Totally asymmetric simple exclusion process on networks

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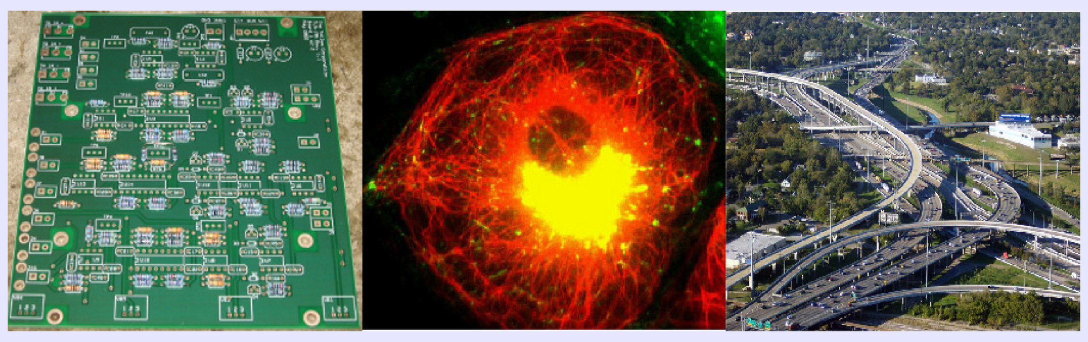
1 Transport processes on networks

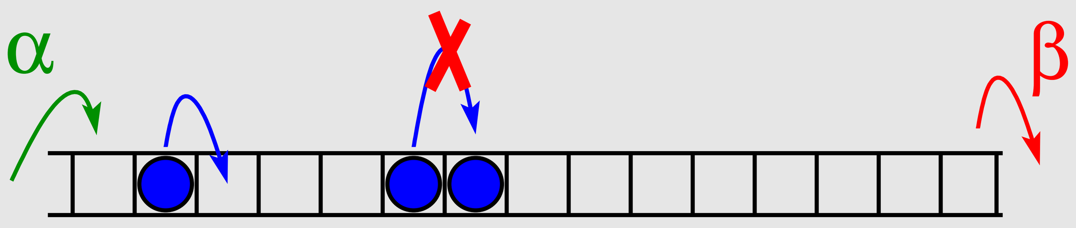


- Do the transport characteristics depend significantly on the **topology** of a network?
- Driven non-interacting diffusive systems have a rich history in physics e.g. *Kirchoff (1847)*[1] In these systems network structure plays a minor role.
- What happens in strongly **interacting** systems, e.g. vehicular traffic, proteins moving on the cytoskeleton?

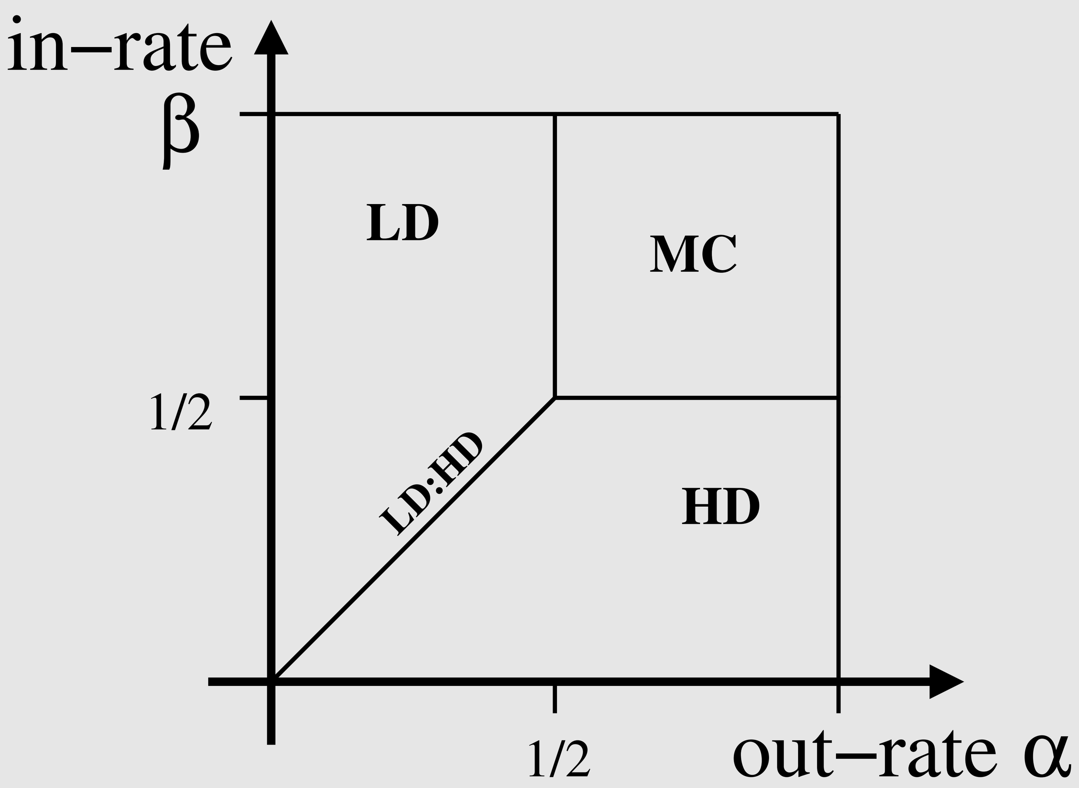
2 TASEP: a paradigmatic model

2.1 Phase diagram

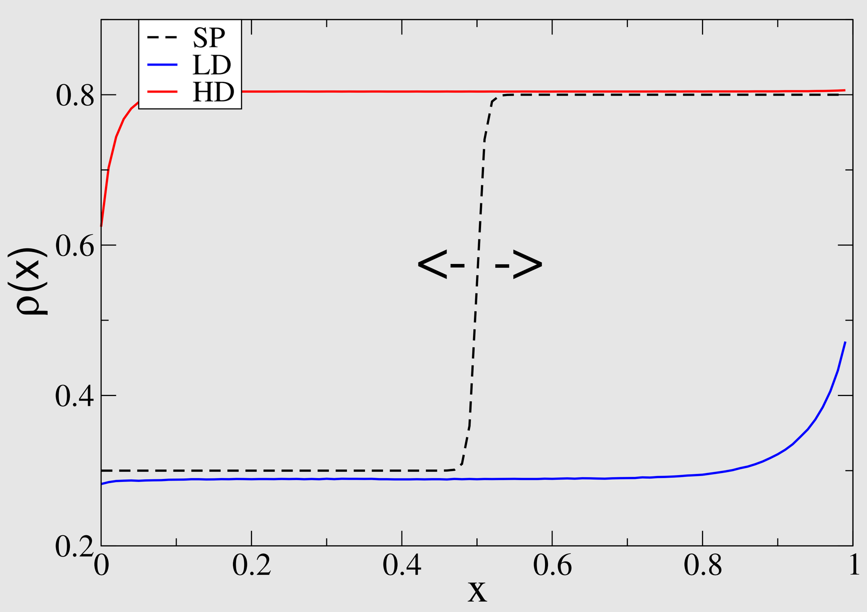
Minimal model for stochastic out-of-equilibruim transport with **excluded volume interactions** between the particles [2]



The one-dimensional TASEP has a first-order phase transitions between a homogenuous **low density** (LD) phase and a **high density** phase (HD). On the coexistence line a shock between a high and a low density region diffuses through the system. We speak of a **shock** phase (SP or LD:HD).



Three phases:



2.2 Current-density profiles

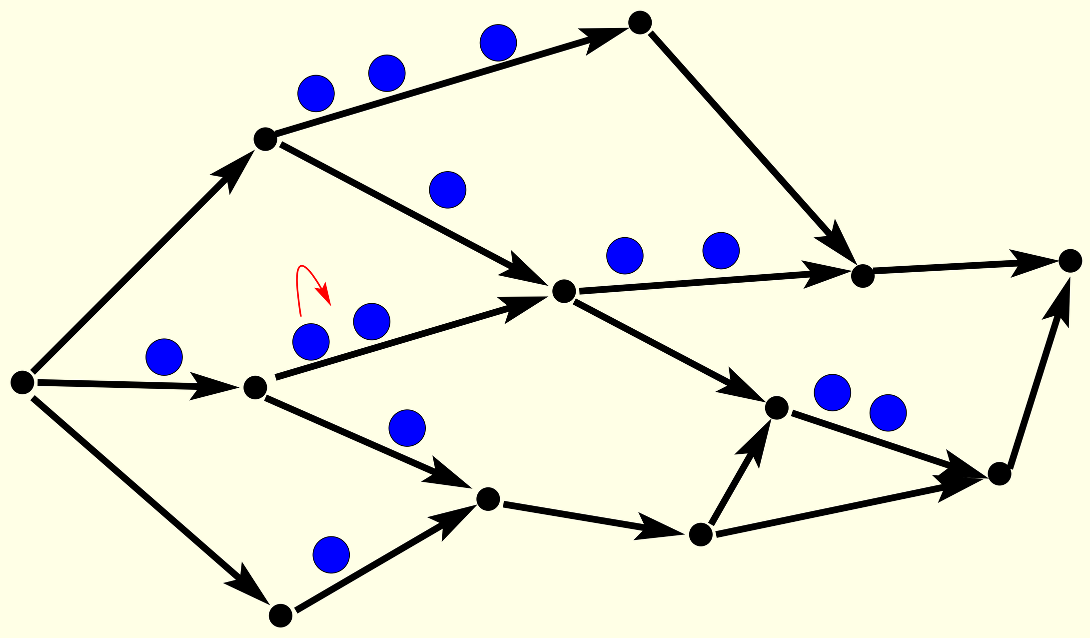
The **density** ρ follows

$$\rho(\alpha,\beta) = \begin{cases} \alpha & \alpha \leq \beta, \alpha < 1/2 \quad \text{(LD)} \\ 1-\beta & \beta \leq \alpha, \beta < 1/2 \quad \text{(HD)} \\ 1/2 & \alpha, \beta \geq 1/2 \quad \text{(MC)} \end{cases}$$

with the **current** J given by the parabolic current-density relation

$$J(\alpha,\beta) = \rho(\alpha,\beta)\left(1-\rho(\alpha,\beta)\right).$$

3 Generalizing TASEP to complex networks



MF algorithm [4] which solves the **continuity** equation

$$\frac{\partial \rho_v}{\partial t} = \sum_{v'\to v} J_{(v',v)} - \sum_{v\leftarrow v''} J_{(v,v'')},$$

using **effective rates** [3]

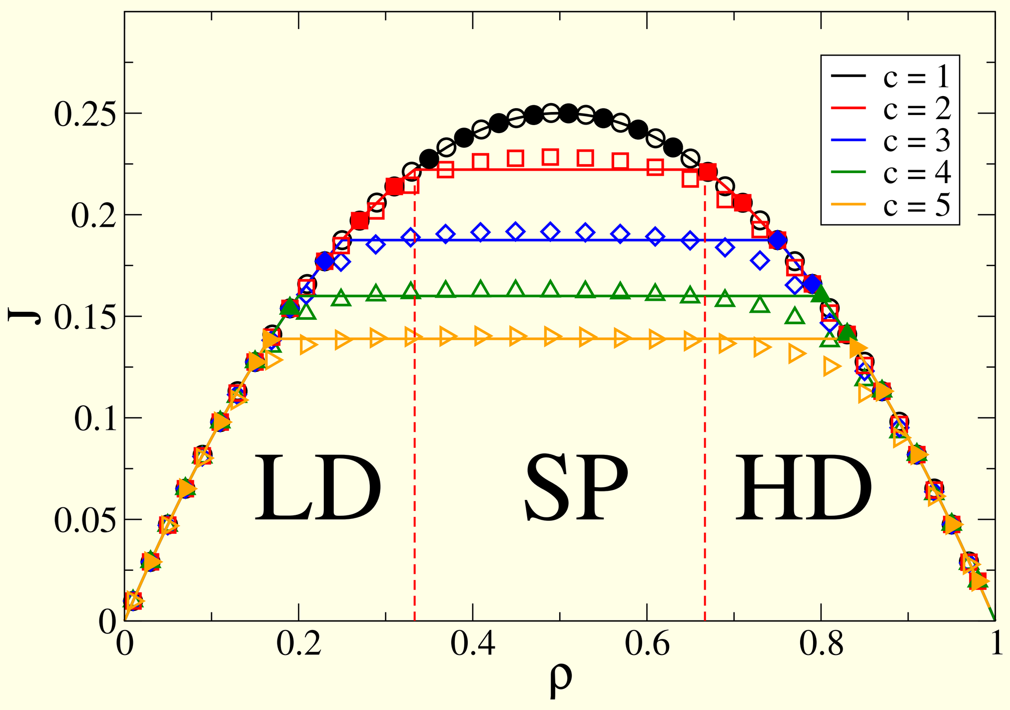
$$\alpha = \rho_{v'}/k^{\text{out}}_{v'} \quad \text{and} \quad \beta = 1-\rho_{v''},$$

and **homogeneity** in the segments

$$\frac{\partial \rho_v}{\partial t} = \sum_{v'\to v} J\left(\frac{\rho_{v'}}{k^{\text{out}}_{v'}}, 1-\rho_v\right) - \sum_{v''\leftarrow v} J\left(\frac{\rho_v}{k^{\text{out}}_{v}}, 1-\rho_{v''}\right).$$

3.1 Regular networks

The current-density profile $J(\rho)$ is a truncated parabola



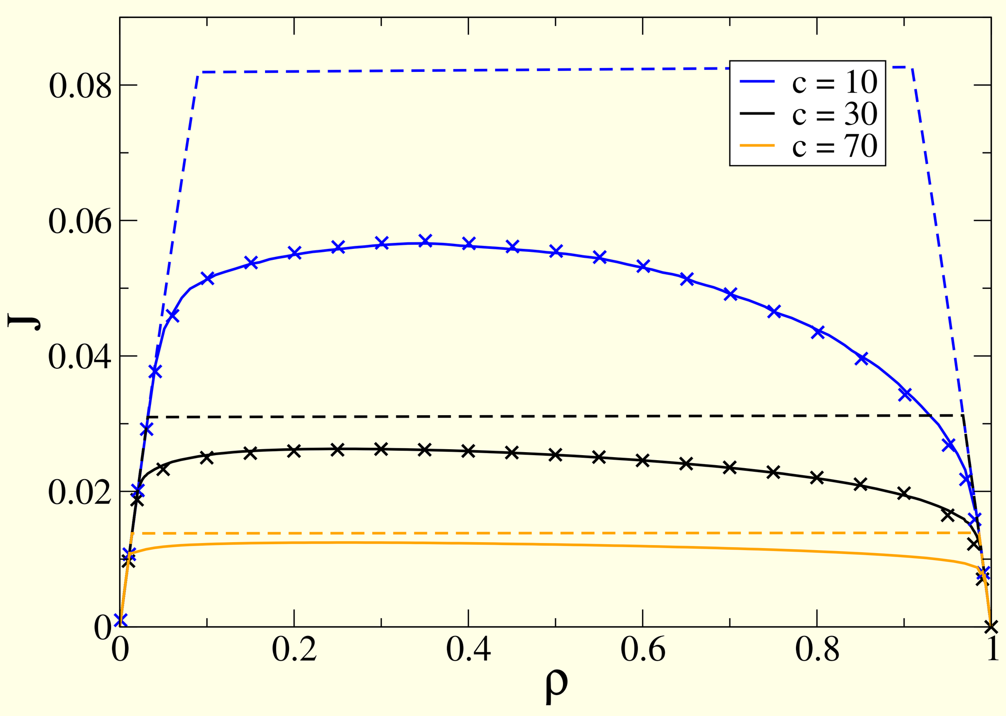
The **plateau** corresponds with a SP and widens, when the number of links leaving from the junctions c, increases.

$$J(\rho) = \begin{cases} \frac{c}{(c+1)^2} & \text{for} \quad \rho^* < \rho < 1-\rho^*[\rho^*, 1-\rho^*] \\ \rho(1-\rho) & \text{otherwise} \end{cases},$$

where $\rho^* = 1/(c+1)$.

3.2 Irregular networks

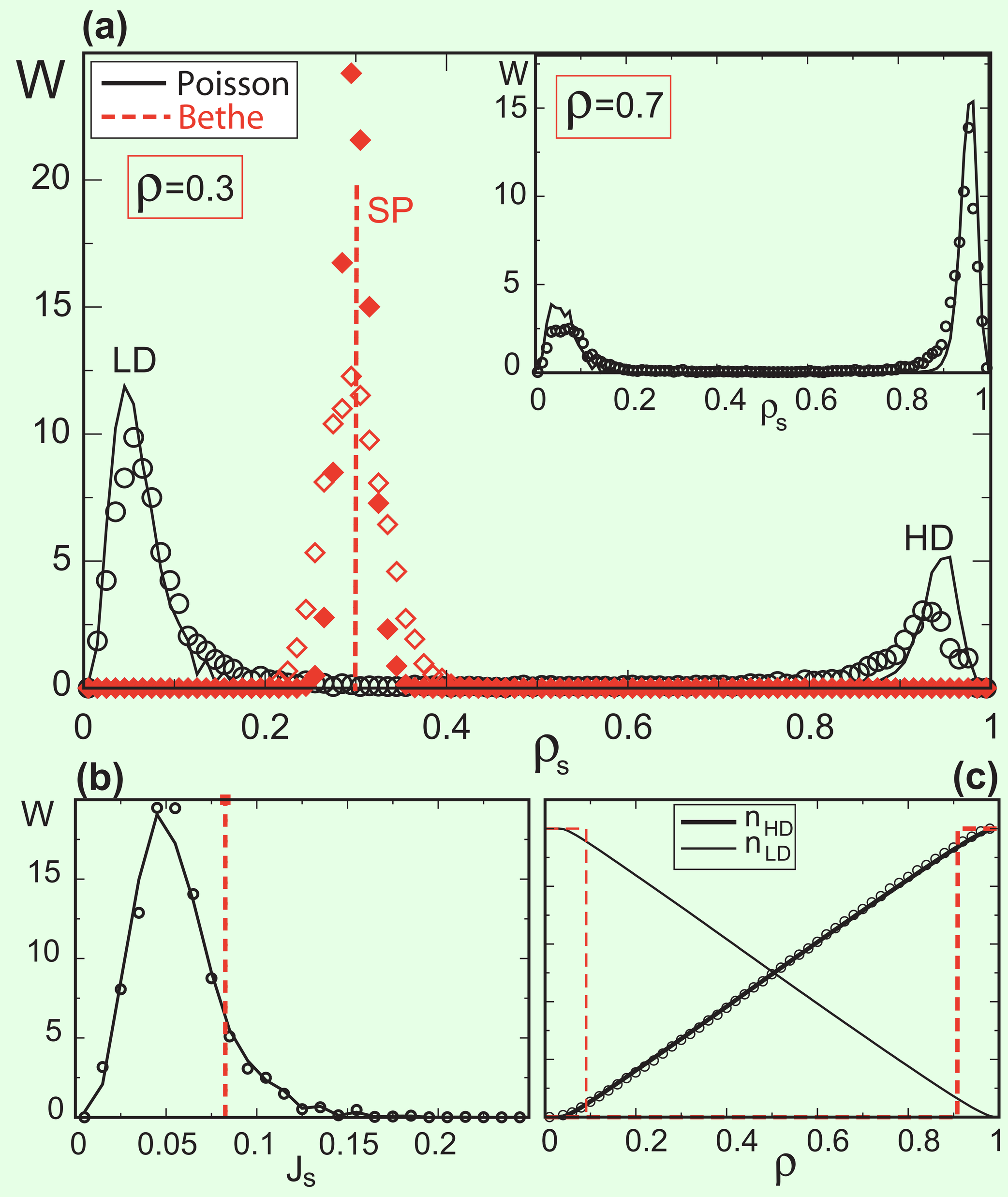
The MF algorithm determines the sample to sample fluctuations



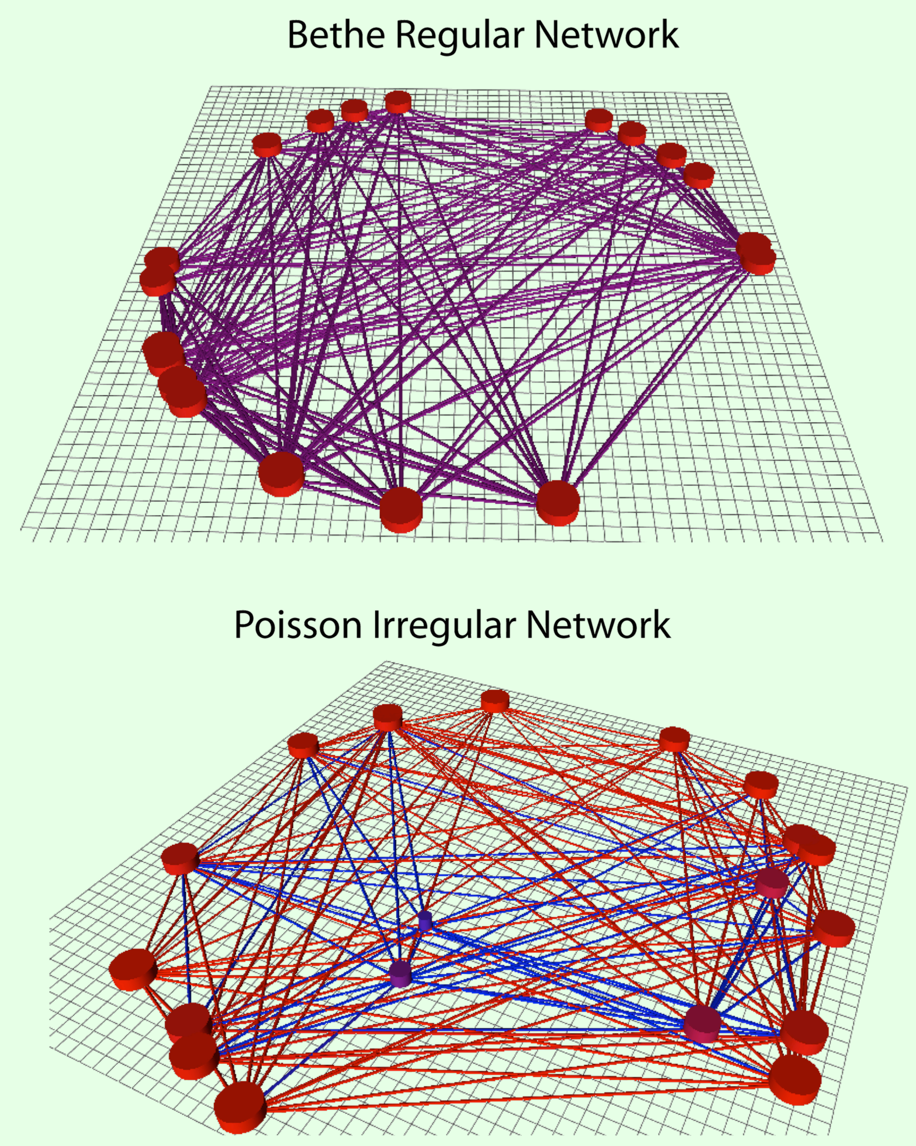
The **plateau**-like region is **absent**! Does the SP not appear in irregular networks? (dashed lines indicate the current-density profile for the corresponding regular graph)

4 Regular vs. Irregular networks

4.1 Distribution of currents



4.2 Graphical representation



Average density $\rho = 0.5$. We thank A. Chesseron for the artwork.

4.3 General networks at high connectivites

Using the concept of **bottlenecks** (i.e. the junction nodes will accumulate particles and have $\rho \approx 1$) the bimodal/unimodal distributions can be rationalized for general networks with high c.

5 Conclusions

Main results:

- Our mean field method determines how transport characteristics of large scale networks depend on their topology
- Regular systems: unimodal density distributions, all segments in shock phase
- Irregular systems: bimodal density distributions, a fraction of segments at very low density and a fraction at very high density

Interesting open questions:

- How general are our results? Is TASEP not a too simple model to account for transport in biological processes? Can we find bimodality in real data?
- What is the benefit of having a bimodal distribution? Can the cytoskeleton regulate the fraction of LD and HD segments to optimize certain functions?

References

[1] G. Kirchoff, *Ueber die auflsung der gleichungen, auf welche man bei der untersuchung der linearen vertheilung galvanischer strme gefhrt wird*, Ann. Phys. Chem, pp. 497-508 (1847)

[2] C. T. MacDonald, J. H. Gibbs, and A. C. Pipkin, "*Kinetics of Biopolymerization on Nucleic Acid Templates*, Biopolymers **6**, 1 (1968).

[3] B. Embley, A. Parmeggiani, N. Kern, *Understanding totally asymmetric simple-exclusion-process transport on networks: generic analysis via effective rates and explicit vertices*, Phys. Rev. E **80**, 041128 (2009)

[4] I. Neri, N. Kern, A. Parmeggiani, *Totally Asymmetric Simple Exclusion Process on Networks*, Phys. Rev. Lett. **107**, 068702 (2011)