Quantum vacuum: Renormalization group and anomalies in Cosmology

$$\mathcal{L} = \sum_{i=0}^{5} f_i \mathcal{L}_i = f_0 R + f_1 R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} + f_2 R_{\alpha\beta} R^{\alpha\beta} + f_3 R^2 + f_4 \phi \Box R + f_5 \phi \Delta \phi$$

$$f_0 = -\frac{{M_{\rm P}}^2}{16\pi}$$

$$f_4 = -\frac{4\pi\sqrt{-b}}{3}$$

$$f_5 = \frac{1}{2}$$

$$f_1 = a_1 + a_2 - \frac{b+\omega}{2\sqrt{-b}}\varphi + \frac{\omega}{2\sqrt{-b}}\psi$$

$$f_2 = -2a_1 - 4a_2 + \frac{\omega + 2b}{\sqrt{-b}}\varphi - \frac{\omega}{\sqrt{-b}}\psi$$

$$f_3 = \frac{a_1}{3} + a_2 - \frac{3c+2b}{36} - \frac{3b+\omega}{6\sqrt{-b}}\varphi + \frac{\omega}{6\sqrt{-b}}\psi$$

Filipe de O. Salles, Ilya L. Shapiro and PP, *Phys. Rev.* **D97** 064044 (2018) [hep-th 1801.00063]

$$
\begin{aligned}
&\left(2f_1+\tfrac{f_2}{2}\right)h^{(\mathrm{IV})}+\left[3H(4f_1+f_2)+4\dot f_1+\dot f_2\right]h^{(\mathrm{III})}+\Big[3H^2\left(6f_1+\tfrac{f_2}{2}-4f_3\right)\\
&+H\left(16\dot f_1+\tfrac{9}{2}\dot f_2\right)+6\dot H(f_1-f_3)+2\ddot f_1+\tfrac{1}{2}(\ddot f_2+f_0+f_4\ddot\varphi)+\tfrac{3}{2}f_4H\dot\varphi-\tfrac{2}{3}f_5\dot\varphi^2\Big]\ddot h\\
&-(4f_1+f_2)\frac{\nabla^2\ddot h}{a^2}+\Big[\dot H(4\dot f_1-6\dot f_3)-21H\dot H\left(\tfrac{1}{2}f_2+2f_3\right)-\ddot H\left(\tfrac{3}{2}f_2+6f_3\right)\\
&+3H^2\left(4\dot f_1+\tfrac{1}{2}\dot f_2-4\dot f_3\right)-9H^3(f_2+4f_3)+H\left(4\ddot f_1+\tfrac{3}{2}\ddot f_2\right)+\tfrac{3}{2}f_4\dot\varphi(3H^2+\dot H)\\
&+H\left(3f_4\ddot\varphi+\tfrac{3}{2}f_0-2f_5\dot\varphi^2\right)+\tfrac{1}{2}f_4\,\dddot\varphi-\tfrac{4}{3}f_5\dot\varphi\ddot\varphi\Big]\dot h-\left[H(4f_1+f_2)+4\dot f_1+\dot f_2\right]\frac{\nabla^2\dot h}{a^2}\\
&+\Big[5f_4H\,\dddot\varphi+f_4\,\ddddot\varphi-(36\dot HH^2+18\dot H^2+24H\ddot H+4\,\dddot H\,)(f_1+f_2+3f_3)\\
&-H\dot H(32\dot f_1+36\dot f_2+120\dot f_3)-8\ddot H(\dot f_1+\dot f_2+3\dot f_3)-H^2(4\ddot f_1+6\ddot f_2+24\ddot f_3)\\
&-4\dot H(\ddot f_1+\ddot f_2+3\ddot f_3)-9f_4\dot\varphi(H^3+H\dot H)+f_4\ddot\varphi(3H^2+5\dot H)-H^3(8\dot f_1+12\dot f_2+48\dot f_3)\\
&+f_5\dot\varphi^2\left(\tfrac{1}{2}H^2+\tfrac{1}{3}\dot H\right)+\tfrac{2}{3}f_5H\dot\varphi\ddot\varphi-\tfrac{1}{6}f_5\ddot\varphi^2+\tfrac{1}{3}f_5\dot\varphi\,\dddot\varphi+f_0(2\dot H+3H^2)\Big]h\\
&+\Big[2(2H^2+\dot H)(f_1+f_2+3f_3)+H\left(2\dot f_1+\tfrac{1}{2}\dot f_2\right)\\
&-\tfrac{1}{2}(\ddot f_2+f_4\ddot\varphi+f_0+3f_4H\dot\varphi)-\tfrac{1}{3}f_5\dot\varphi^2\Big]\frac{\nabla^2 h}{a^2}+\left(2f_1+\frac{1}{2}f_2\right)\frac{\nabla^4 h}{a^4}=0
\end{aligned}
$$

Filipe de O. Salles, Ilya L. Shapiro and PP, *Phys. Rev.* **D97** 064044 (2018) [hep-th 1801.00063]

A simple method for singularity avoidance and some consequences

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Motivations: (quantum) cosmology

Homogeneous & isotropic metric (FLRW): $$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]$$

spatial curvature ($\mathcal{K}$)

Hubble rate $H \equiv \frac{\dot{a}}{a}$

Matter component: perfect fluid $$T_{\mu\nu} = p g_{\mu\nu} + (\rho + p)\, u_\mu u_\nu$$

equation of state $p = w\rho$ →

$$\begin{cases} w = 0 & \text{dust} \\ w = \frac{1}{3} & \text{radiation} \end{cases}$$

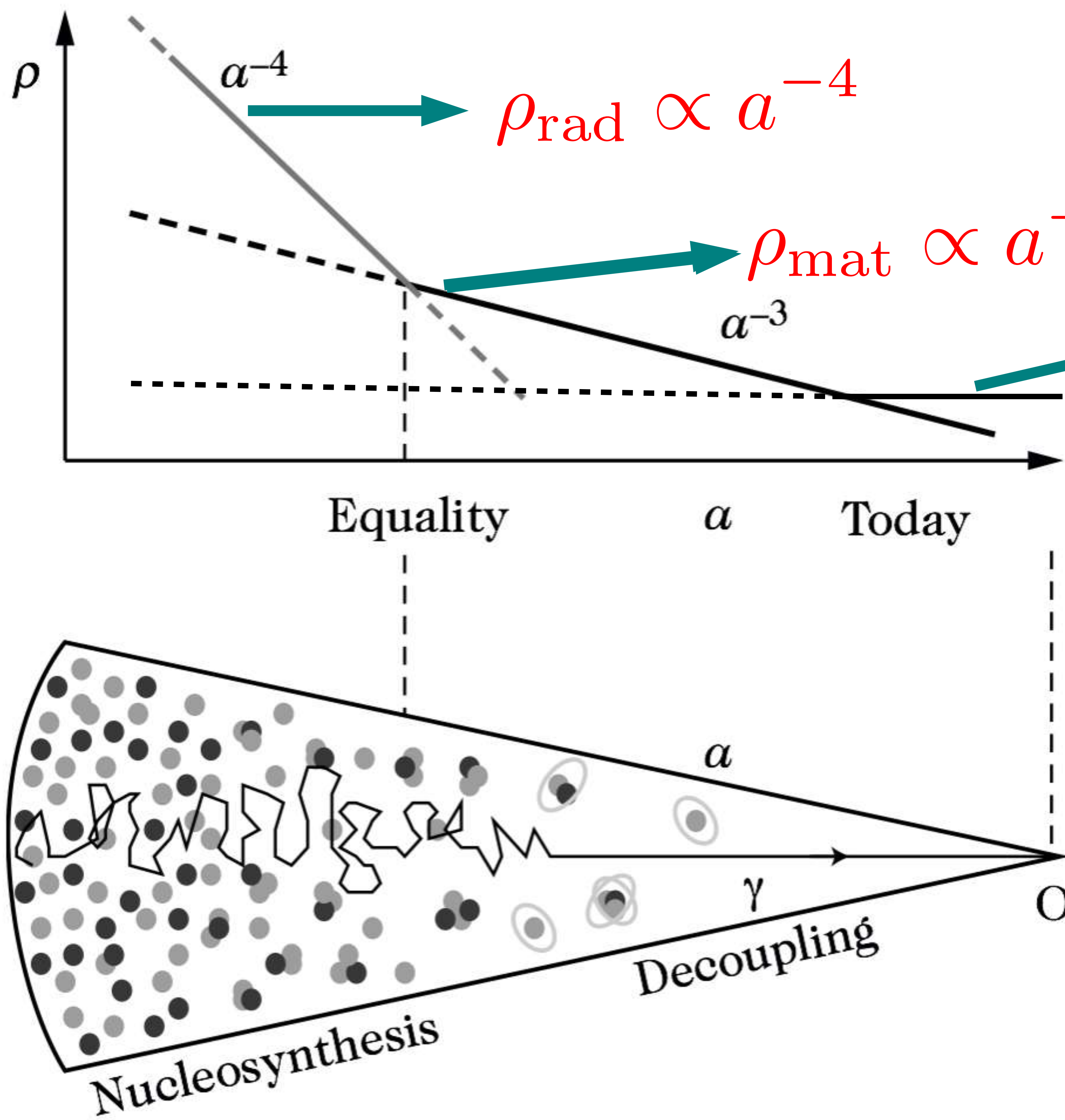
+ cosmological constant = Einstein equations

$$\begin{cases} H^2 + \frac{\mathcal{K}}{a^2} = \frac{1}{3}\left(8\pi G_{\mathrm{N}}\rho + \Lambda\right) \\ \frac{\ddot{a}}{a} = \frac{1}{3}\left[\Lambda - 4\pi G_{\mathrm{N}}\left(\rho + 3p\right)\right] \end{cases}$$

Particular solution: dust and radiation

integrate conservation equation

$$\rho\left[a\left(t\right)\right] = \rho_{\rm ini}\exp\left\{-3\int\left[1+w(a)\right]\mathrm{d}\,\ln a\right\} \underset{w\to{\rm cst}}{=} \rho_{\rm ini}\left(\frac{a}{a_{\rm ini}}\right)^{-3(1+w)}$$



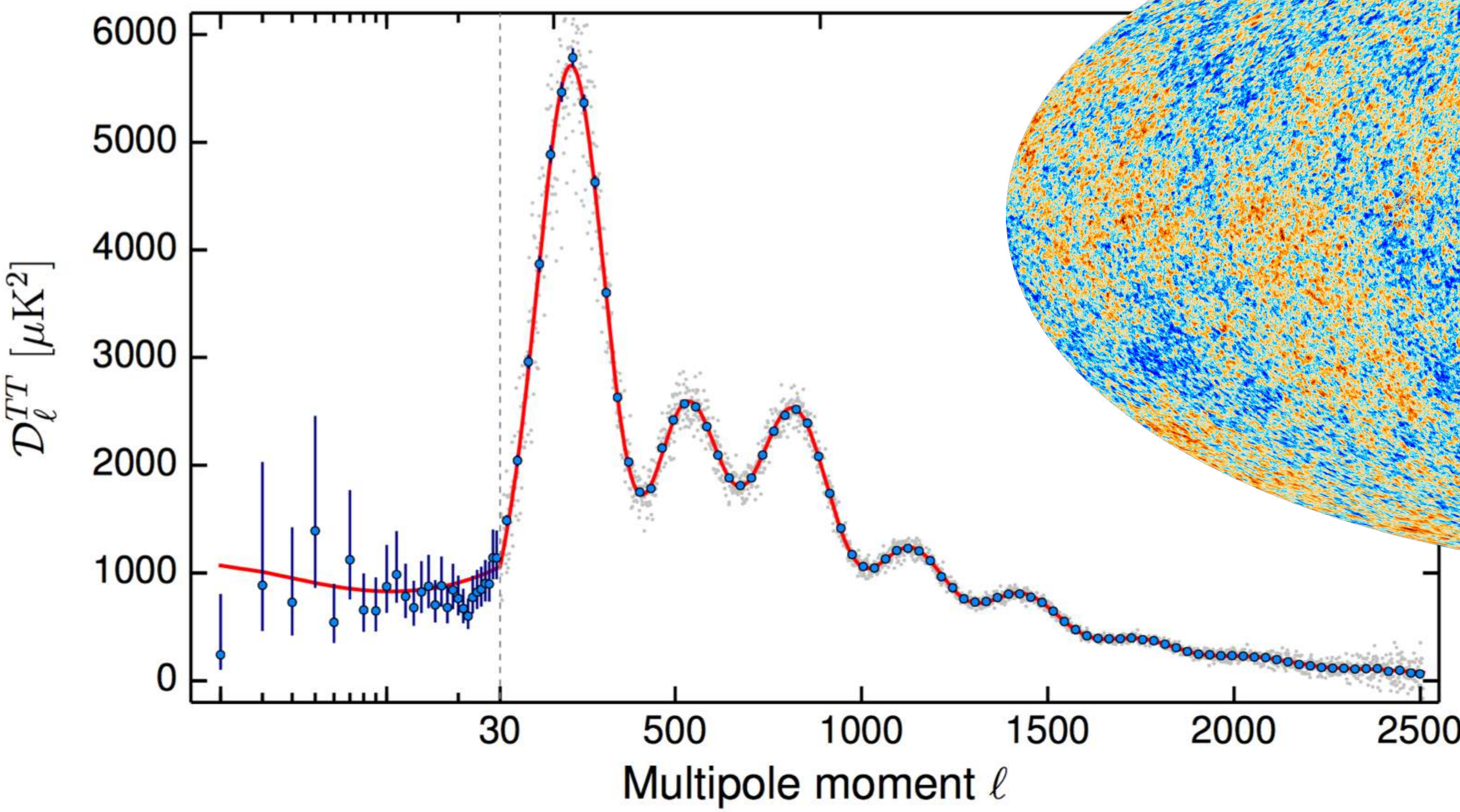
$\rho_{\rm rad} \propto a^{-4}$

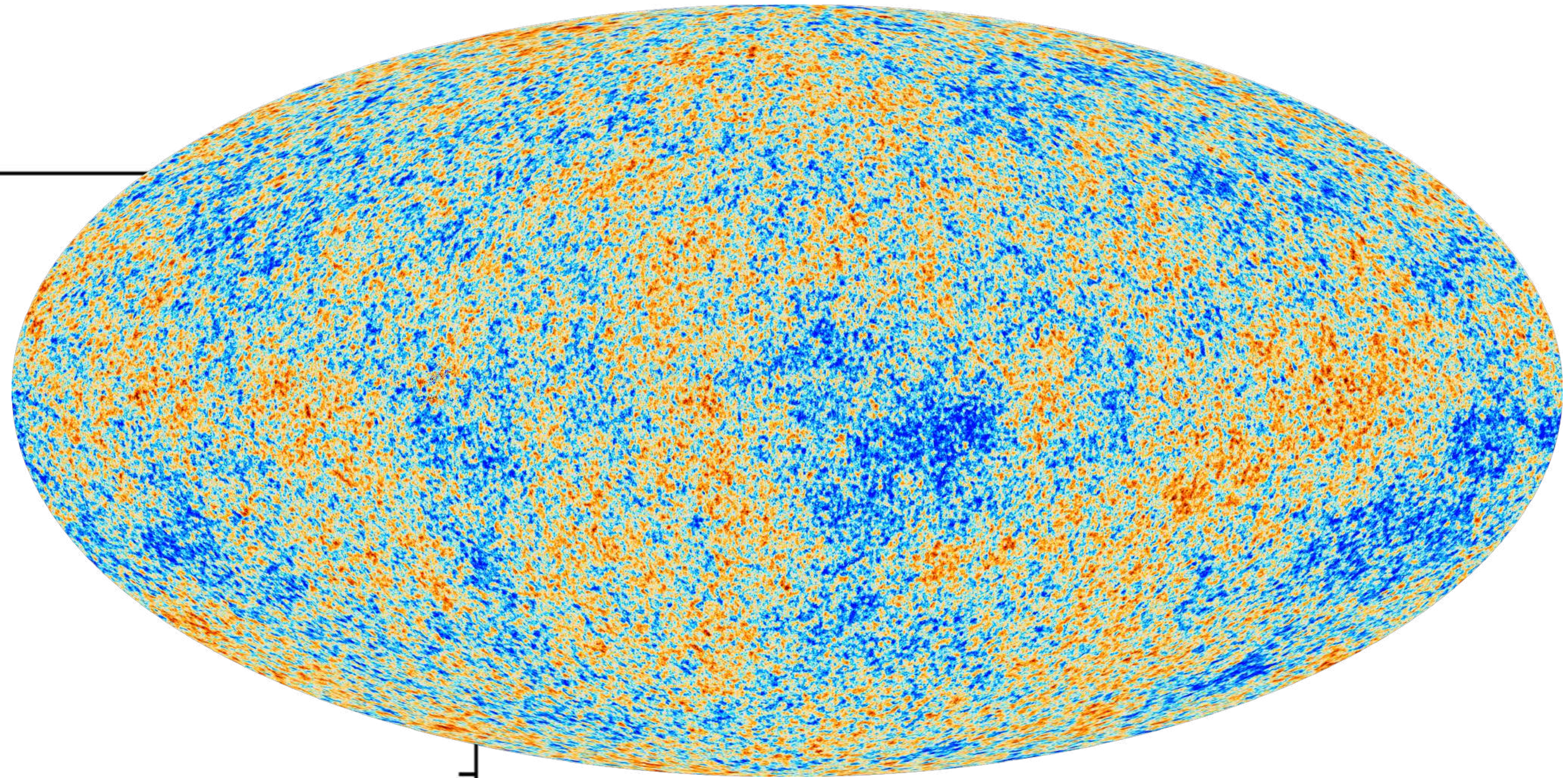
$\rho_{\rm mat} \propto a^{-3}$

$\rho_\Lambda \propto a^0$

Phenomenologically valid description for 14 Gyrs!!!!

Planck 2015





$\Omega_{\mathcal{K}} = 0.000 \pm 0.005$

$n_{\rm s} = 0.9639 \pm 0.0047$ almost scale invariant

excluded

quantum vacuum fluctuations of a single scalar d.o.f

$f_{\rm NL}^{\rm loc} = 0.8 \pm 5$

$f_{\rm NL}^{\rm eq} = -4 \pm 43$

$f_{\rm NL}^{\rm ort} = -26 \pm 21$

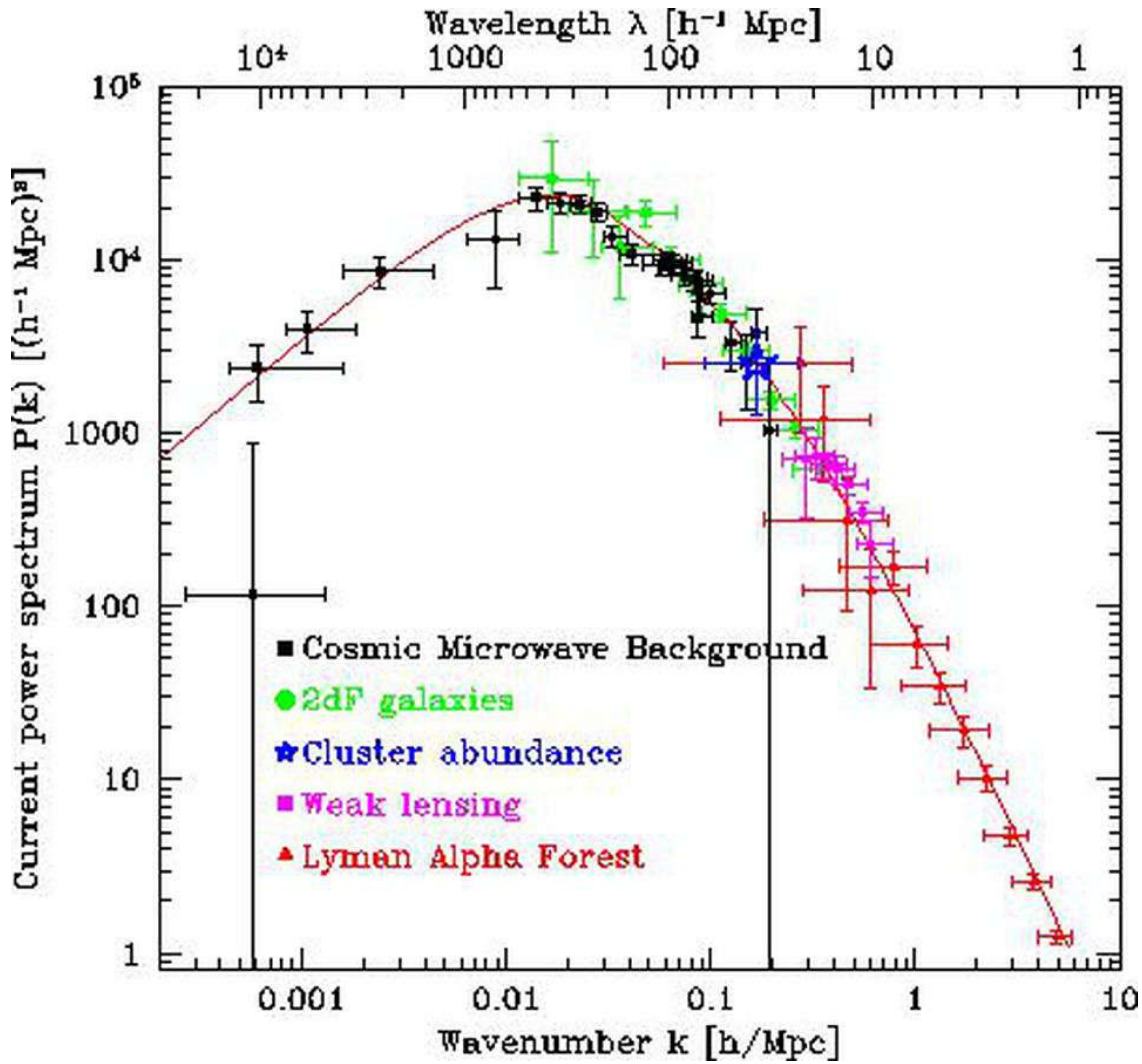
gaussian signal

compatible with ***INFLATION***

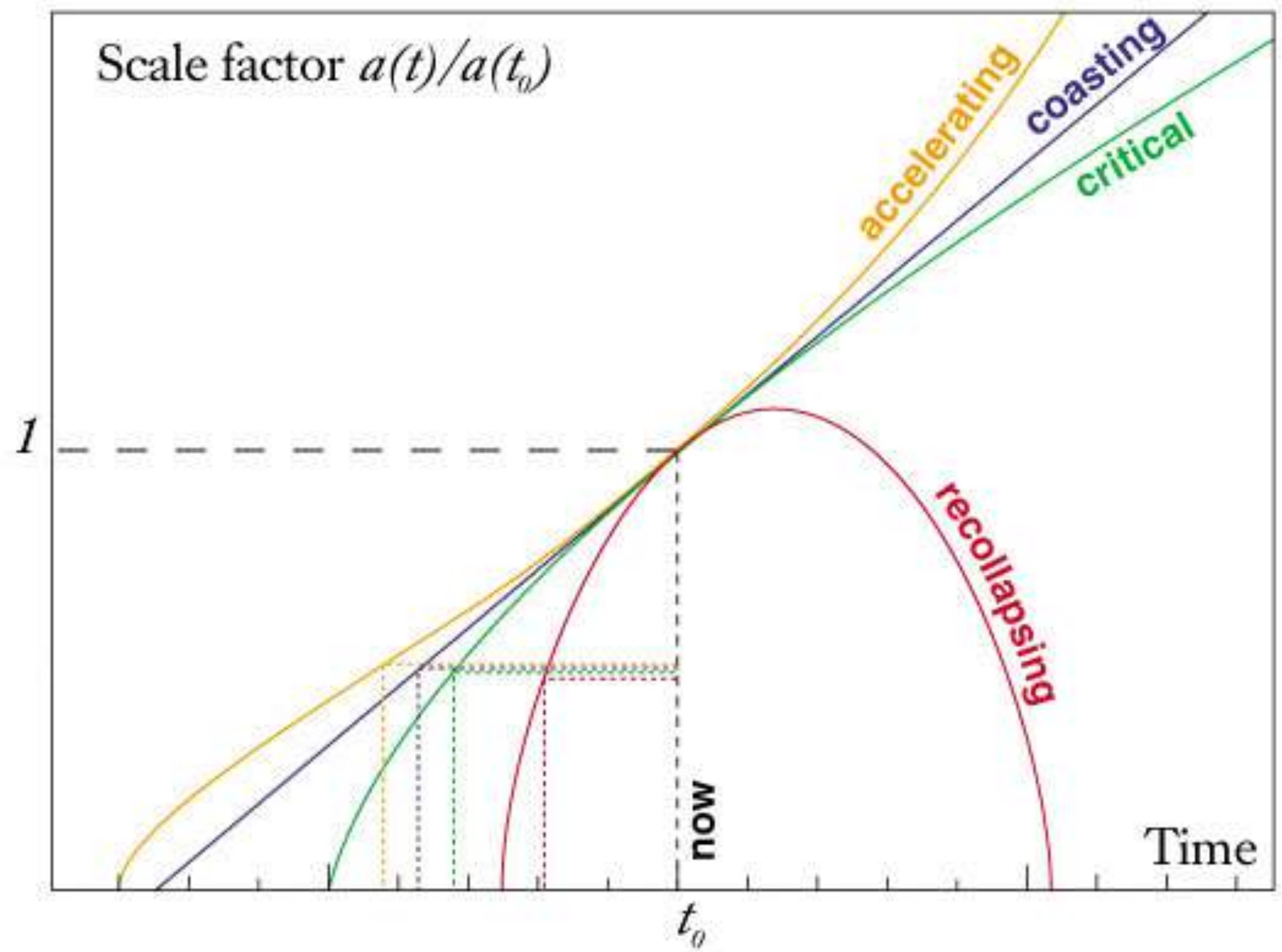
isocurvature $\lesssim 1\%$

$r < 0.08$

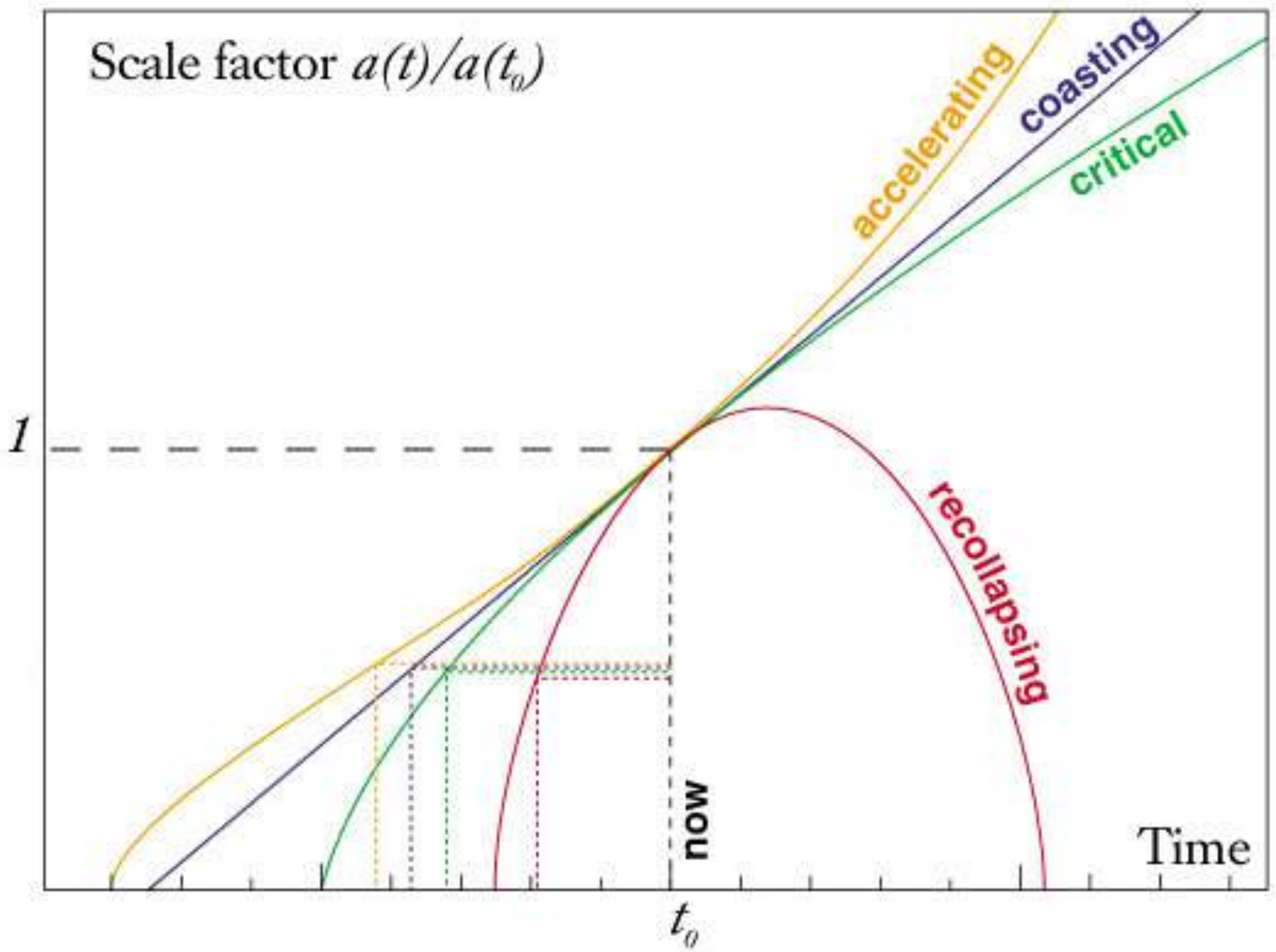
Numerical simulation for large scale structure formation...



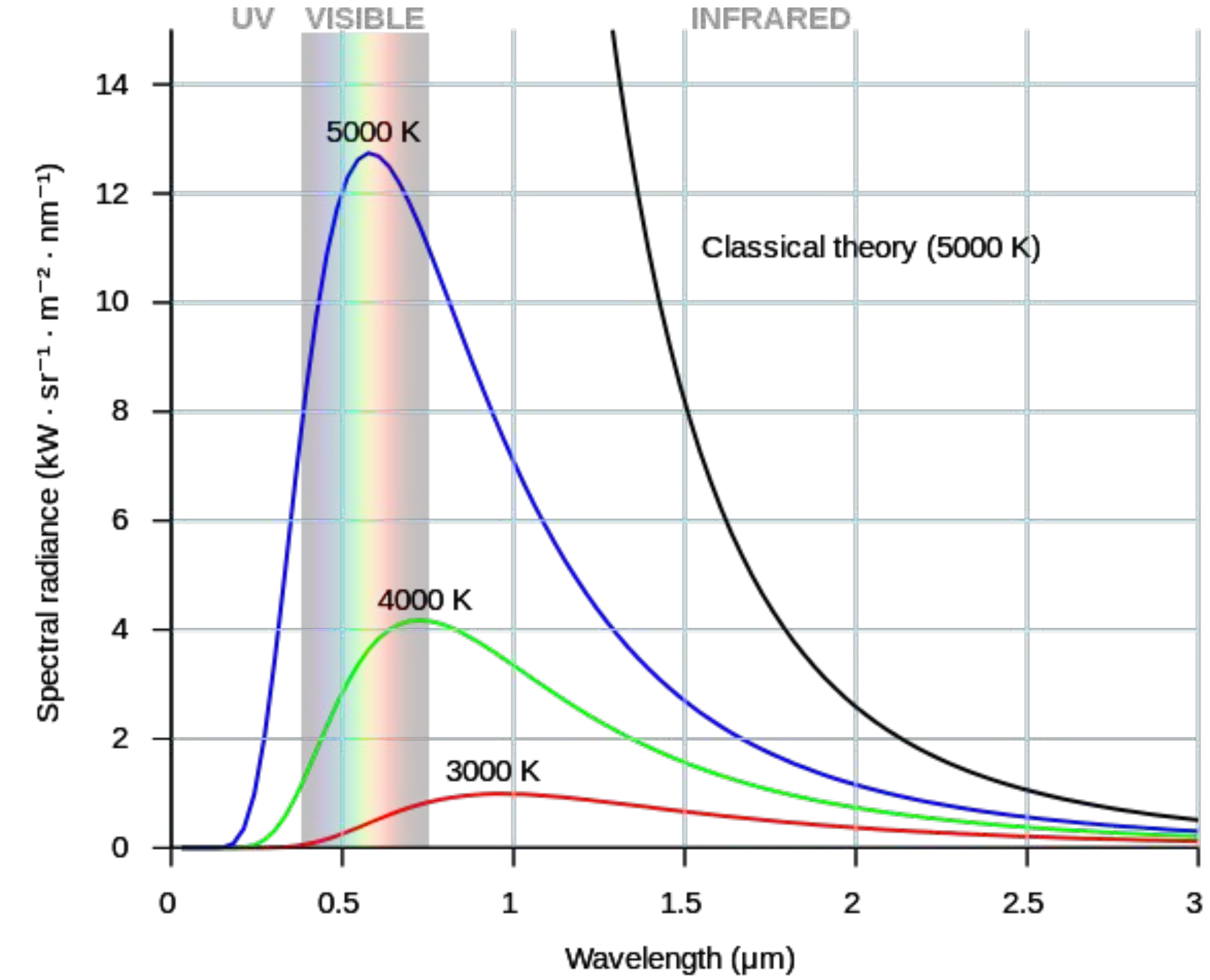
A central problem (though not often formulated thus...): the singularity

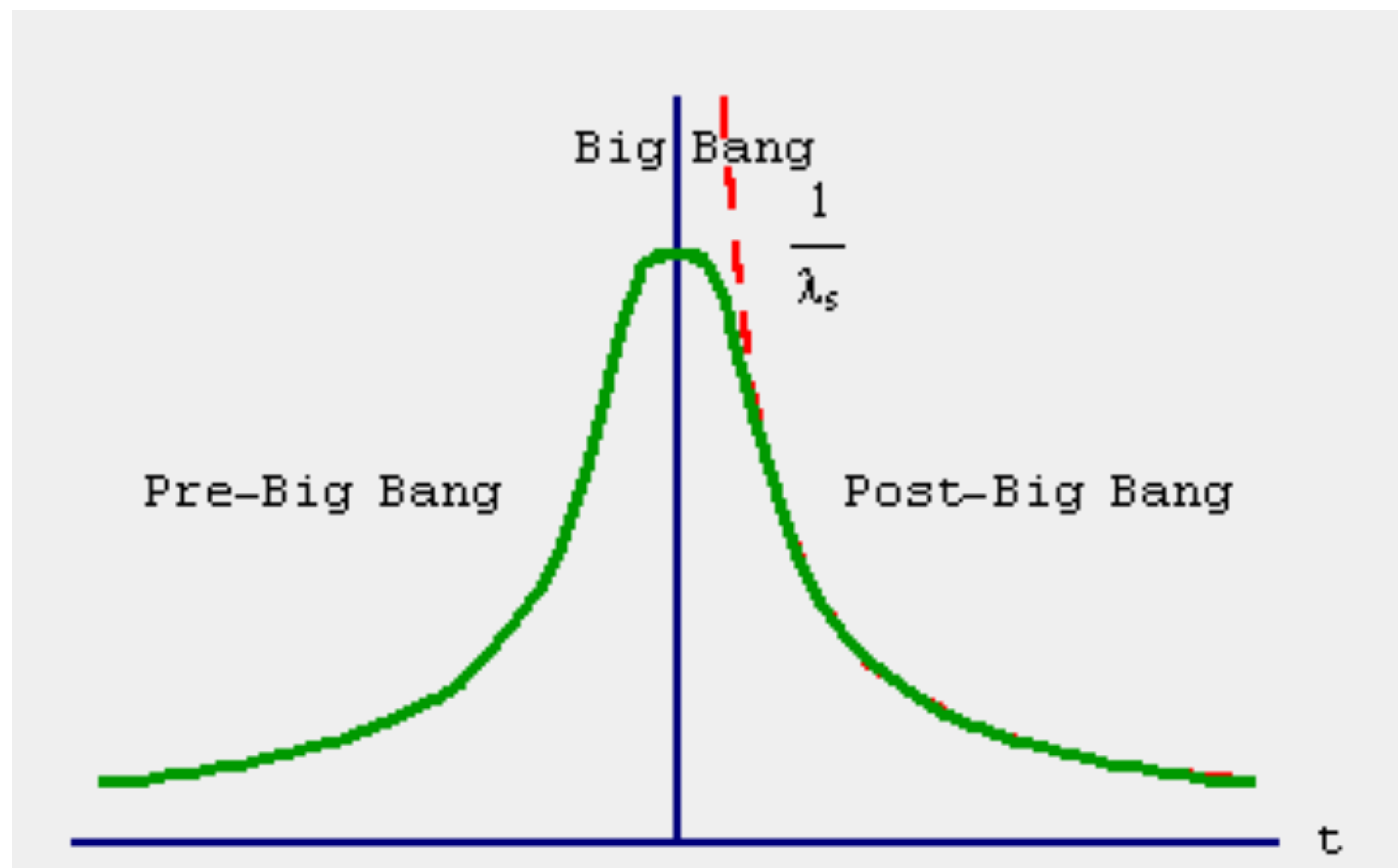


Singularity problem...



a quantum effect?





Quantum cosmology

Hamiltonian GR (3+1)

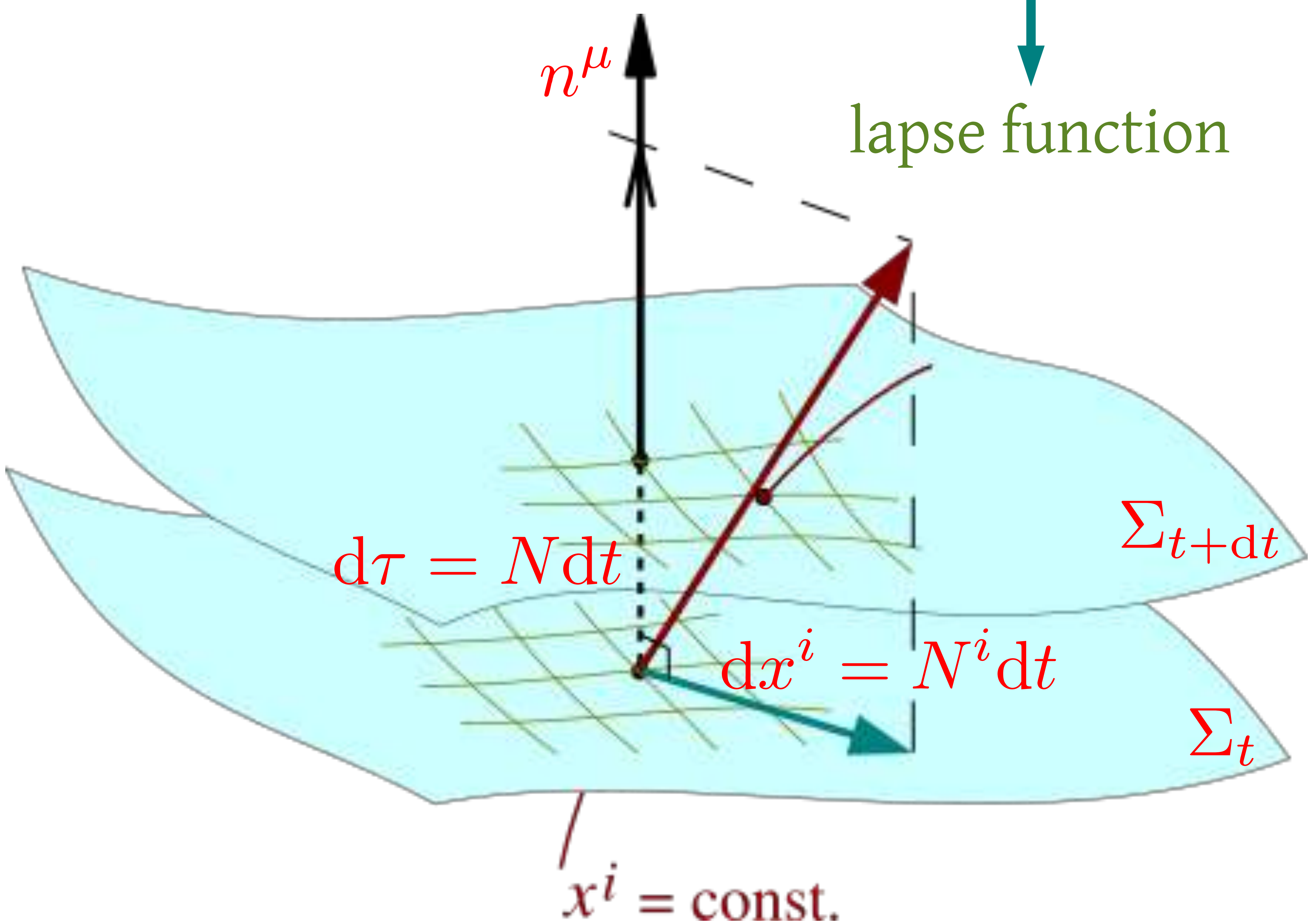
$$ds^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right)$$

lapse function

intrinsic metric = first fundamental form

shift vector

intrinsic curvature tensor $^3R^i{}_{jkl}(h)$



extrinsic curvature = second fundamental form:

$$K_{ij} = -\nabla_j^{(h)} n_i = \frac{1}{2N}\left(\nabla_j^{(h)} N_i + \nabla_i^{(h)} N_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action (Einstein-Hilbert, compact space):

$$\mathcal{S} = \frac{1}{16\pi G_{\mathrm{N}}} \left[\int_{\mathcal{M}} \sqrt{-g} \, (R - 2\Lambda) \, \mathrm{d}^4 x + 2 \int_{\partial\mathcal{M}} \sqrt{h} K^i{}_i \mathrm{d}^3 x \right] + \mathcal{S}_{\mathrm{matter}} \left[\Phi(x) \right]$$

$$\longrightarrow \mathcal{S} = \int L \mathrm{d}t = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}t \left[\int \mathrm{d}^3 x \, N \sqrt{h} \left(K_{ij} K^{ij} - K^2 + {}^3R - 2\Lambda \right) + L_{\mathrm{matter}} \right]$$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\mathrm{N}}} \left(K^{ij} - h^{ij} K \right)$$

$$\pi^{\Phi} \equiv \frac{\delta L}{\delta \dot{\Phi}} = -\frac{\sqrt{h}}{N} \left(\dot{\Phi} - N \frac{\partial \Phi}{\partial x^i} \right)$$

$$\left. \begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{N}} \approx 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{N}^i} \approx 0 \end{aligned} \right\} \text{primary constraints}$$

Hamiltonian $$H \equiv \int d^3x \left(\pi^0 \dot{N} + \pi^i \dot{N}_i + \pi^{ij} \dot{h}_{ij} + \pi^\Phi \dot{\Phi}\right) - L$$

$$= \int d^3x \left(\pi^0 \dot{N} + \pi^i \dot{N}_i + N\mathcal{H} + N_i \mathcal{H}^i\right)$$

$$\mathcal{H} = \frac{1}{\sqrt{h}} \left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right) \pi^{ij}\pi^{kl} - \sqrt{h}\,{}^3\!R$$

$$\mathcal{H}^i = -2\sqrt{h}\nabla_j \left(\frac{\pi^{ij}}{\sqrt{h}}\right)$$

variation wrt lapse: $\mathcal{H} = 0 \to$ Hamiltonian constraint
variation wrt shift: $\mathcal{H}^i = 0 \to$ momentum constraint
$\Longrightarrow$ classical description complete

Superspace & canonical quantization

relevant configuration space $\mathrm{Riem}(\Sigma) \equiv \{h_{ij}(x^\mu), \Phi(x^\mu) | x \in \Sigma\}$

→ parameters

GR $\Longrightarrow$ invariance/diffeomorphisms $\Longrightarrow$ $\mathrm{Conf} = \dfrac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}(\Sigma)}$: superspace

Wave functional $\Psi[h_{ij}(x), \Phi(x)] = \langle h_{ij}, \Phi | \Psi \rangle$

+ Dirac canonical quantization procedure

$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}} \qquad \pi^{\Phi} \to -i\frac{\delta}{\delta \Phi} \qquad \pi^{0} \to -i\frac{\delta}{\delta N} \qquad \pi^{i} \to -i\frac{\delta}{\delta N_i}$$

primary constraints

$$\begin{cases} \hat{\pi}^0 = -i\dfrac{\delta\Psi}{\delta N} = 0 \\ \hat{\pi}^i = -i\dfrac{\delta\Psi}{\delta N_i} = 0 \end{cases}$$

momentum $\hat{\mathcal{H}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\dfrac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\rm N}\hat{T}^{0i}\Psi$

same Ψ for configurations related by a coordinate transformation

Hamiltonian

$$\hat{\mathcal{H}}\Psi = \left[-16\pi G_{\rm N}\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G_{\rm N}}\left(-{}^3R + 2\Lambda + 16\pi G_{\rm N}\hat{T}^{00}\right)\right]\Psi = 0$$

Wheeler - De Witt equation

$$\mathcal{G}_{ijkl} = \frac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

De Witt metric

primary constraints $$\begin{cases} \hat{\pi}^0 = -i\dfrac{\delta\Psi}{\delta N} = 0 \\ \hat{\pi}^i = -i\dfrac{\delta\Psi}{\delta N_i} = 0 \end{cases}$$

momentum $\hat{\mathcal{H}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\dfrac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\rm N}\hat{T}^{0i}\Psi$

same Ψ for configurations related by a coordinate transformation

Hamiltonian

$$\hat{\mathcal{H}}\Psi = 0$$

time-independent Schrödinger equation

mini-superspace

restrict attention from an infinite dimensional configuration space to a 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j \quad \mapsto \quad a^2(t)\left[\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)\right]$$

WDW equation becomes Schrödinger like for $\Psi\left[a(t),\phi(t)\right]$

Conceptual & technical issues

infinite # d.o.f. to a few: mathematical consistency?

freeze momenta... Heisenberg uncertainties?

$[\text{quantization}, \text{minisuperspace}] \neq 0$

mini-superspace

restrict attention from an infinite dimensional configuration space to a 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j \quad \mapsto \quad a^2(t)\left[\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)\right]$$

WDW equation becomes Schrödinger like for $\Psi\left[a(t),\phi(t)\right]$

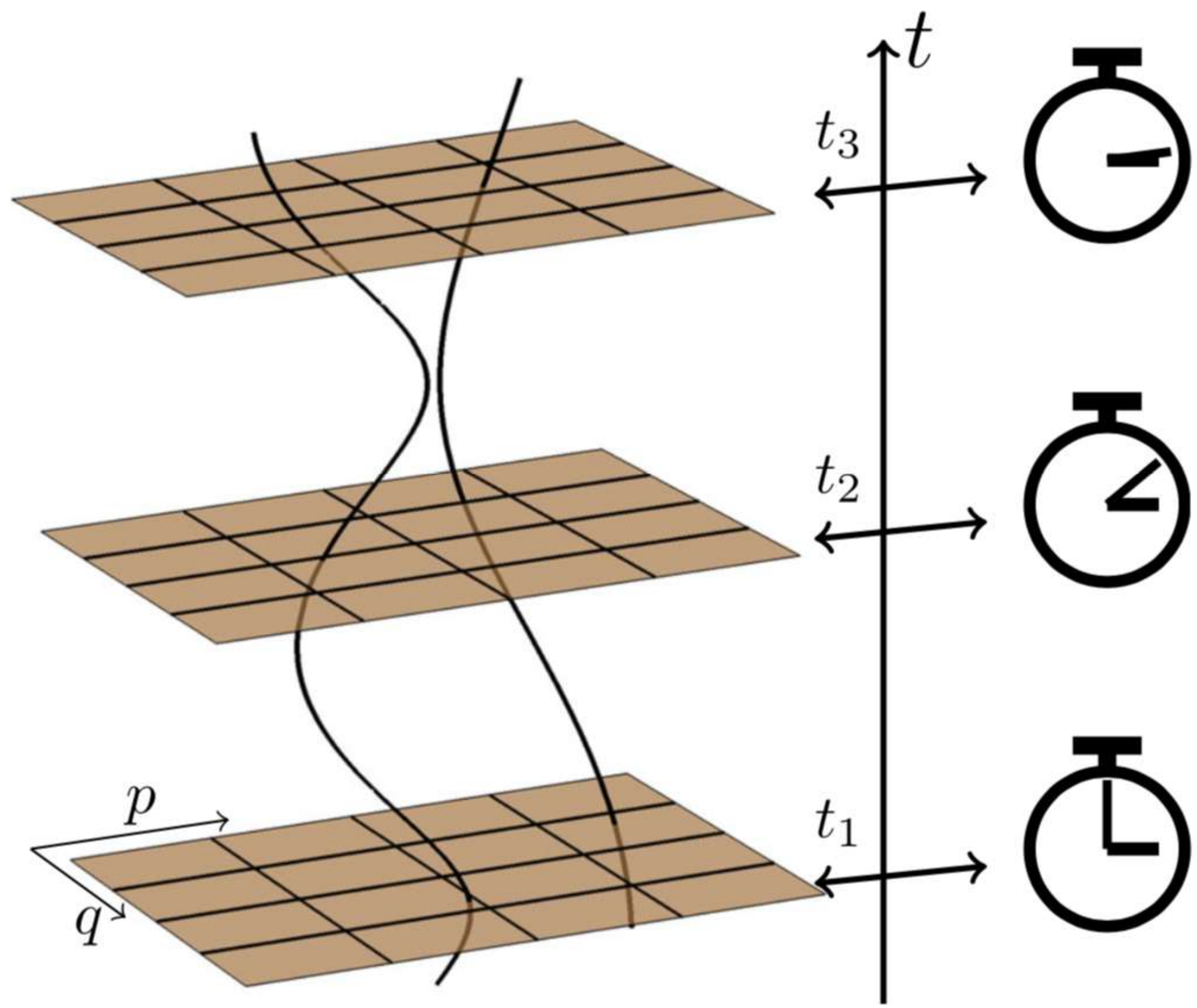
Conceptual & technical issues

ACTUALLY MAKE CALCULATIONS!

The clock issue in quantum cosmology

→ GR = constrained system: lack of external time

→ arbitrary degree of freedom: internal clock



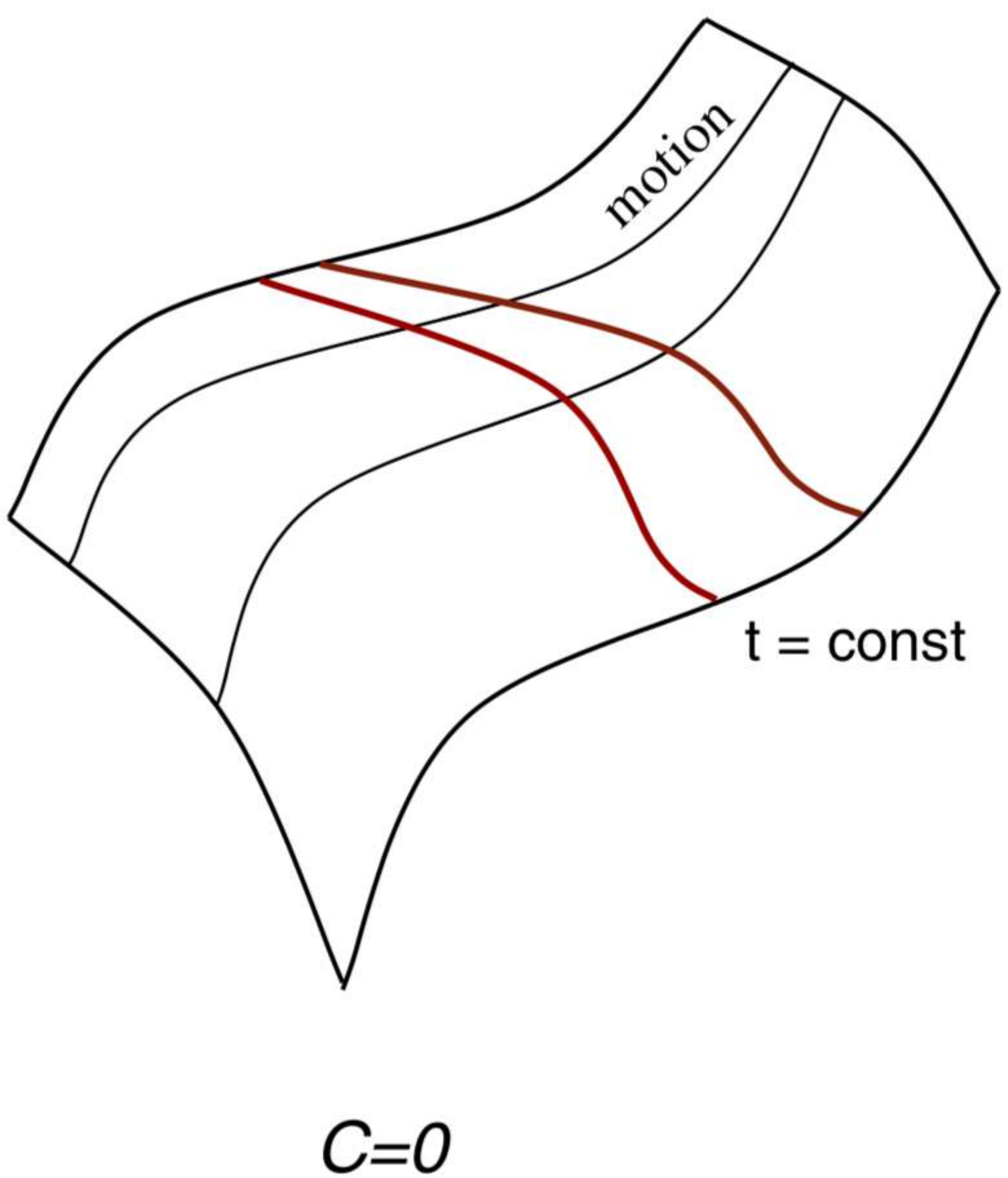
Classical system q_i & p_i

Constraint $C(q_i, p_i) = 0$ & $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathcal{O}(q_i, p_i) = \{\mathcal{O}, C\}_{\mathrm{P.B}}$

evolution parameter (time) | observable

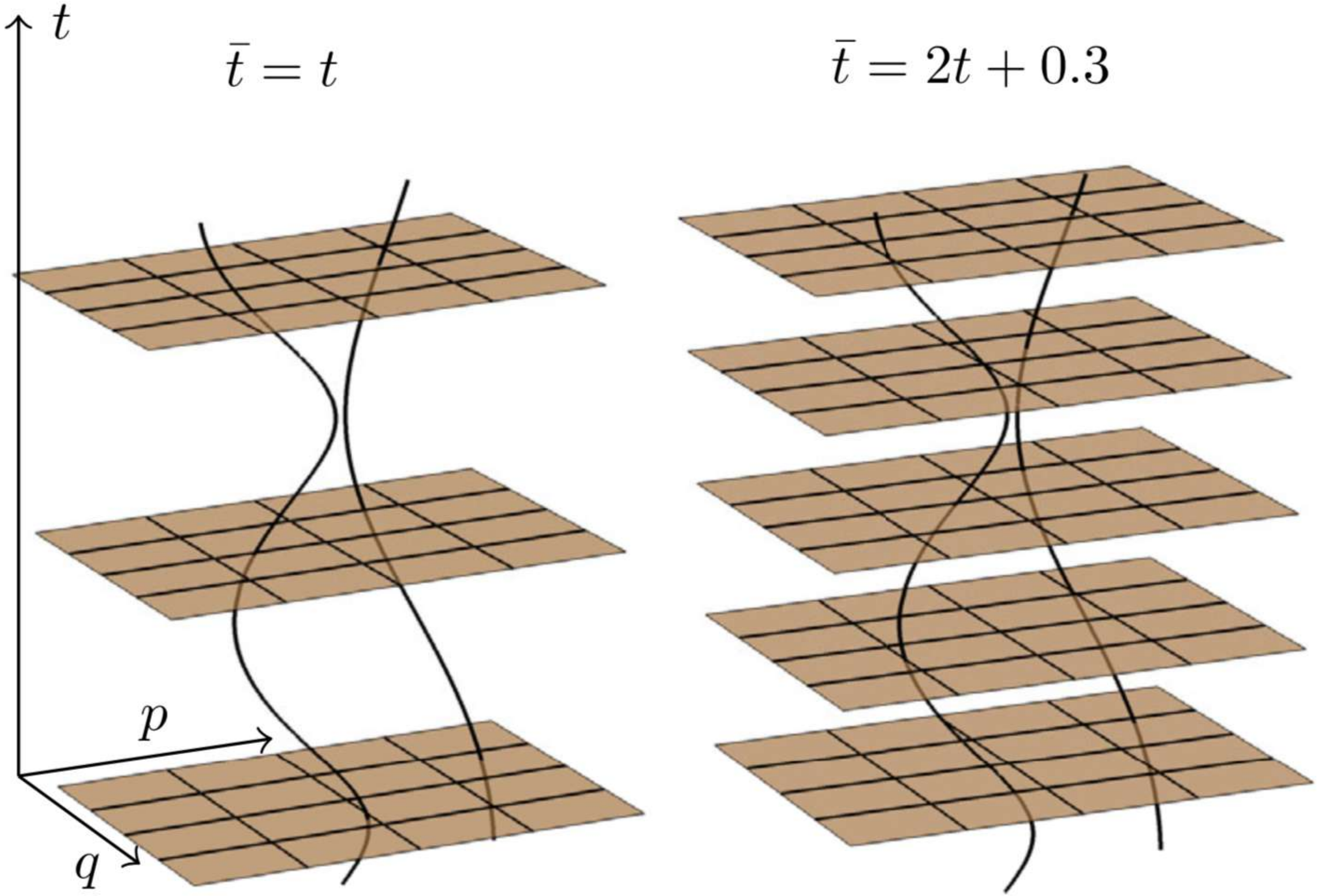
Time parametrization invariance $\tau \to \tau'$ → $N(q_i, p_i, \tau)$

arbitrary non vanishing lapse function

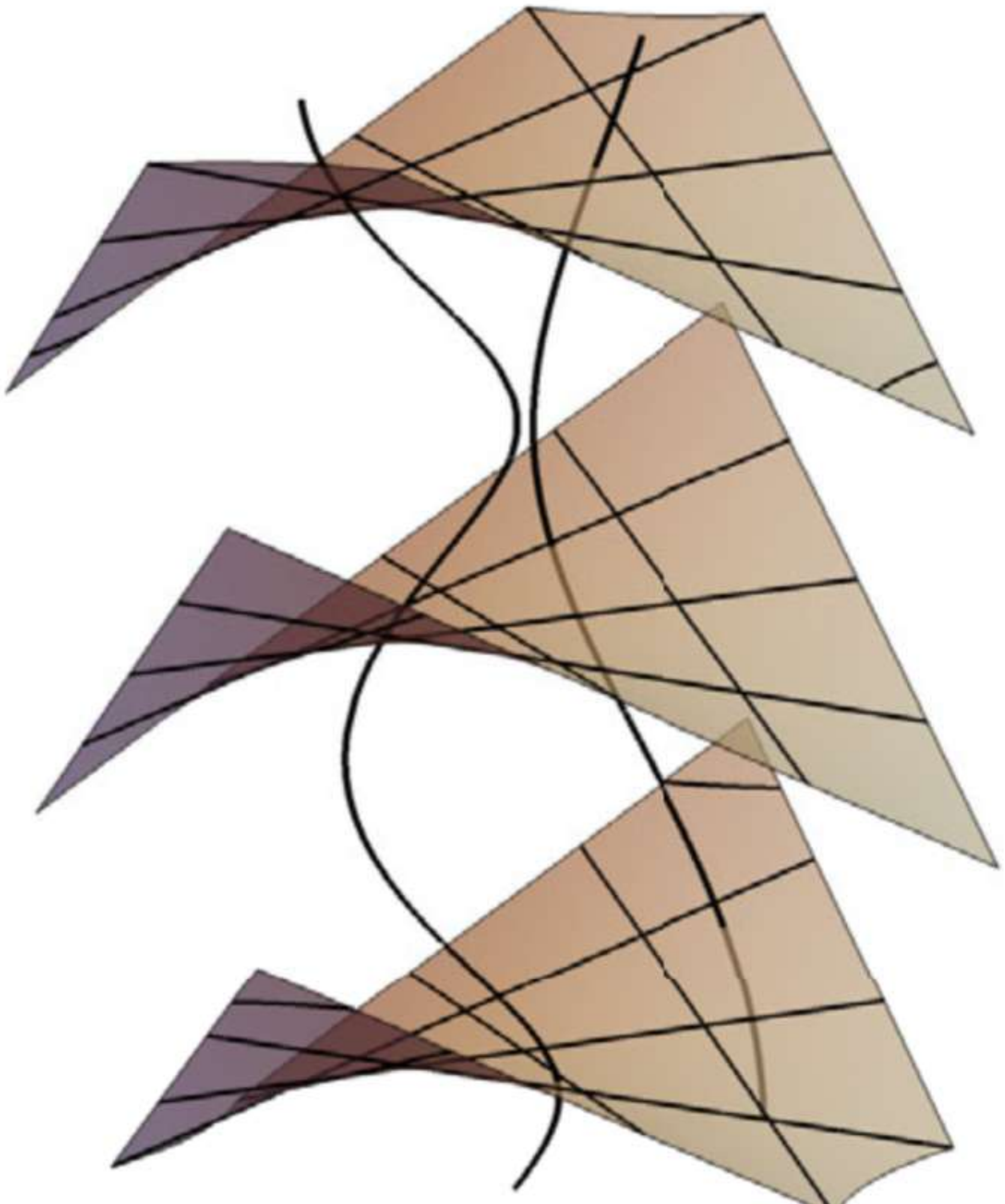


$\mathrm{d}\tau = N\mathrm{d}\tau' \implies \frac{\mathrm{d}}{\mathrm{d}\tau'}\mathcal{O}(q_i, p_i) = \{\mathcal{O}, NC\}_{\mathrm{P.B}}$

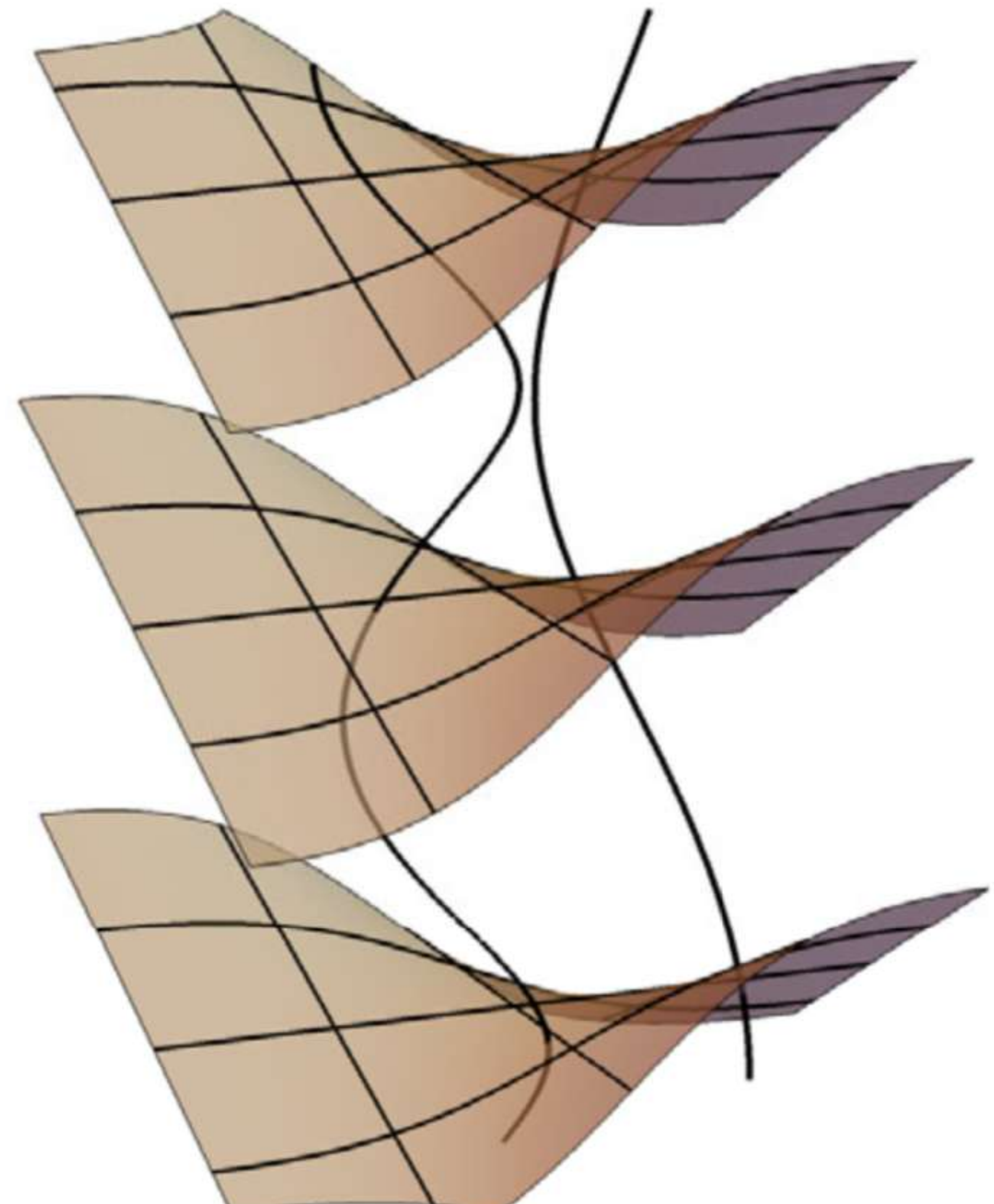
hamiltonian $H = NC$


t
$\bar{t} = t$
$\bar{t} = 2t + 0.3$
p
q

$$\bar{t} = t + qp$$



$$\bar{t} = t - \frac{3qp}{3p^2+1}$$



Quantum system $\hat{C}\Psi(q_i) = 0$

if $\hat{C}$ linear in $\hat{p}_1 = -i\frac{\partial}{\partial q_1}$

↙ will become time

→ $\hat{C} = \hat{p}_1 + \hat{H}(p_2, \cdots, p_n, \{q_i\})$

$\hat{C}\Psi(q_i) = 0 \Longrightarrow$ time dependent Schrödinger equation

Bianchi I case $$\mathrm{d}s^2 = -N^2\mathrm{d}\tau^2 + \sum_{i=1}^{3} a_i^2 \left(\mathrm{d}x^i\right)^2$$

Scale factors
$$\begin{cases} a_1 = e^{\beta_0+\beta_+ + \sqrt{3}\beta_-} \\ a_2 = e^{\beta_0+\beta_+ - \sqrt{3}\beta_-} \\ a_3 = e^{\beta_0 - 2\beta_+} \end{cases}$$

Volume $V \equiv a_1 a_2 a_3 = e^{3\beta_0}$

$$\mathrm{d}\beta_0 = \frac{1}{3} e^{-3\beta_0}\mathrm{d}V$$

Action $$\mathcal{S} = \int \mathrm{d}\tau \Big(\underbrace{p_0\dot\beta_0 + p_+\dot\beta_+ + p_-\dot\beta_-}_{\frac{\mathrm{d}\theta}{\mathrm{d}\tau}} - NC\Big)$$

$\frac{\mathrm{d}\theta}{\mathrm{d}\tau}$ → canonical one-form

NC → H

constraint $$C = \frac{e^{-3\beta_0}}{24}\left(-p_0^2 + p_+^2 + p_-^2\right)$$

ensure canonical one-form remains canonical $p_V \equiv \frac{\mathrm{e}^{-3\beta_0}}{3} p_0$

↓

$$\mathrm{d}\theta = p_V \mathrm{d}V + p_+ \mathrm{d}\beta_+ + p_- \mathrm{d}\beta_-$$

constraint $C = \frac{3V}{8}\left(-p_V^2 + \frac{p_+^2 + p_-^2}{9V^2}\right)$

cyclic variable $\dot{p}_\pm = 0$ set $p_+ = k\cos\alpha$ and $p_- = k\sin\alpha$

$$\longrightarrow \mathrm{d}\theta = p_V \mathrm{d}V + p_k \mathrm{d}k + p_\alpha \mathrm{d}\alpha + \underbrace{\mathrm{d}\left(k\cos\alpha\beta_+ + k\sin\alpha\beta_-\right)}$$

$\to$ exact… ignore!

$$\begin{aligned} p_k &\equiv -\left(\cos\alpha\beta_+ + \sin\alpha\beta_-\right), \\ p_\alpha &\equiv \left(k\sin\alpha\beta_+ - k\cos\alpha\beta_-\right) \end{aligned}$$

neither α nor P_α in $H = NC$

the system reduces to

$$\left\{\begin{array}{lcl} \mathrm{d}\theta &=& p_V \mathrm{d}V + p_k \mathrm{d}k \\ C &=& \frac{3V}{8}\left(-p_V^2 + \frac{k^2}{9V^2}\right) \end{array}\right.$$

Hamilton equations

$$\begin{aligned} \dot{k} &= 0 \\ \dot{p}_k &= -N\frac{k}{12V} \\ \dot{V} &= -N\frac{3Vp_V}{4} \\ \dot{p}_V &= -N\left[\frac{3}{8}\left(-p_V^2 + \frac{k^2}{9V^2}\right) - \frac{k^2}{12V^2}\right] \end{aligned}$$

+ constraint

$$\frac{3V}{8}\left(-p_V^2 + \frac{k^2}{9V^2}\right) = 0$$

→ closed for V and p_V

Choosing a time

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(9\frac{p_k}{k}\right) = -\frac{3}{4}\frac{N}{V}$$ monotonically increasing function

valid time choice $\tau = \frac{9p_k}{k} \quad \Longrightarrow \quad N = -\frac{4}{3}V$

Solving directly in the action $\mathcal{S} = \int \mathrm{d}\theta = \int \mathrm{d}\tau \left(p_V \dot{V} - \frac{V^2 p_V^2}{2}\right)$

H

classical unconstrained one dimensional system

$$\frac{\mathrm{d}}{\mathrm{d}\tau}(V p_V) = 0 \quad \Longrightarrow \quad V p_V = V_0 p_{V0}$$

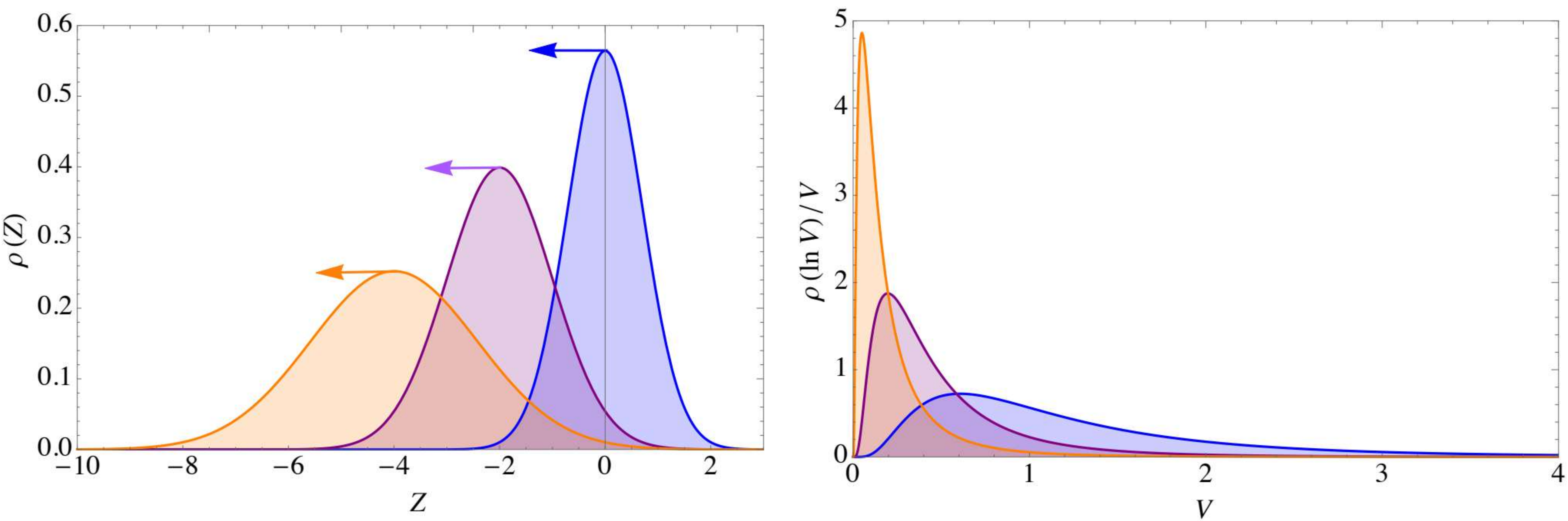
$V = V_0 \mathrm{e}^{(V p_V)\tau}$ and $p_V = p_{V0}\mathrm{e}^{-(V p_V)\cdot\tau}$

symmetric ordering choice

$$H = V^2 p_V^2 \quad \mapsto \quad \hat{H} = \sqrt{V}\frac{1}{i}\partial_V\sqrt{V}\cdot\sqrt{V}\frac{1}{i}\partial_V\sqrt{V}$$

coordinate transformation $V \mapsto Z = \ln V$

$$\longrightarrow \quad U\hat{H}U^{-1} = -\partial_Z^2, \quad \text{and} \quad Z \in \mathbb{R}$$



slow-gauge time

$$\mathrm{d}\theta = (V p_V)\mathrm{d}V - \left(\frac{V^2 p_V^2}{2}\right)\mathrm{d}\left(\frac{9p_k}{k} + \frac{V - \ln V}{V p_V}\right) + \mathrm{d}\left(\frac{9p_k}{2k}V^2 p_V^2 + \frac{1}{2}V \ln V p_V - \frac{1}{2}V^2 p_V\right)$$

→ Action

$$\mathcal{S} = \int \mathrm{d}\theta = \int \mathrm{d}\eta \left(V p_V \acute{V} - \frac{V^2 p_V^2}{2}\right)$$

new time variable $\eta \equiv \frac{9p_k}{k} + \frac{V - \ln V}{V p_V}$

$\acute{V} \equiv \mathrm{d}V/\mathrm{d}\eta$

canonical if

$\pi_V = p_V V$

↓

$$H = \frac{1}{2}V^2 p_V^2 = \frac{1}{2}\pi_V^2$$

freely moving particle...

$(V, \pi_V) \in \mathbb{R}_+ \times \mathbb{R}$

on the half line

Quantization: a gaussian wave packet

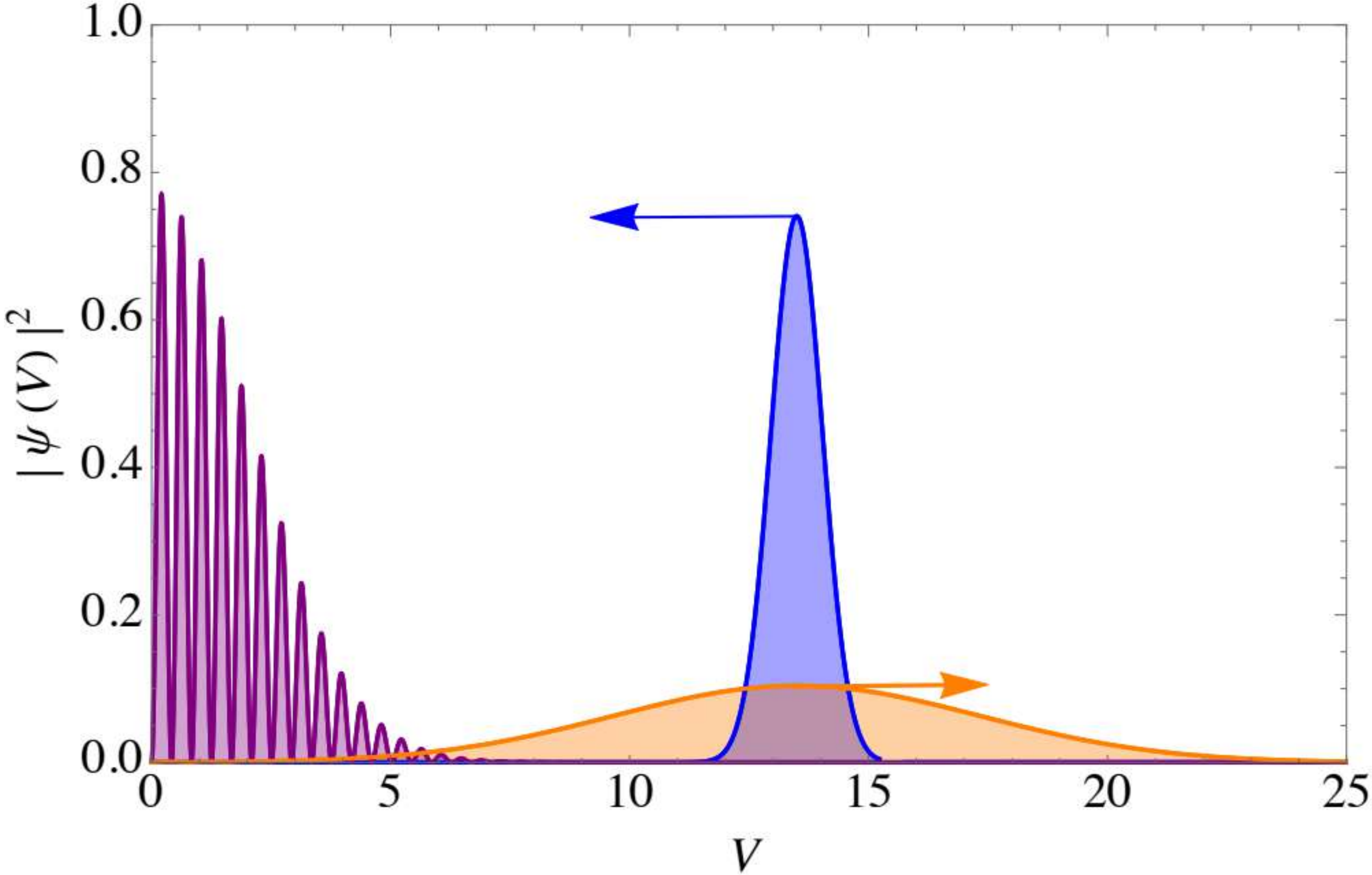
$$u(V,\eta) = \frac{\mathrm{e}^{-k^2/4}}{\sqrt{1+4i\eta}} \exp\left[-\frac{(V-ik/2)^2}{1+4i\eta}\right]$$

implement boundary conditions to ensure self-adjointness

$$\psi(V,\eta) = \frac{u(V+V_0,\eta) - u(-V+V_0,\eta)}{\left[\sqrt{\pi/2}\left(1-\mathrm{e}^{-V_0^2-k^2/2}\right)\right]^{1/2}}$$

→ ***solves the Schrödinger equation***

$$i\frac{\partial}{\partial\eta}\psi = -\,\triangle_D\,\psi$$



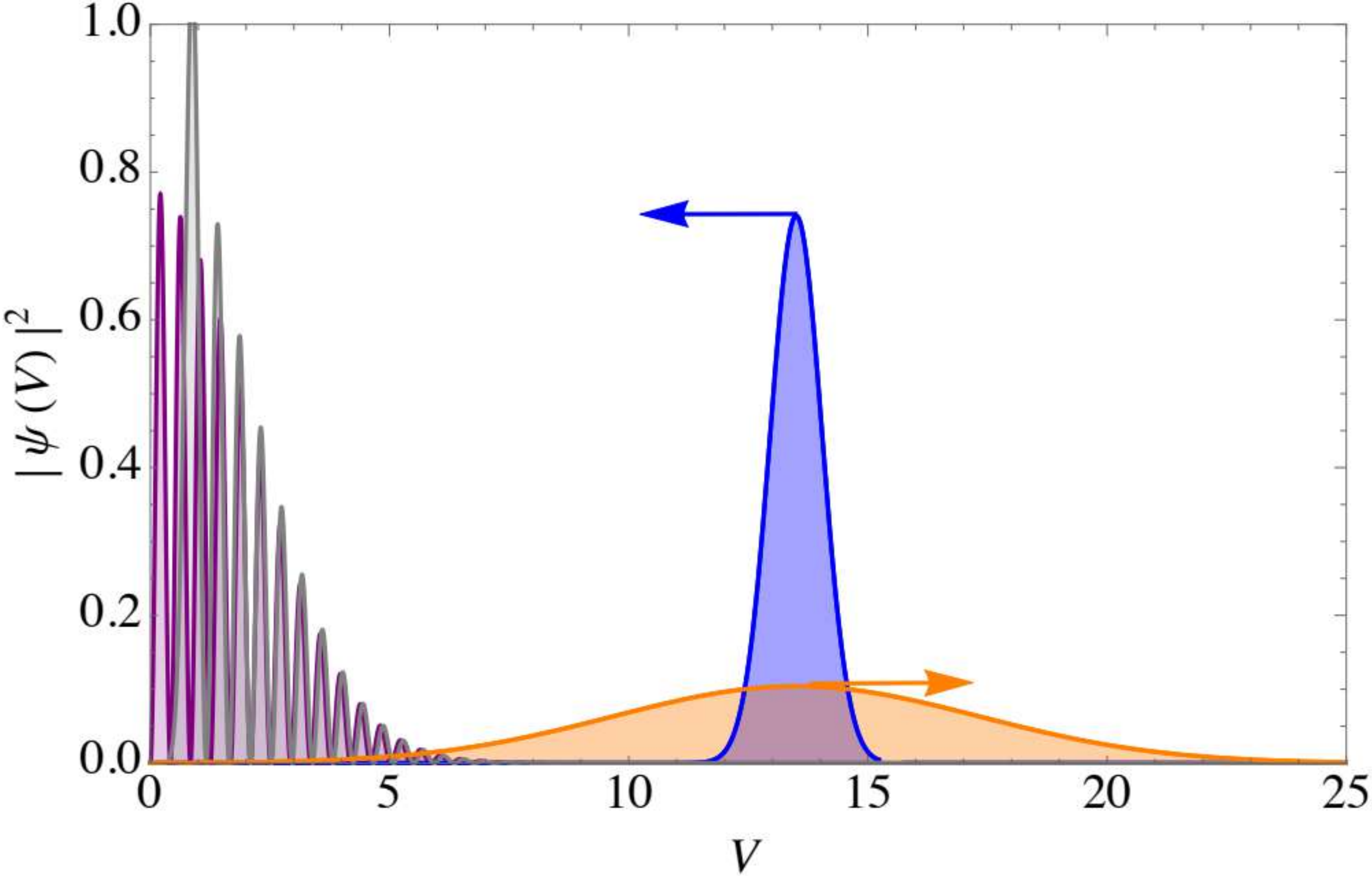
1.0
0.8
0.6
0.4
0.2
0.0
$|\psi(V)|^2$
0
5
10
15
20
25
V

Operator ordering ambiguity $\pi_V^2 \mapsto \hat{V}^s \hat{\pi}_V \hat{V}^{-2s} \hat{\pi}_V \hat{V}^s$

→ $\pi_V^2 \mapsto \hat{\pi}_V^2 + s\hat{V}^{-2}$

↓

self-adjoint hamiltonian on the half-line $s > 3/4$



1.0
0.8
0.6
0.4
0.2
0.0
$|\psi(V)|^2$
0
5
10
15
20
25
V

Closed algebra of operators

$$\begin{cases} [\hat{V}^2, \hat{H}] &= 4i\hat{D}, \\ [\hat{D}, \hat{H}] &= 2i\hat{H}, \\ [\hat{V}^2, \hat{D}] &= 2i\hat{V}^2, \end{cases}$$

$$\hat{D} \equiv \frac{1}{2}\left(\hat{V}\hat{\pi}_V + \hat{\pi}_V\hat{V}\right)$$

→ Heisenberg equations of motion

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{V}^2 = -i[\hat{V}^2, \hat{H}] = 4\hat{D}$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{D} = -i[\hat{D}, \hat{H}] = 2\hat{H}$$

→ Heisenberg equations of motion

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{V}^2 = -i[\hat{V}^2, \hat{H}] = 4\hat{D}$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{D} = -i[\hat{D}, \hat{H}] = 2\hat{H}$$

→ solution as time-dependent operators

$$\hat{D}(\eta) = 2\hat{H}\eta + \hat{D}(0) \quad \Longrightarrow \quad \hat{V}^2 = 4\hat{\eta}^2 + 4\hat{D}(0)\eta + \hat{V}^2(0)$$

expectation values follows similar equations…

→ ***semi-classical variables***

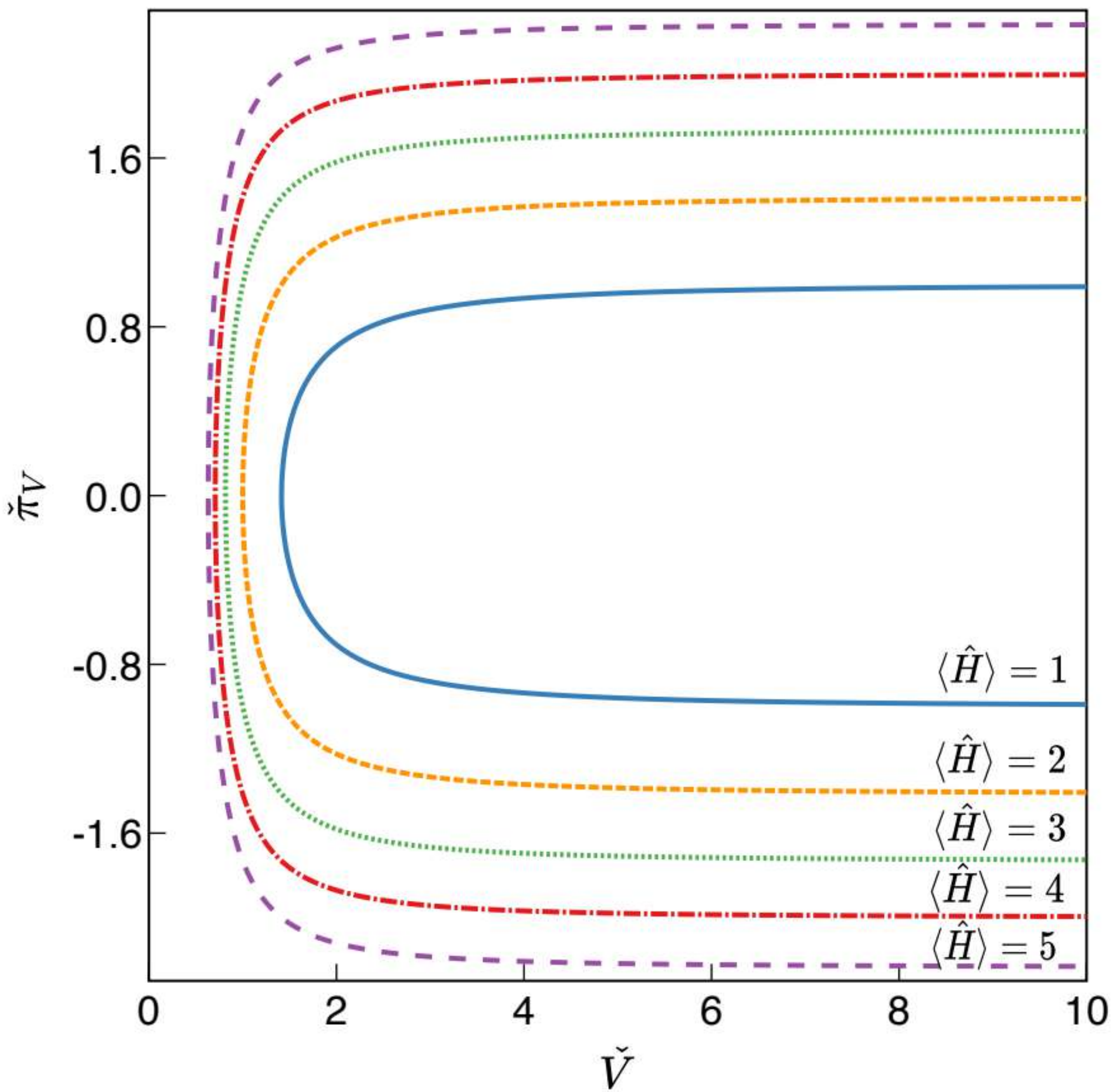
$$\check{V}(t) = \sqrt{\langle \hat{V}^2(t) \rangle}$$

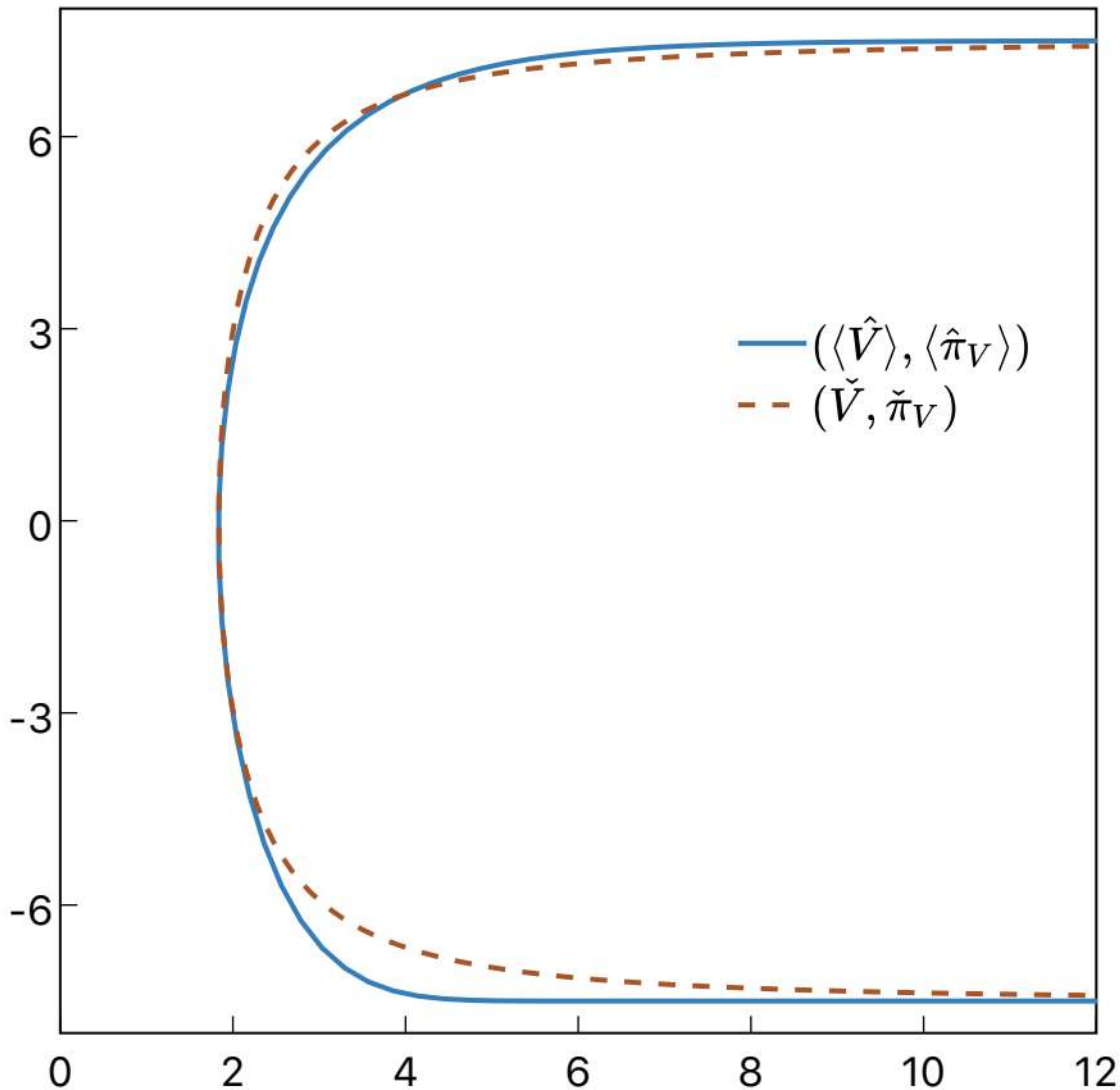
$$\check{\pi}_V(t) = \frac{\langle \hat{D}(t) \rangle}{\check{V}(t)}$$

→ *phase space solution*

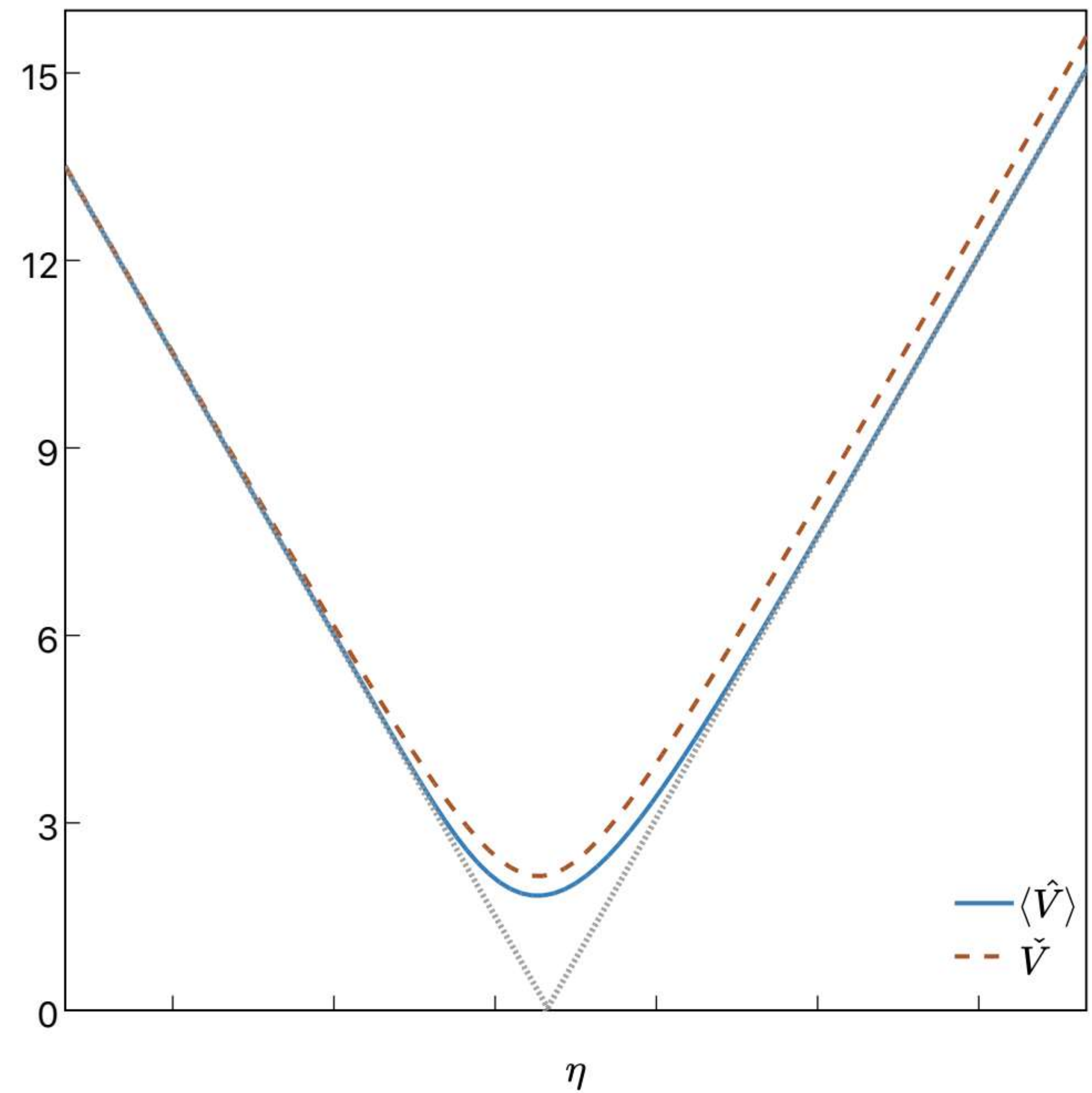
$$\check{V}(t) = \sqrt{4\langle\hat{H}\rangle t^2 + V_0^2},$$
$$\check{\pi}_V(t) = \frac{2\langle\hat{H}\rangle t}{\sqrt{4\langle\hat{H}\rangle t^2 + V_0^2}}.$$

NO SINGULARITY





6
3
0
-3
-6
0
2
4
6
8
10
12
$(\langle\hat{V}\rangle, \langle\hat{\pi}_V\rangle)$
$(\check{V}, \check{\pi}_V)$



15
12
9
6
3
0
η
$\langle \hat{V} \rangle$
$\check{V}$

Changing the time variable $\eta' = \eta'(\eta, V, \pi_V)$

redefining the dynamical variables in the process

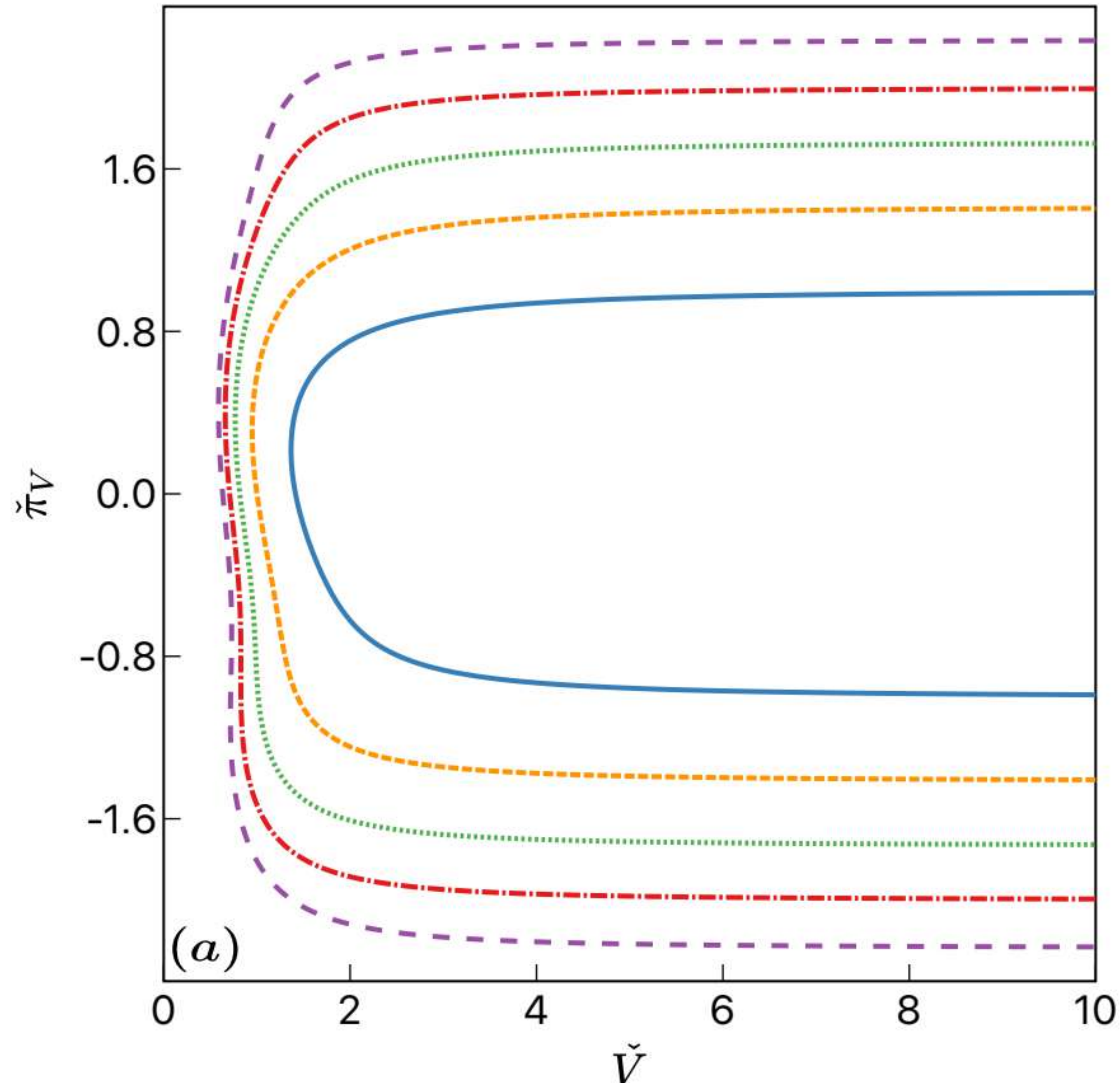
$\pi'_V = \pi_V$ & $V' = V + \pi_V(\eta' - \eta)$ no change of range...

change the canonical one-form

$$\mathrm{d}\theta = \pi_V \mathrm{d}V - \frac{\pi_V^2}{2}\mathrm{d}\eta = \pi'_V \mathrm{d}V' - \frac{\pi_V'^2}{2}\mathrm{d}\eta' + \mathrm{d}\left[(\eta - \eta')\,\frac{\pi_V'^2}{2}\right]$$

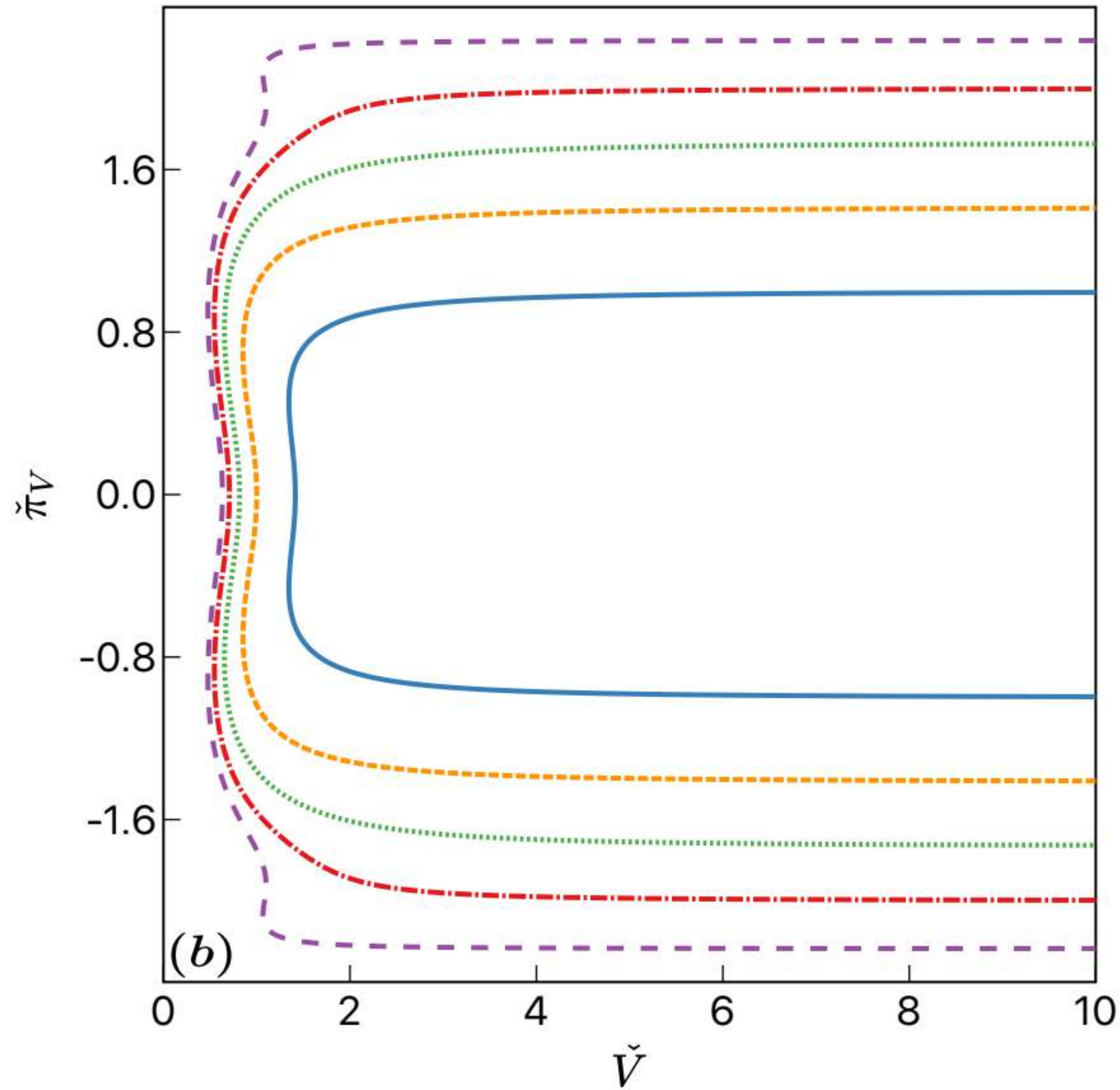
→ ***same system!***

delay function $\Delta(V, \pi_V) = \eta' - \eta$ no dependency on time



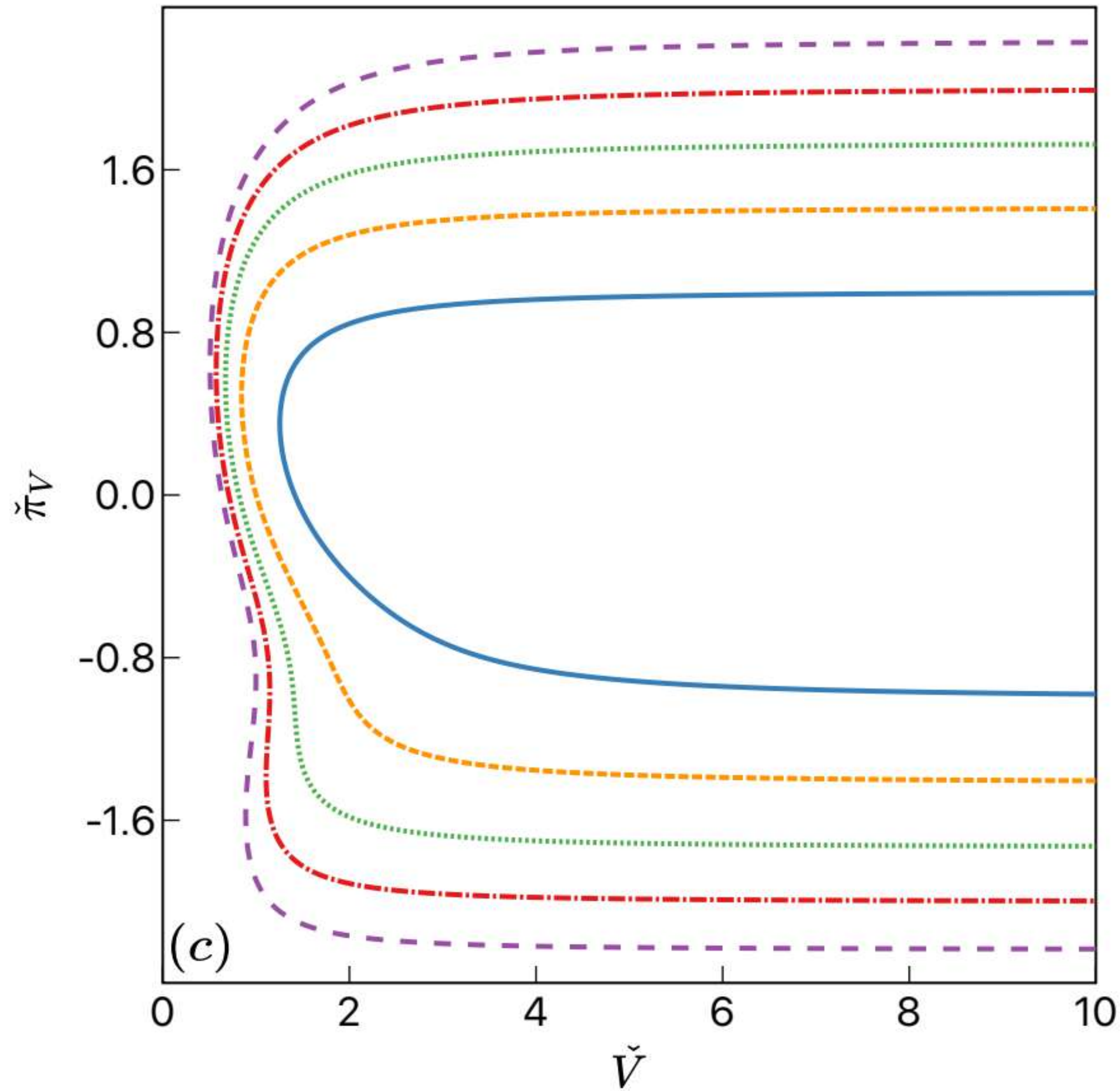
Delay function

$\Delta = V e^{-2|\pi_V|/3} \sin(3V\pi_V)/(10\pi_V)$



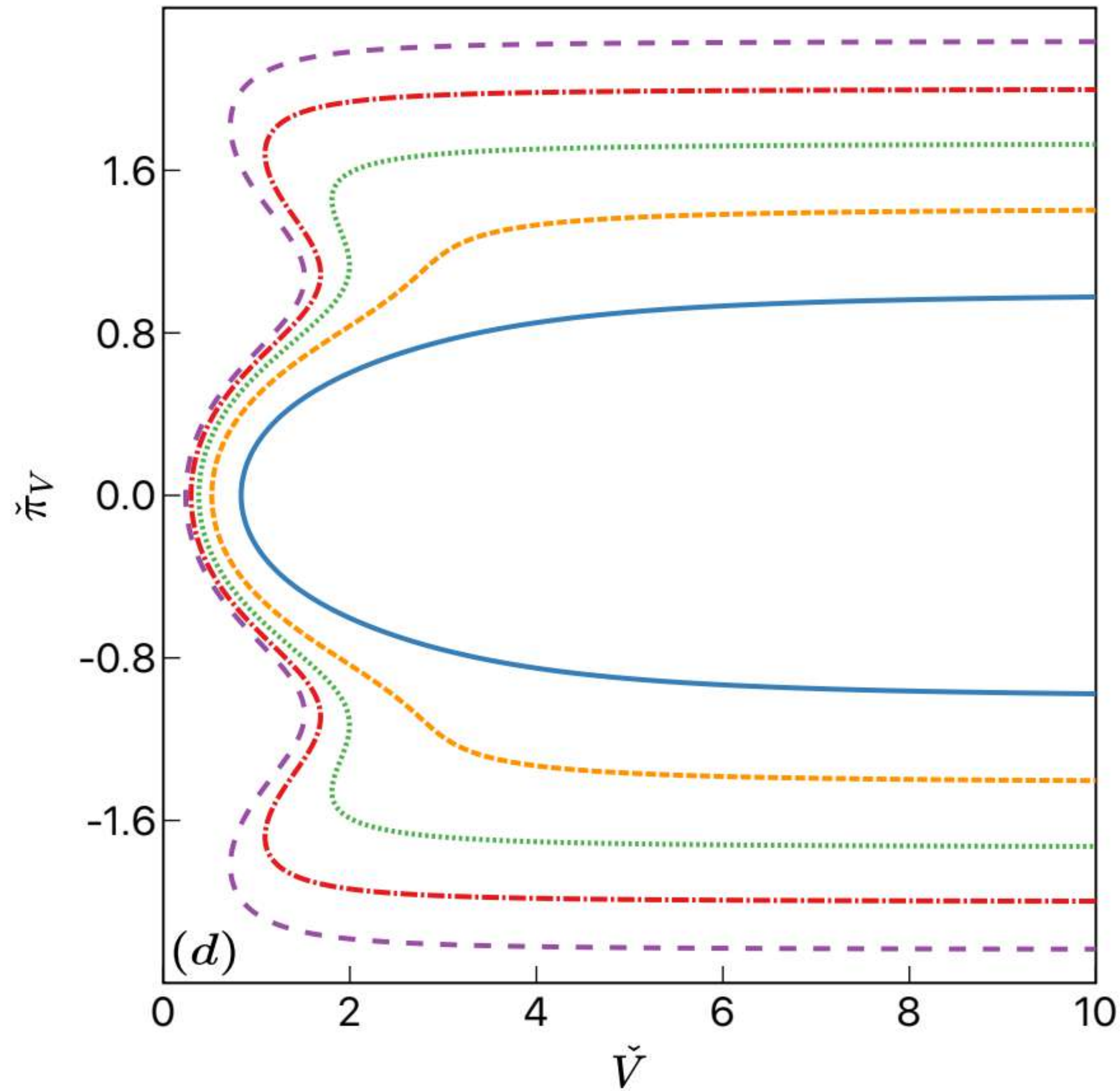
Delay function

$$\Delta = V(\pi_V - 10^{-0.2}\pi_V^3 + \pi_V^5/10)$$



Delay function

$$\Delta = 10^{-0.5} V \sin(2\pi_V)/\pi_V$$



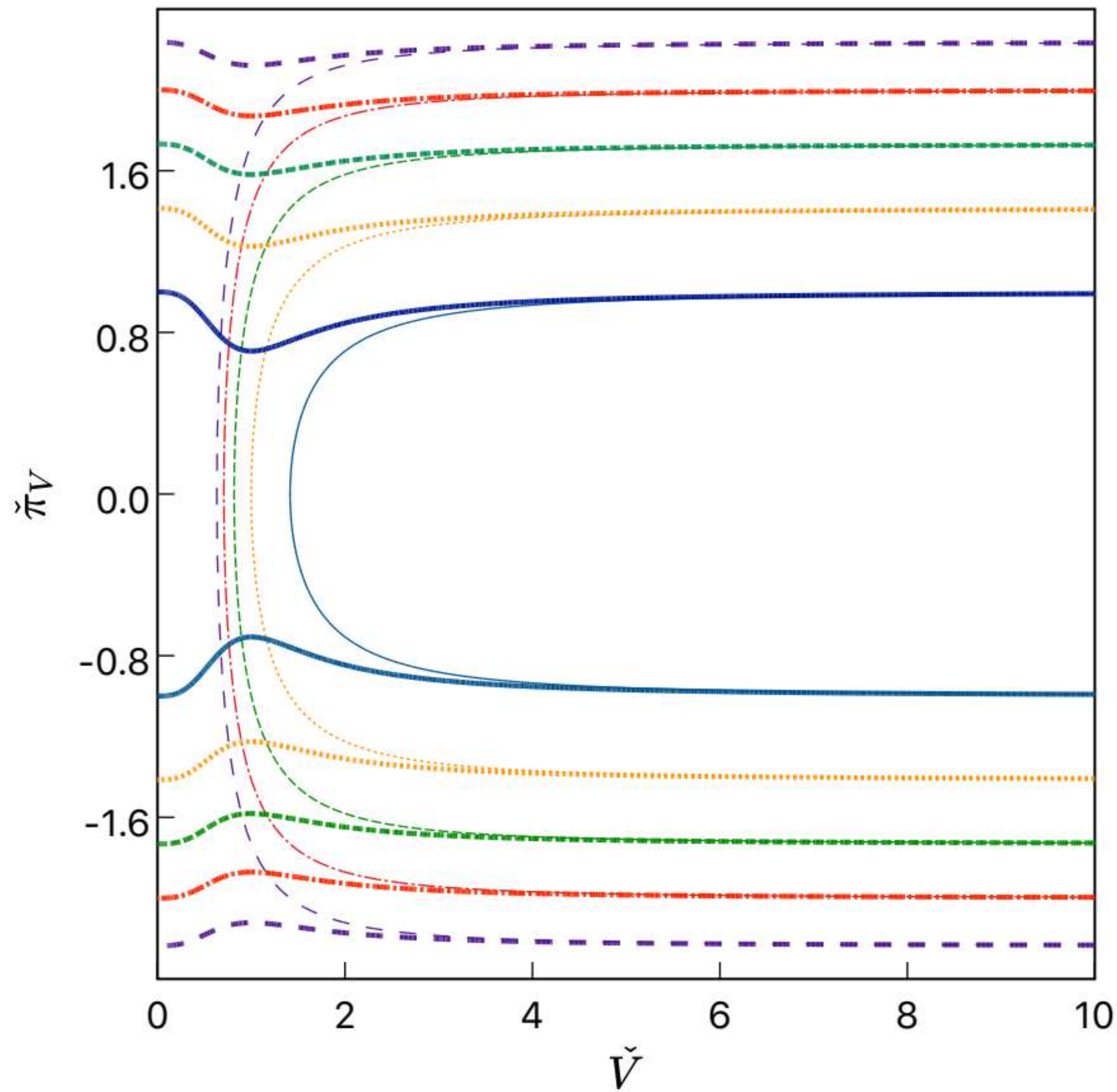
Delay function

$\Delta = 10^{-0.5}(V+1)\cos(3\pi_V)/\pi_V$

Delay function (slow to fast)

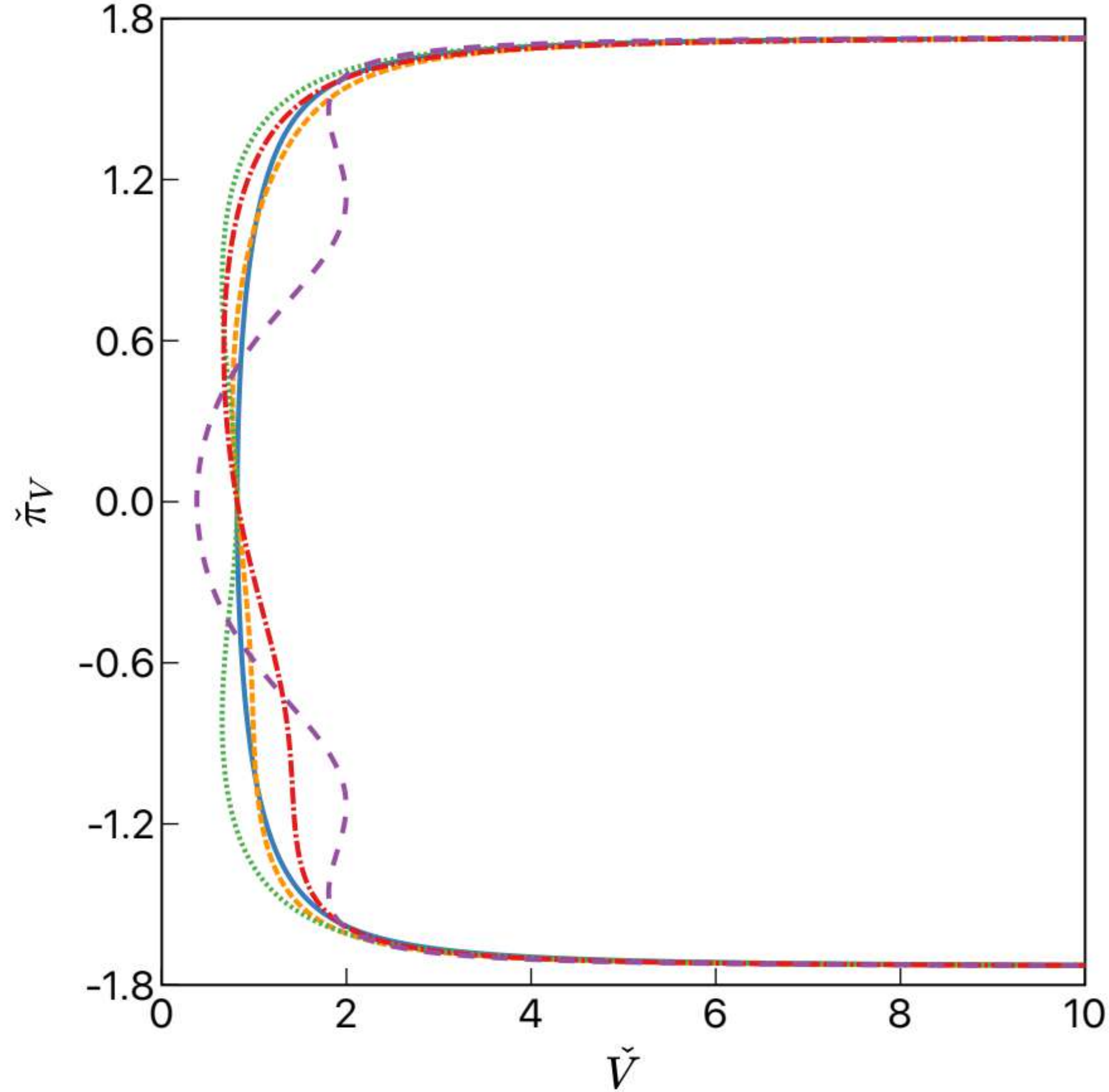
[regular to singular]

$$\Delta_{\mathrm{slow}\to\mathrm{fast}} = \frac{V - \ln V}{V p_V}$$



Comparison between different delay functions

Same asymptotics



S. Vitenti, PP & A. Valentini, *Phys. Rev.* **D 100**, 043506 (2019) [astro-ph 1901.08885]
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Conclusions

- Internal clock formulation of QM & QC
- Clock issue in QC can be approached by WDW and set constraints on time
- Asymptotics may solve the problem… perturbations???
- Other trajectory approach = same asymptotics
- Out-of-Quantum-Equilibrium

modified power spectrum

→ ***Planck best-fit…***

$\ell_{\mathrm{NEW}} \simeq 2000\mathrm{Mpc}$