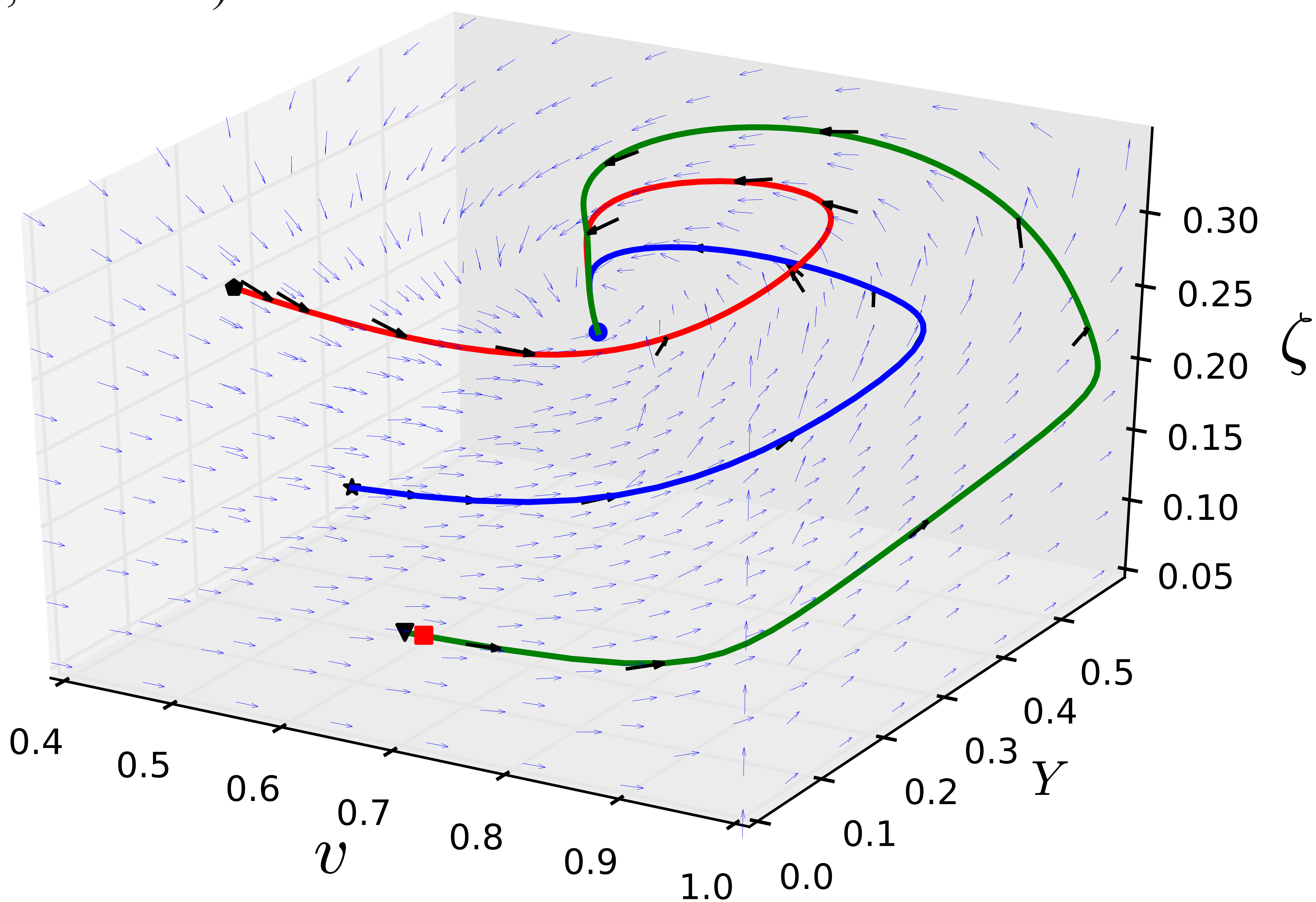


Dynamical solutions

No leakage $g = 1 + 2bY$

$c_o = 0.23 \quad k(v) = k_o = 0.6 \quad b = 0.4$

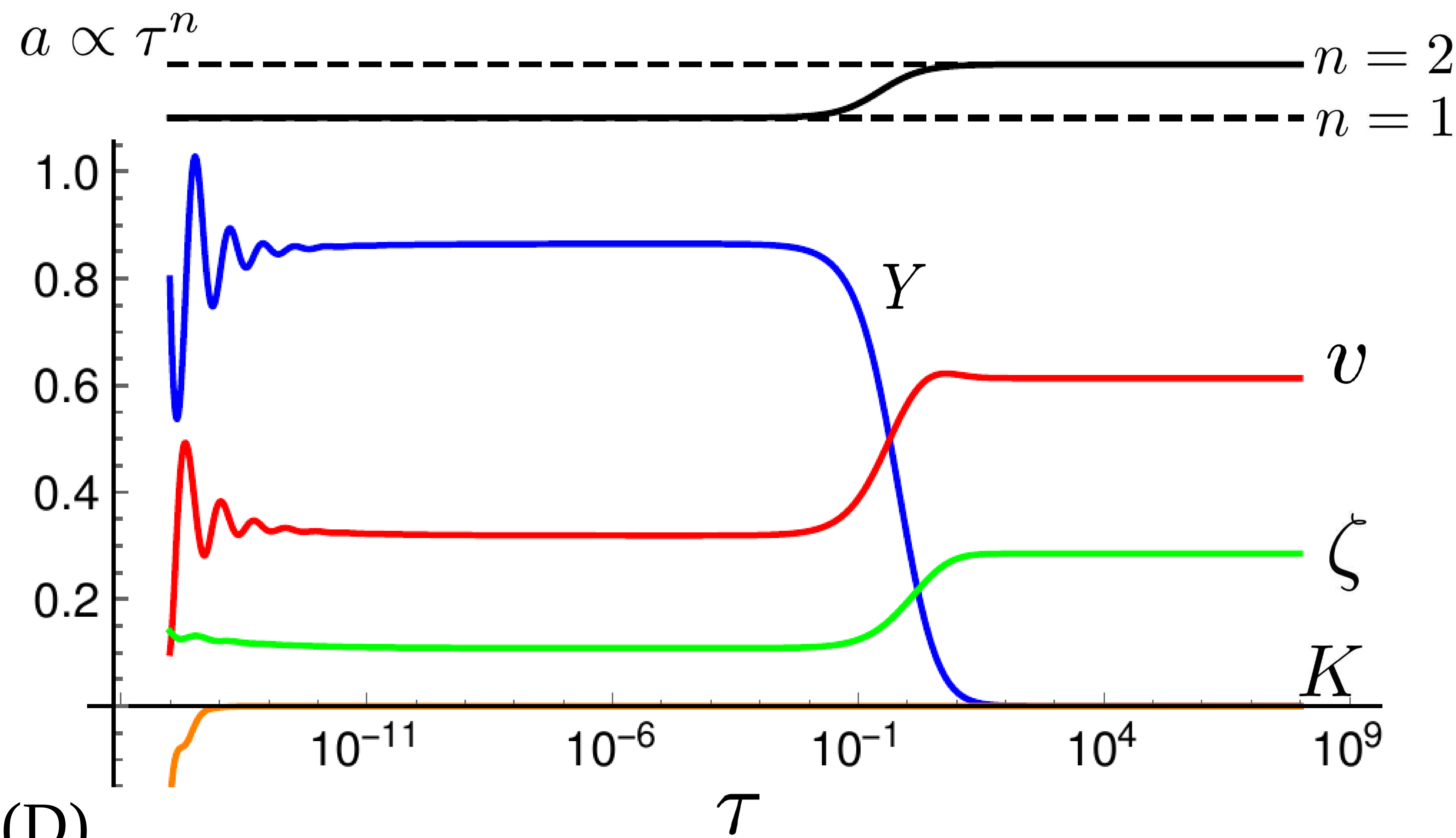
Attractor ($n = 1$, radiation)



Dynamical solutions

With leakage

$c_o = 0.23 \quad k(v) = k_o = 0.7 \quad b = 0 \quad A = 0.6$

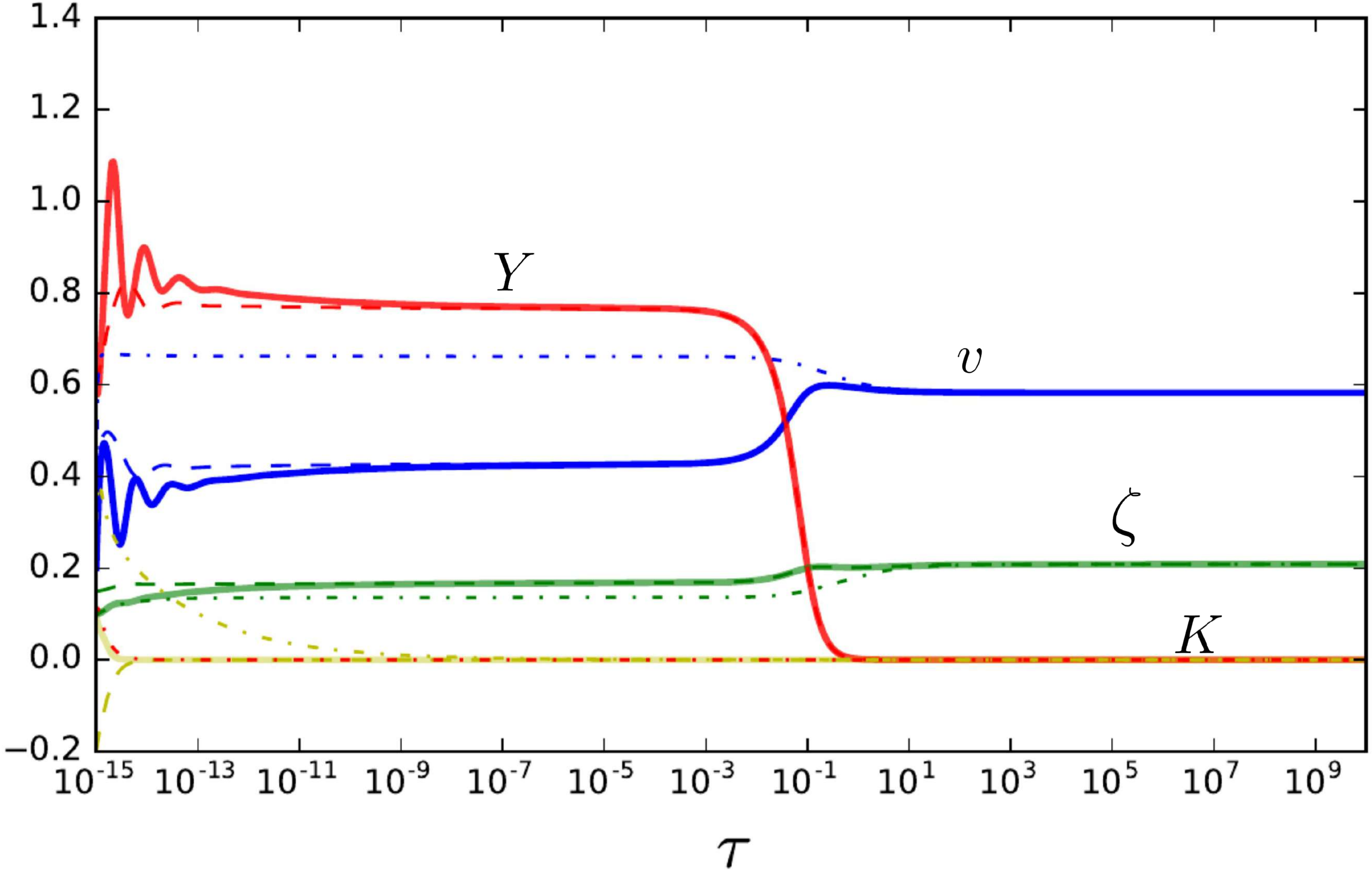


(D)

Dynamical solutions

With leakage $g = 1 + 2bY$

$c_o = 0.23 \quad b = 0 \quad A = 0.25$

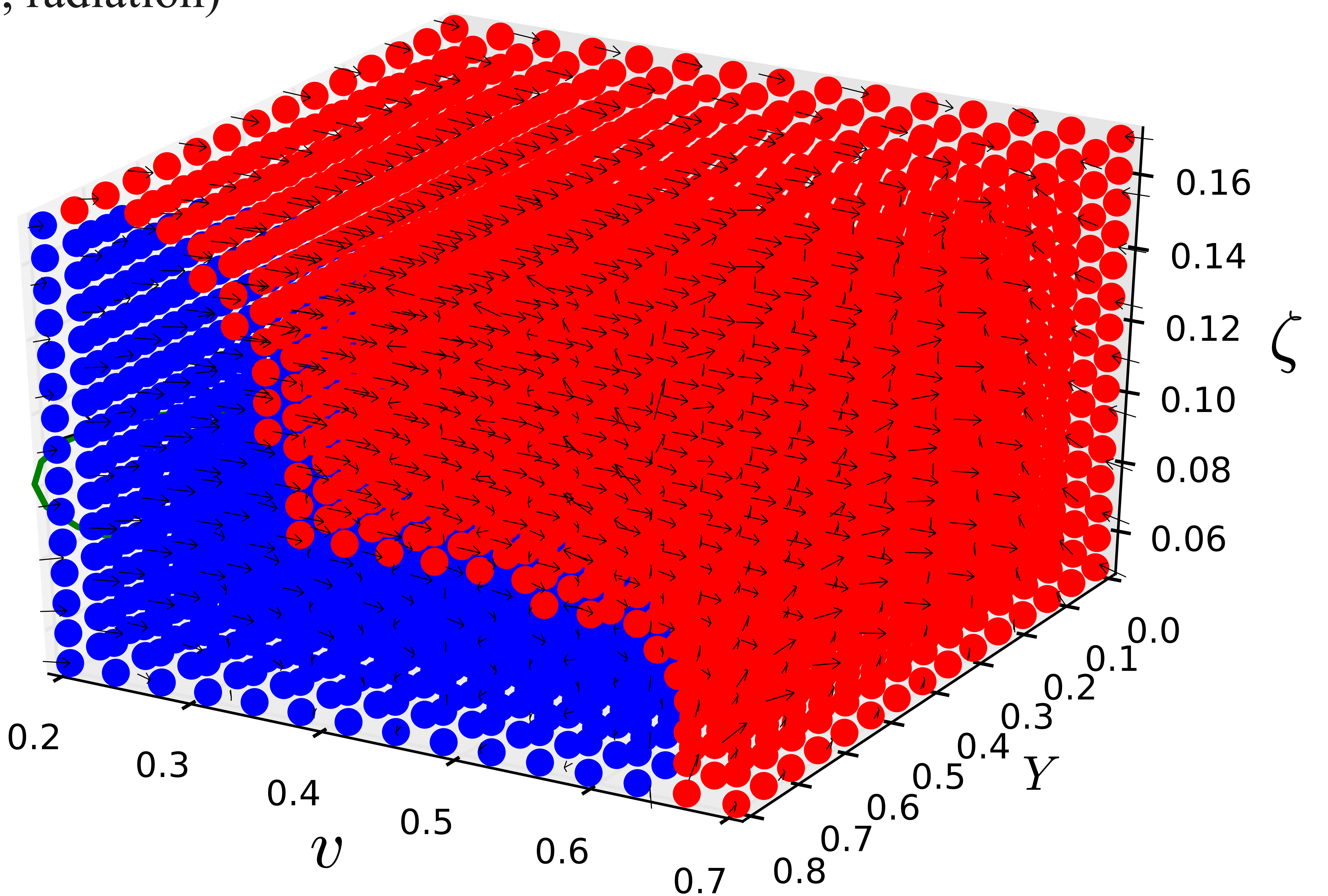


Dynamical solutions

With leakage $g = 1 + 2bY$

$c_o = 0.23 \quad b = 0 \quad A = 0.25$

Attractor ($n = 1$, radiation)

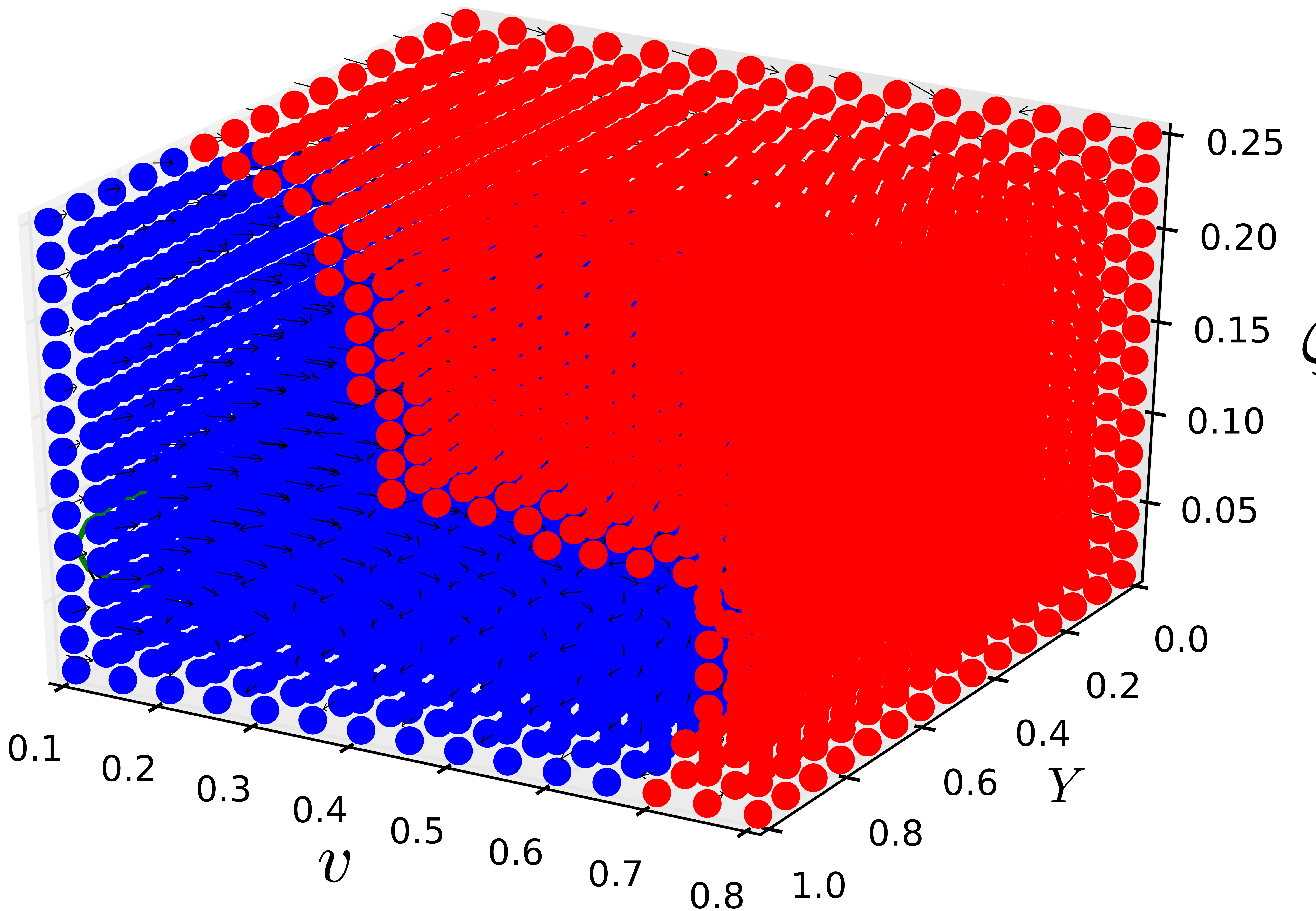


Dynamical solutions

With leakage $g = 1 + 2bY$

$c_o = 0.23 \quad b = 0 \quad A = 0.15$

$n = 1$

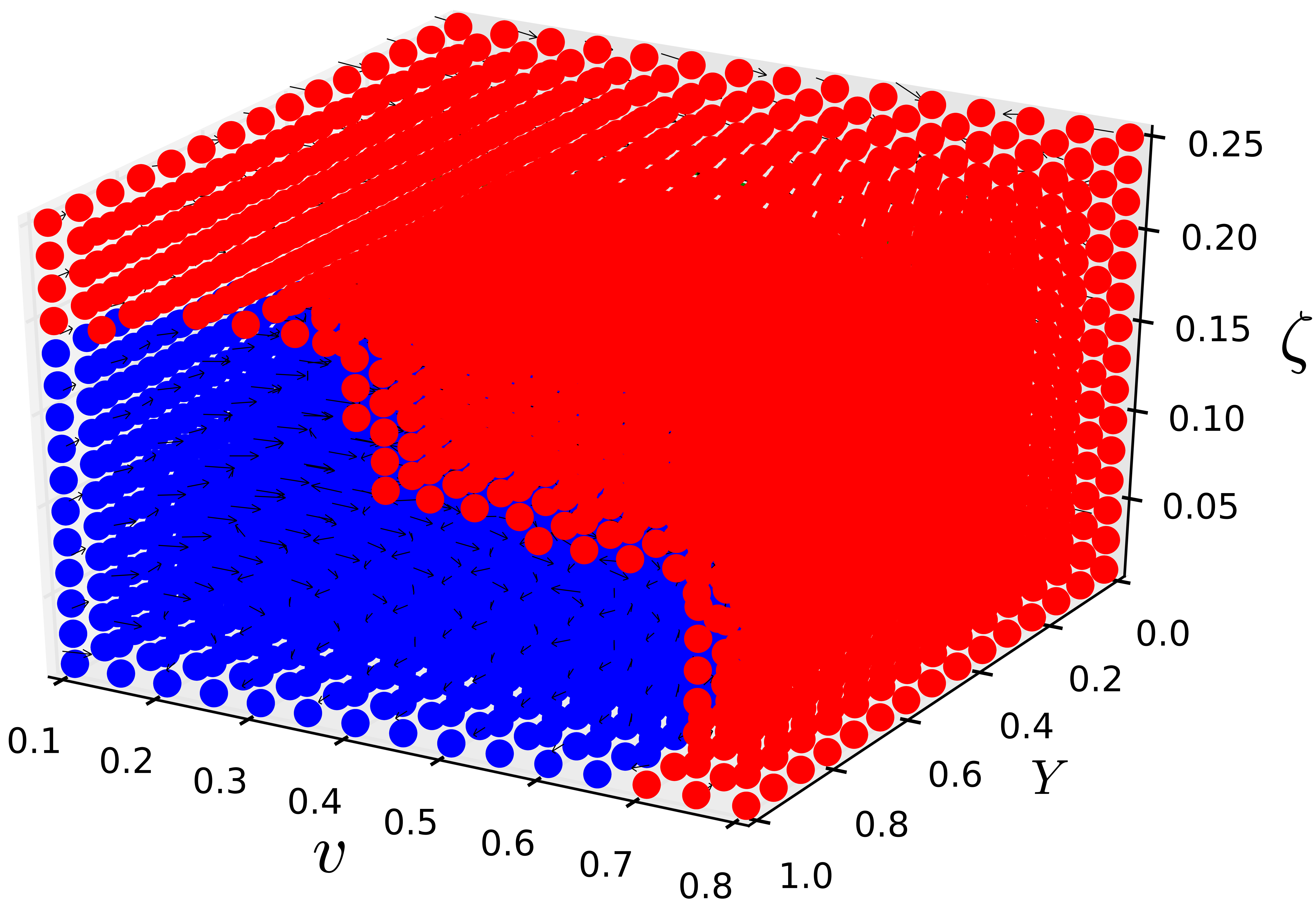


Dynamical solutions

With leakage $g = 1 + 2bY$

$c_o = 0.23 \quad b = 0 \quad A = 0.15$

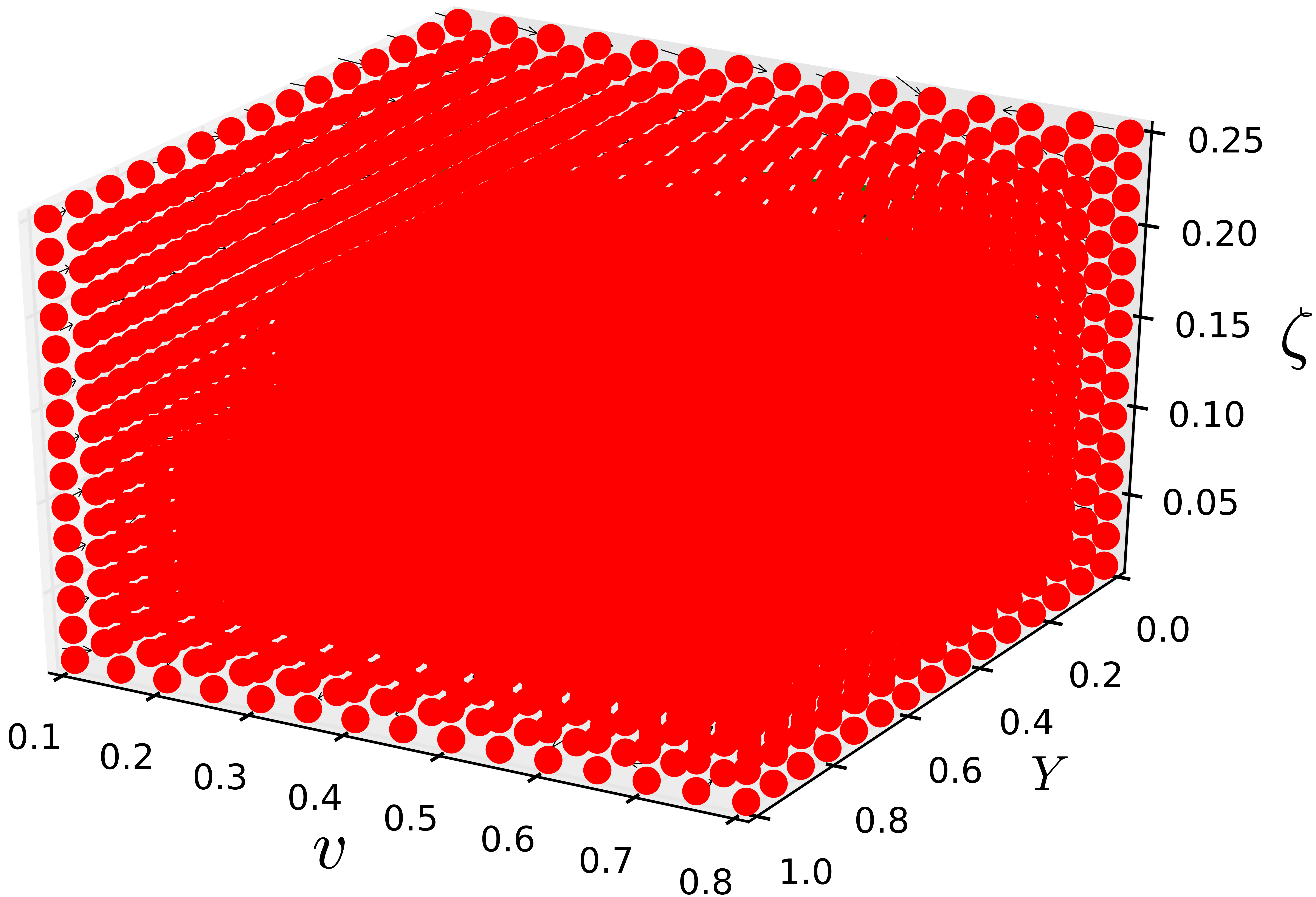
$n = 1.2$



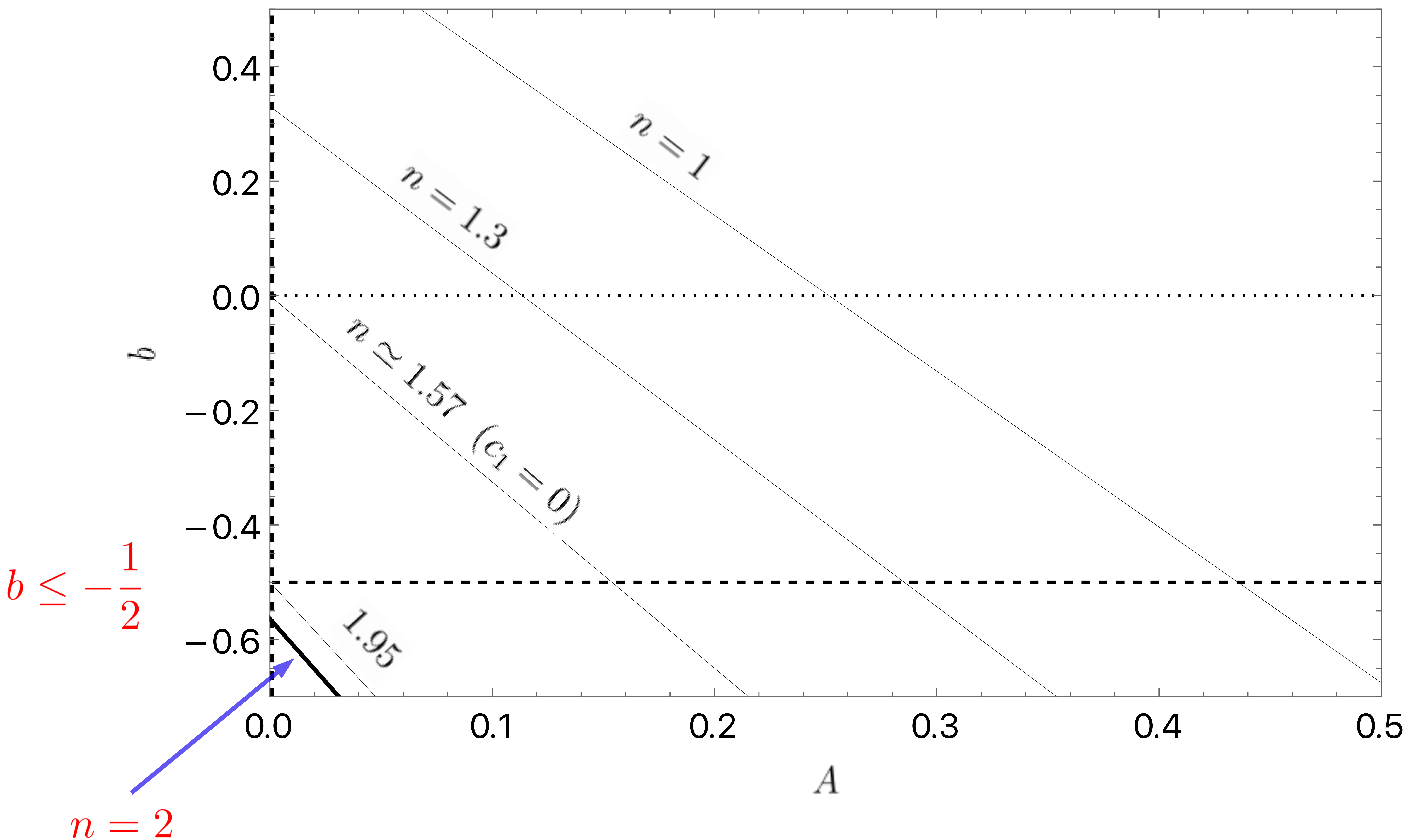
With leakage $g = 1 + 2bY$

$c_o = 0.23 \quad b = 0 \quad A = 0.15$

$n = 1.3$



Constraints



CONCLUSIONS

- Current-carrying cosmic string models w/ analytic e.o.s
- A general formalism to describe integrated quantities

$$L_{\rm C} = \zeta\tau$$

$$v = \sqrt{\langle \dot{\boldsymbol{X}}^2 \rangle}$$

$$Q \text{ and } J \qquad \Longleftrightarrow \qquad K \text{ and } Y$$

- Scaling solutions $\zeta \to \zeta_{\rm SC}\,,\ v \to v_{\rm SC}\,,\ K \to K_{\rm SC}$ and $Y \to Y_{\rm SC}$
- Charged configurations possible only for radiation era

  Stability analysis
- Leakage needed to prevent charge domination
- Matter domination $\Longrightarrow$ Nambu-Goto string network!

} **Linear equation of state…**

- Non linear cases???

THANK YOU FOR YOUR ATTENTION