

# Goos-Hänchen effect in the gaps of photonic crystals

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We show the existence of a Goos-Hänchen effect when a monochromatic beam illuminates a photonic crystal inside a photonic band gap.

*OCIS codes:* 050.0050,230.4170,240.7040      © 2006 Optical Society of

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When a light beam illuminates the interface between two homogeneous media under total internal reflection, the barycenter of the reflected beam does not coincide with that of the incident one: this is the Goos-Hänchen effect.<sup>1</sup> This phenomenon has been analyzed in its many guises, both theoretically<sup>2,3,4,5</sup> and experimentally.<sup>1,6,7,8</sup> In its original form, the incident beam was to come from the medium with higher index, in order to obtain total internal reflection. In this communication, we show that there is also a Goos-Hänchen shift when a monochromatic beam illuminates a photonic crystal, that is, a periodically structured device exhibiting photonic band gaps.<sup>9,10</sup> Since the beams considered in nanophotonic devices are usually very narrow, this effect should be taken into account when designing such structures when the photonic band gap phenomenon is involved. The Goos-Hänchen effect is linked to the variation of the phase of the reflection coefficient with the angle of incidence. In the case of total internal reflection, the existence of evanescent waves explains the variations of the phase. Such an effect can be expected in photonic crystals in photonic band gaps, where the Bloch waves behave much like evanescent ones. The main difficulty is here to consider the correct reflection coefficient.

We deal with 1D (for instance a stack of Bragg mirrors) or 2D photonic crystals (for instance a stack of diffraction gratings periodic in the  $x$  direction), which are finite in the  $y$  direction (located between the  $y = 0$  and the  $y = -h$  planes) and infinite in the  $x$  and  $z$  directions (see fig. 1). We consider harmonic fields with a time dependence of  $\exp(-i\omega t)$ . We denote  $\lambda$  the wavelength in vacuum and  $k_0 = \frac{2\pi}{\lambda}$  the wavenumber in vacuum. Considering only  $z$  invariant fields, the problem of diffraction is reduced to the study of the two usual polarized cases :  $E_{||}$  (electric field linearly

polarized along  $z$ ) and  $H_{||}$  (magnetic field linearly polarized along  $z$ ).

The photonic crystal is illuminated by an incident Gaussian beam:<sup>5</sup>

$$u^i(x, y) = \int A(\alpha, W) e^{i(\alpha x + \sqrt{k_0^2 - \alpha^2} y)} d\alpha \quad (1)$$

where

$$A(\alpha, W) = \frac{W}{2\sqrt{\pi}} e^{-\frac{W^2}{4}(\alpha - \alpha_0)^2}, \quad (2)$$

and  $\alpha_0 = k_0 \sin \theta_0$ , the angle  $\theta_0$  being the mean angle of incidence of the beam (fig. 1).

For a 1D crystal, there is only one reflected (and hence transmitted) order of diffraction. So that for an incident plane wave of wavevector  $\mathbf{k} = k_0(\sin \theta, -\cos \theta)$ , the field outside the crystal can be written :

$$\begin{aligned} u(x, y) &= e^{ik_0(x \sin \theta - y \cos \theta)} + r(k_0, \theta) e^{ik_0(x \sin \theta + y \cos \theta)}, \quad \text{for } y \geq 0 \\ u(x, y) &= t(k_0, \theta) e^{ik_0(x \sin \theta - (y+h) \cos \theta)}, \quad \text{for } y \leq -h \end{aligned}$$

For a 2D crystal, when the period along the  $x$  axis is smaller than  $\frac{1}{2}\lambda$ , there is only one reflected (and hence one transmitted) *propagating* order of diffraction for each plane wave constituting the beam. Since there are evanescent waves, the above expressions for the field are not rigorous any more - they represent an approximation, which holds far enough from the crystal.

It is then possible to characterize the electromagnetic properties of the structure by simply deriving its transfer matrix, the considered structure being a 1D or a 2D

crystal. More precisely,<sup>11</sup> there exists only one real matrix  $\mathbf{T}(k_0, \theta)$  such that :

$$\mathbf{T} \begin{pmatrix} 1 + r \\ i\beta_0(1 - r) \end{pmatrix} = t \begin{pmatrix} 1 \\ i\beta_0 \end{pmatrix}, \quad (3)$$

where we denote  $\beta_0 = k_0 \cos \theta$ . This matrix gives an effective description of the medium, as seen by the incident field.

The matrix  $\mathbf{T}$  is real and has a determinant<sup>11</sup> which is equal to 1. The eigenvalues of  $\mathbf{T}$  are thus the roots of the polynomial  $X^2 - tr(\mathbf{T})X + 1$ , which has real roots if  $|tr(\mathbf{T})| > 2$ . The product of these roots is equal two 1. One of the eigenvalues is smaller than one in modulus and we will denote it  $\mu$ . The other one is then equal to  $\mu^{-1}$ .

For a 1D photonic crystal, let us denote  $T_0$  the transfer matrix for a period. For the whole structure containing  $N$  periods, the transfer matrix  $T$  is equal to  $T_0^N$ . Let us denote  $\kappa$  the eigenvalue of  $T_0$  whose modulus is smaller than one ( $\mu = \kappa^N$ ). Then the amplitude of the field is simply decreased by a factor  $\kappa$  each time it crosses a period of the structure. The field thus behaves like an evanescent wave without being one strictly speaking.

Finally, for 1D and 2D structures  $|tr(\mathbf{T})| > 2$  implies that  $(k_0, \theta)$  is in a forbidden band. In this case, let us denote  $\mathbf{v} = (v_1, v_2)$  (respectively  $\mathbf{w} = (w_1, w_2)$ ) an eigenvector associated with  $\mu$  (resp.  $\mu^{-1}$ ). Solving equation (3), we obtain the following form for the reflection and transmission coefficients

$$r(k_0, \theta) = \frac{(\mu - 1)f}{\mu^2 - g^{-1}f}, \quad t(k_0, \theta) = \frac{\mu(1 - g^{-1}f)}{\mu^2 - g^{-1}f} \quad (4)$$

where the functions  $f$  and  $g$  are defined by

$$g(k_0, \theta) = \frac{i\beta_0 v_1 - v_2}{i\beta_0 v_1 + v_2}, \quad f(k_0, \theta) = \frac{i\beta_0 w_1 - w_2}{i\beta_0 w_1 + w_2} \quad (5)$$

Since  $\mu < 1$ , then  $\mu^2 \frac{g}{f} < 1$  and  $\frac{1}{1 - \mu^2 \frac{g}{f}}$  can be considered as an infinite sum. Thus the coefficients become<sup>12</sup>

$$r(k_0, \theta) = g + (g - f) \sum_{m=1}^{+\infty} \mu^{2m} g^m f^{-m} \quad (6)$$

$$t(k_0, \theta) = (1 - gf^{-1}) \mu \sum_{m=0}^{+\infty} \mu^{2m} g^m f^{-m} \quad (7)$$

The physical meaning of these series is well-known, it represents the multiple reflections<sup>12</sup> inside the photonic crystals on the  $y = 0$  and  $y = -h$  planes, leading to the fact that infinitely many beams are transmitted and reflected (though of course, with rapidly decreasing amplitude). Here we are only interested in the first reflected beam. The above result means that this beam behaves for 1D crystal as if the structure was semi-infinite (since  $g$  is the reflection coefficient of the semi-infinite crystal). For a 2D crystal,  $g$  is not exactly the reflection coefficient of a semi-infinite structure though it tends towards this coefficient when  $h \rightarrow +\infty$ .

The beam can finally be written

$$u^d(x, y) = \int A(k_0 \sin \theta, W) g(k_0, \theta) e^{ik_0(x \sin \theta + y \cos \theta)} \cos \theta d\theta. \quad (8)$$

As  $\mathbf{T}$  is a real matrix and  $\mu$  is real as well,  $v_1$  and  $v_2$  are real too and hence  $|g| = 1$ . Then  $g$  can be written under the form

$$g(k_0, \theta) = \exp(i\phi(k_0, \theta)). \quad (9)$$

The Goos-Hänchen shift is the distance between the centers of the incident and reflected beams. Since the center of the incident beam is located at  $x = 0$  the shift can

be written :

$$G_r = \frac{\int x |u^d(x, 0)|^2 dx}{\int |u^d(x, 0)|^2 dx}, \quad (10)$$

using Parseval-Plancherel lemma, we get :

$$G_r = -\frac{\int A^2(\theta, k_0 W) \partial_\alpha \phi(\alpha) \cos \theta d\theta}{\int A^2(\theta, k_0 W) \cos \theta d\theta} \quad (11)$$

and assuming a sufficiently large waist, we get :

$$G_r \sim - (k_0 \cos \theta)^{-1} \frac{\partial \phi}{\partial \theta}. \quad (12)$$

This result is formally identical to that obtained for homogeneous media.

**Remark :** In fact, it can be shown that expression (6) is still valid for  $(k_0, \theta)$  outside the gap, in which case  $g$  is still defined as in (5) but is no longer of modulus one. In this case  $\bar{g} = f$  and  $g$  is chosen such that  $|g| < 1$ . Moreover  $g$  is a continuous function of  $(k_0, \theta)$ .

We have computed the Goos-Hänchen shift for a 1D photonic crystal, illuminated by a gaussian beam presenting a 50 degrees angle of incidence. The crystal is presented figure and the shift versus the wavelength on figure . As could be expected, the shift is important in the gaps, and more particularly, presents a peak at the left side of each gap, due to a swift variation of the phase of  $g$ . This phenomenon has much in common with what happens in the case of total internal reflexion. The phase of the reflection coefficient indeed presents such a behavior near the limit angle. Let us consider the Goos-Hänchen shift when  $\lambda/d = 10$  is fixed and when the angle of incidence may vary. It can be seen on figure that small angles corresponds to couples  $(k_0, \theta)$  outside

the gap. But when the angle of incidence increases, the structure enters the gap.<sup>13</sup> At the very edge of the gap, the phase is subject to rapid variations, leading to a great shift of the outgoing beam just like in the case of total internal reflection.<sup>5</sup>

We have theoretically demonstrated the existence of a Goos-Hänchen effect in the gaps of photonic crystals and provided theoretical tools to deal with such an effect. We have exhibited the function defined by equation (5) which is the correct reflection coefficient to be considered. Our numerical computations for a one dimensional photonic crystal show that the shift can indeed be found for values of  $\lambda$  and  $\theta$  in a gap. The shift is very important when entering the gap by making either  $\lambda$  or  $\theta$  vary. In the latter case, the phenomenon has much in common with the total internal reflection near the limit angle. This effect could play an important role in structures such as that described in<sup>14</sup> where a fine knowledge of the trajectories of the reflected or refracted beams is needed for the structure to work properly.

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## List of figure caption

Fig. 1. One period of the photonic crystal consists in two layers of height  $d$  and of permittivity  $\varepsilon_1 = 11.56$  and  $\varepsilon_2 = 1$ . The Goos-Hänchen shift is the distance between the centers of the incident and reflected beams. schema.eps.

Fig. 2. Goos-Hänchen shift for a gaussian beam of waist  $10\lambda$  under a  $50^\circ$  angle of incidence for a wavelength  $\lambda/d \in [4.5, 18]$  (the height of a layer being of size 1). The dash-dot line represents  $|g|$  so that the gaps, characterized by  $|g| = 1$ , can easily be identified by the reader. The dashed line represents the derivative of the phase of  $g$  which can barely be distinguished of the shift for a gaussian beam (solid line). lambda.eps

Fig. 3. The Goos-Hänchen shift (normalized by  $\lambda = 10$ ) for a gaussian beam is presented for an angle of incidence  $\theta \in [0, 70]$ . The dash-dotted line represents  $|g|$ . angle.eps.

# Goos-Hanchen Shift





