Halo statistics from excursion set theory with correlated steps

arXiv: 1201.3876 with Ravi Sheth — and work in progress w/ R. Sheth and A. Paranjape

- Competitive constraints on primordial NG (and other cosm. param.) from LSS
- # counts (mass function) probes smaller scales than corr. funct.

One needs theory:

- Sims are heavy
- GR effects? QCDM? Mod. gravity??
- motivated fits to simulations

Moreover:

- Interdisciplinary playground (QFT and Stats to Astro)
- Beautiful example of poor assumptions giving good predictions
- Use of linear theory to make non-linear statements

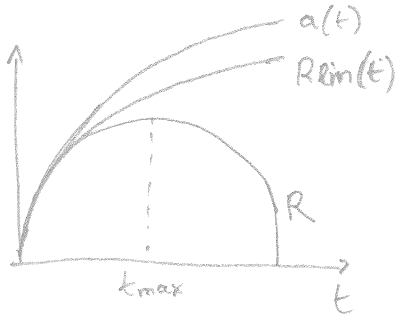
Primordial NG: $g_{ij} = a^2 e^{2\zeta}(\delta_{ij} + \gamma_{ij})$; $\gamma_{ii} = 0$ $\partial_i \gamma_{ij} = 0$

$\zeta = \zeta_G + f_{NL}^{local}(\zeta_G^2 - \langle \zeta_G^2 \rangle)$ $\qquad \zeta \sim \phi$ (+ transfer function)

$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}} \propto \nabla^2 \phi \sim \nabla^2 \zeta$ is a probe of f_{NL}

$\langle \delta^3 \rangle_c \sim 3 f_{NL} \langle (\nabla^2 \zeta)^2 \nabla^2 \zeta^2 \rangle$

Spherical collapse: $\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_{in}\left(\frac{R_{in}}{R}\right)^3 - \frac{k}{R^2}$



$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \approx \frac{\delta\rho}{\bar{\rho}} \approx -3\frac{\delta R}{a}$ $\delta_{lin} = \frac{3}{20}\left(\frac{6\pi t}{t_{max}}\right)^{3/2}$

$t_{vir} \sim t_{coll} \sim 2t_{max}$ $\delta_{lin}(t_{vir}) \approx 1.686 \equiv \delta_c$

Objects form when $\delta_{lin} \approx 1.69$ (but true $\delta \sim 100\,\delta_{lin}$)

Indicator for clusters, not estimator of δ

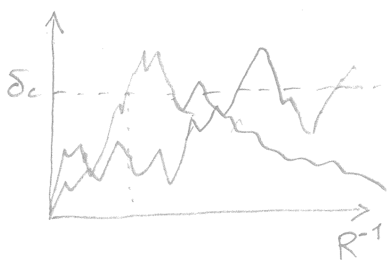
Universe with small inhomogeneities: $\delta_{lin}(x, t_{in})$

Evolve linearly. The real δ will become non-linear and form structures, but i don't care. Clusters still form where $\delta_{lin}(x, t_{end}) > \delta_c$.

With what SIZE? The largest R for which

$$\delta_R(x) = \frac{1}{V}\int d^3y\, W\left(\frac{\vec{y}-\vec{x}}{R}\right)\delta_{lin}(y, t_{end}) > \delta_c \qquad (M \propto R^3)$$

Random walks:



$S_R \equiv \langle \delta_R^2 \rangle$

First crossing at scale S_R $\longleftrightarrow$ halo of mass $M \propto R^3$

Abundance $n(M)$ $\longleftrightarrow$ first crossing rate $f(s)$

Diffusion equation: $\dot{P} = \frac{1}{2}\partial_\delta^2 P + \frac{\langle \delta^3 \rangle_c}{3!}\partial_\delta^3 P + \dots$

Gaussian process: $P(\delta, s) = \frac{e^{-\delta^2/2s}}{\sqrt{2\pi s}}$

Crossing vs First Crossing Rate

Rate $\quad f(s) = -\frac{d}{ds}\int_{-\infty}^{\delta_c} d\delta\, P(\delta, s) \quad (*)$ $\quad$ (Press-Schechter 74)

Gaussian: $\quad f(s) = -\frac{1}{2}\frac{\partial}{\partial \delta_c} P(\delta_c) = \frac{\nu}{2s}\frac{e^{-\nu^2/2}}{\sqrt{2\pi}} \qquad \nu = \frac{\delta_c}{\sqrt{s}}$

NG, but $P(\delta/\sqrt{s})$: $\quad f(s) = -\frac{d\nu}{ds} P(\nu) = \frac{\nu}{2s} P(\nu)$

probe of NG!! $\longrightarrow P(\nu) = \frac{e^{-\frac{\nu^2}{2} + \frac{\langle \delta^3 \rangle}{3!\, s^{3/2}} \nu^3 + \dots}}{\sqrt{2\pi}}$

First crossing, uncorrelated Gaussian walks: $\quad \Pi(\delta, s) = P(\delta, s) - P(2\delta_c - \delta, s)$
(Chandrasekhar, 1943)
Bond et al, 1991

$$f(s) = -\frac{\partial}{\partial \delta_c} P(\delta_c) = \frac{\nu}{s} P(\nu)$$

BUT walks are NG and correlated: $\quad \delta_R = \int dk\, k^2 \hat{W}(kR)\, \delta(\vec{k})$

zig-zags are suppressed, at small s one gets $(*)$
Paranjape and Sheth (2012)

Better approx: $\quad f(s) = \int_0^{+\infty} dv\, v\, P(v, \delta_c) = \underbrace{P(\delta_c)}_{\#\text{ density}} \underbrace{\int_0^{+\infty} dv\, v\, P(v|\delta_c)}_{\text{mean pos. velocity}}$

$$\Rightarrow s f(s) = \frac{\nu}{2} P(\nu) \left[\frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\Gamma \nu}{\sqrt{2}}\right)\right) + \frac{e^{-(\Gamma\nu)^2/2}}{\sqrt{2\pi}\,\Gamma \nu} \right]$$

$$\left(\Gamma^2 = \frac{\gamma^2}{1-\gamma^2} \qquad \gamma^2 = \frac{\langle \delta \delta' \rangle^2}{\langle \delta^2 \rangle \langle \delta'^2 \rangle} = \frac{1}{4s\langle \delta'^2 \rangle} \qquad \langle v^2 - \langle v \rangle^2 | \delta_c \rangle = \frac{1}{4s\Gamma^2} \right)$$

Also in NG case: $\quad P(v, \delta_c) = e^{\mathcal{D}} P_G(v, \delta_c)$

diff. operator in $\frac{\partial}{\partial \delta_c}$ and $\frac{\partial}{\partial v}$

Dangerous to expand in $\mathcal{D}$. single pieces blow up at small s

Still: $\quad f(s) = \left[\frac{d}{ds}\int_{\delta_c}^{+\infty} d\delta\, P(\delta, s)\right] \frac{1 + \mathrm{erf}(\Gamma\nu/\sqrt{2})}{2} + P(\delta_c, s)\left[\frac{e^{-(\Gamma\nu)^2/2}}{\sqrt{2\pi s}\, 2\Gamma} + \dots \right]$

small NG corrections

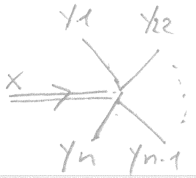
Halo bias

Galaxies depend on the DM environment

$$\rho_h(x) = 1 + \delta_h(x) = \sum_{n=1}^{+\infty} \int d^3y_1 \dots d^3y_n \, b_n(x-y_1, \dots, x-y_n)\, \delta(y_1)\dots\delta(y_n)$$

(Matsubara, 2011)
(Many before)

$$= \sum_{n=1}^{+\infty}$$



Traditionally, bias factors = coeffs. of Taylor exp. of $\langle 1+\delta_h \mid \delta_0, S_0 \rangle$ in powers of δ_0 at $S_0 = 0$ $\left(\Rightarrow \frac{f(S \mid \delta_0, S_0)}{f(S)} \right)$

Also, convolution of the same with $H_n\left(\frac{\delta_0}{\sqrt{S_0}}\right)$ (Szalay, 1988)

In QFT language, what matters is $\langle \rho_h(x)\, \delta(x_1) \dots \delta(x_n) \rangle_{1\text{-PI}}$, 1-PI corr. funct. amputated of ext. propagators.

Given exc. set with corr. steps and the above approx., it returns the conv. with H_n