NNLO hard-thermal-loop thermodynamics for QCD

arXiv:1103.2528, arXiv:1106.0514, …

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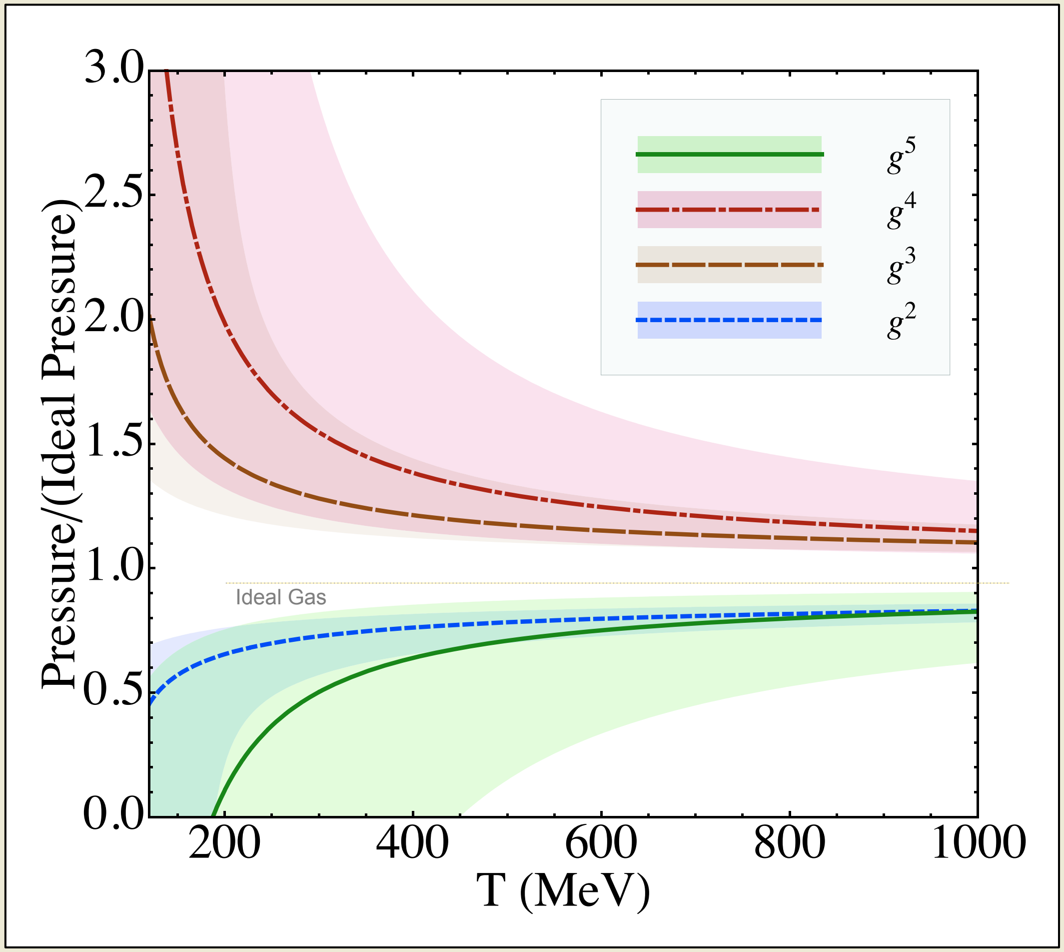
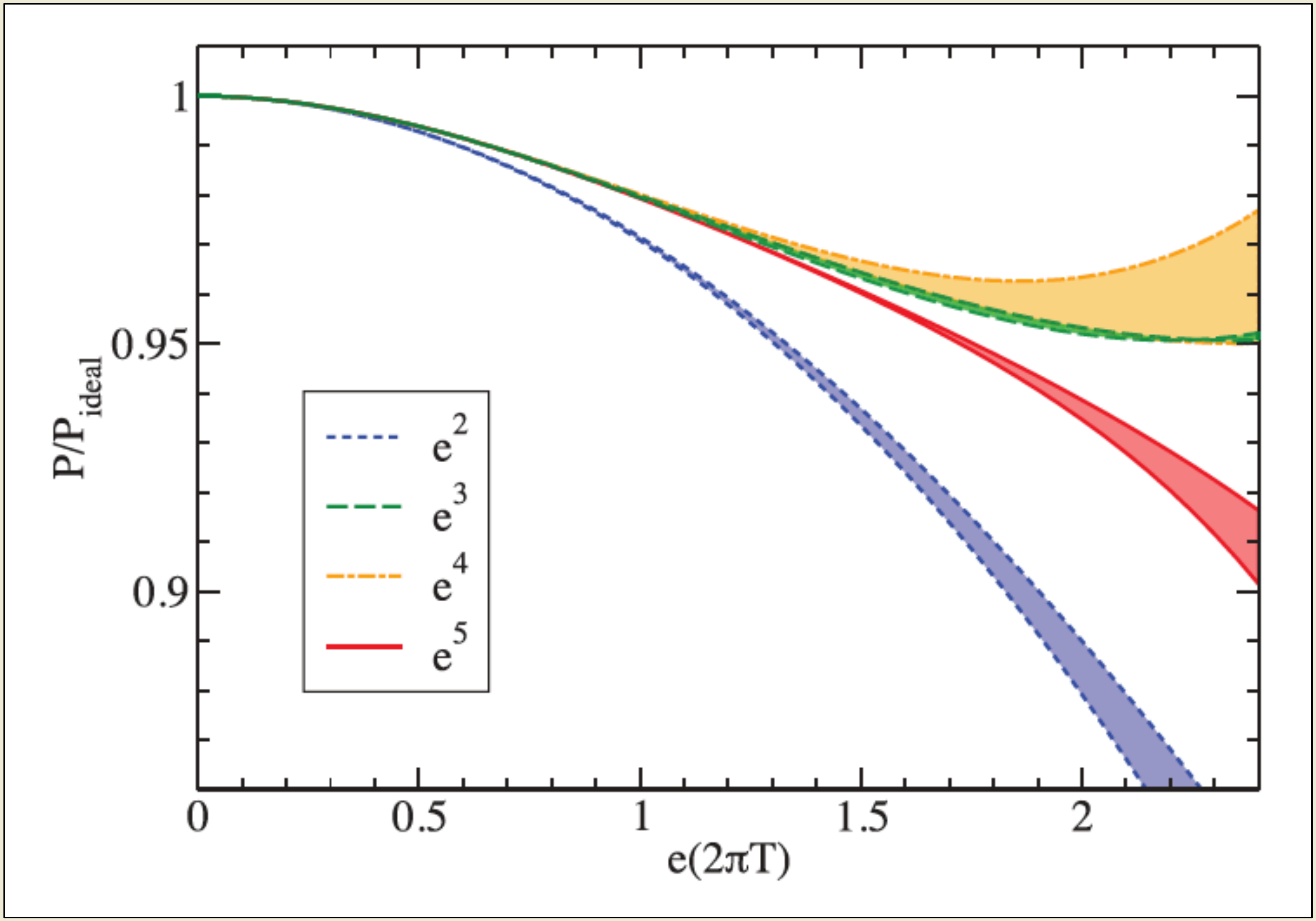
July 16, 2012

Motivation

- QCD free energy known up to three loops since 1994 (Arnold, Zhai, and Khastening)
- Series in g (not g^2) due to plasma screening effects: Debye mass $m_D = gT$
- Very poorly convergent
- Need temperatures on the order of $T \sim 10^5$ GeV
- Poor convergence has been taken as evidence of failure of applicability of pQCD to heavy ion collisions
- Can we do better at T > 300 MeV by using the right DOFs?



Problem is universal … exists in QED, scalar theories, etc.



Simpler Case – Anharmonic Oscillator

- Consider quantum mechanics in an anharmonic potential

$$V(x) = \frac{1}{2}\omega^2 x^2 + \frac{g}{4}x^4 \qquad (\omega^2, g > 0)$$

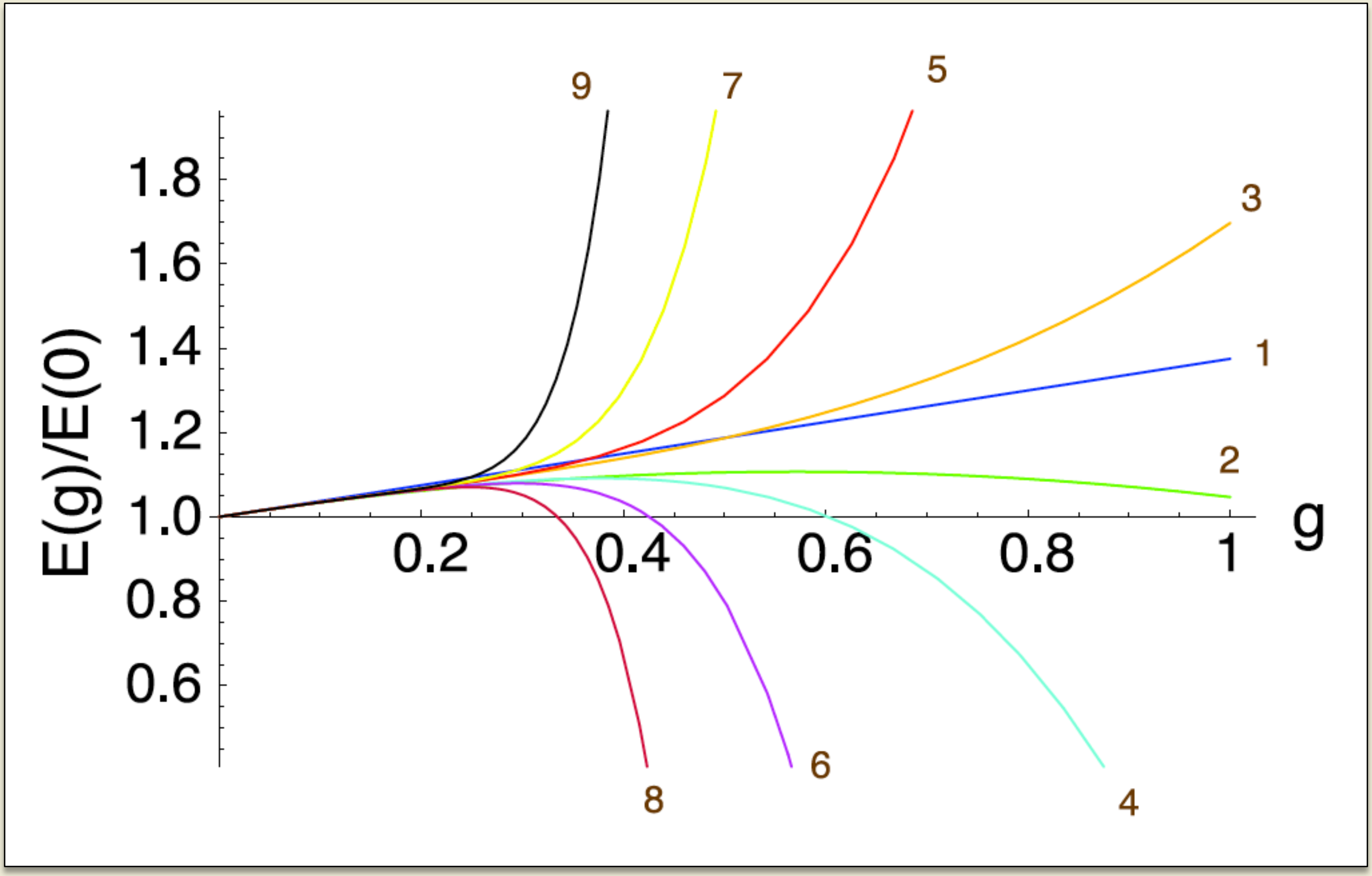
- Weak-coupling expansion of the ground state energy is known up to all orders (Bender and Wu 69/73)

$$E(g) = \omega \sum_{n=0}^{\infty} c_n^{\rm BW} \left(\frac{g}{4\omega^3}\right)^n , \quad c_n^{\rm BW} = \left\{ \frac{1}{2}, \frac{3}{4}, -\frac{21}{8}, \frac{333}{16}, -\frac{30885}{16}, \ldots \right\}$$

$$\lim_{n\to\infty} c_n^{\rm BW} = (-1)^{n+1}\sqrt{\frac{6}{\pi^3}}\,3^n (n-\tfrac{1}{2})!$$

- Because of factorial growth, the expansion is an asymptotic series with zero radius of convergence!

Anharmonic Oscillator



Variational Perturbation Theory

- Split the harmonic term into two pieces and treat the second as part of the interaction

$$\omega^2 \to \Omega^2 + \left(\omega^2 - \Omega^2\right) \quad \Longrightarrow \quad E_N(g,r) = \Omega \sum_{n=0}^{N} c_n(r) \left(\frac{g}{4\Omega^3}\right)^n$$

$$r \equiv \tfrac{2}{g}\left(\omega^2 - \Omega^2\right)$$

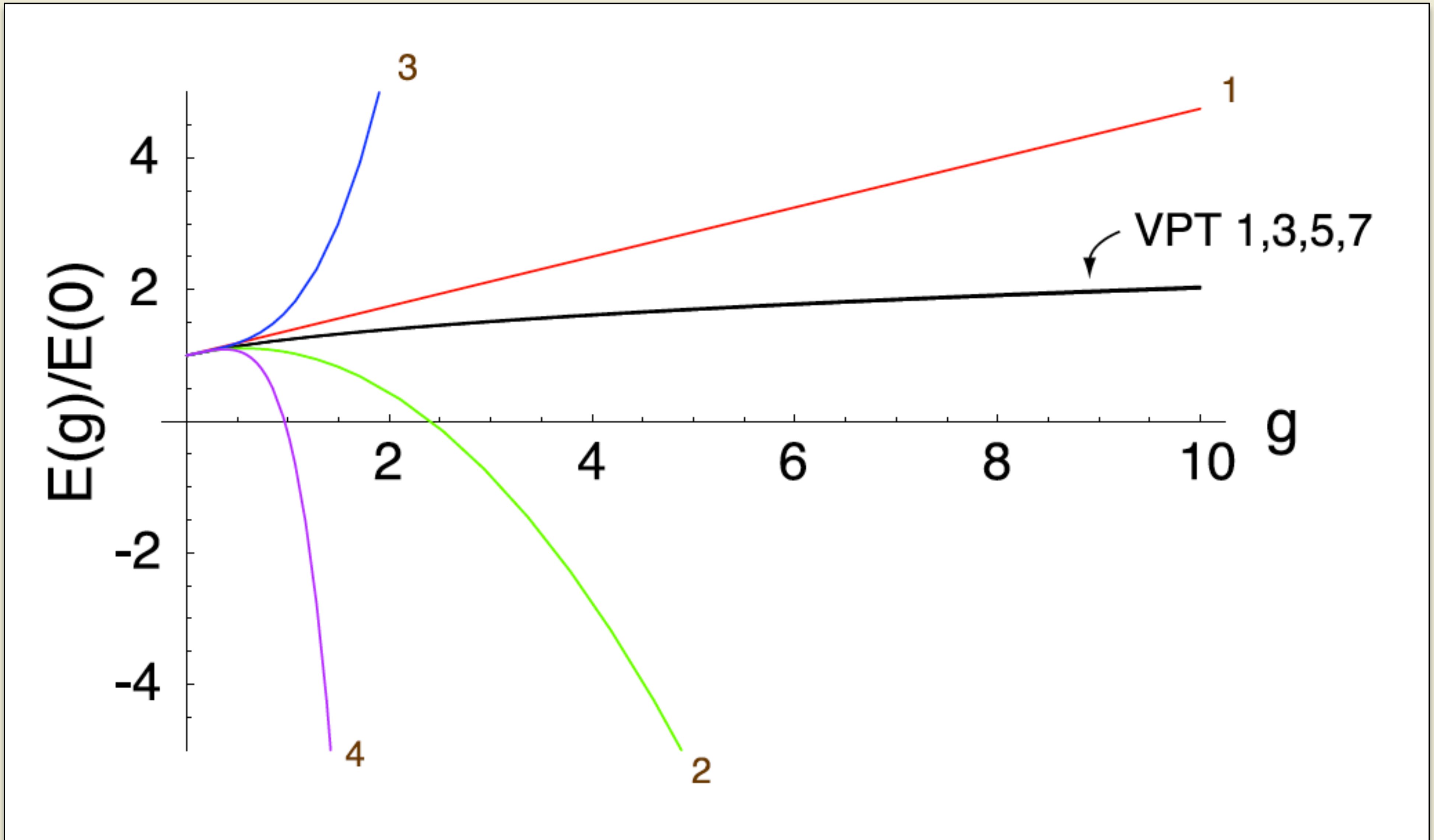
- The coefficients c_n can be computed analytically in this case

$$c_n(r) = \sum_{j=0}^{n} c_j^{\rm BW} \begin{pmatrix} (1-3j)/2 \\ n-j \end{pmatrix} (2r\Omega)^{n-j}$$

- Fix Ω_N by imposing variational condition that ground state energy is minimized

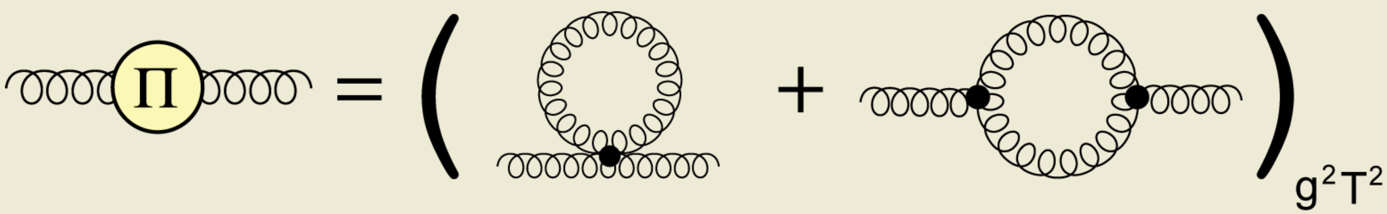
$$\left.\frac{\partial E_N}{\partial \Omega}\right|_{\Omega=\Omega_N} = 0$$

Variational Perturbation Theory

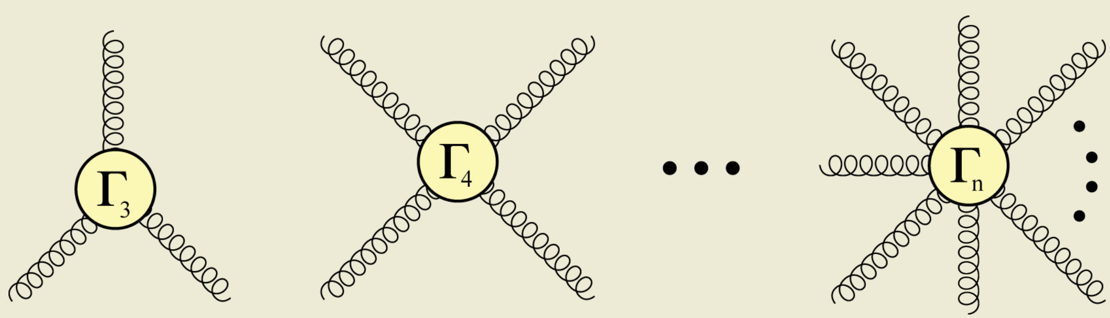


High Temperature QCD Degrees of Freedom

- At high temperatures gluons experience screening
- Ignoring quarks (for now) the polarization tensor for gluons has a “hard thermal loop”



- Since theory must be gauge invariant this implies that there are similar hard thermal loops in all gluon n-point functions



HTL Collective Modes

$$\Delta^{\mu\nu}(p) = -\Delta_T(p) T_p^{\mu\nu} + \Delta_L(p) n_p^\mu n_p^\nu - \xi \frac{p^\mu p^\nu}{(p^2)^2} \qquad \text{covariant}$$

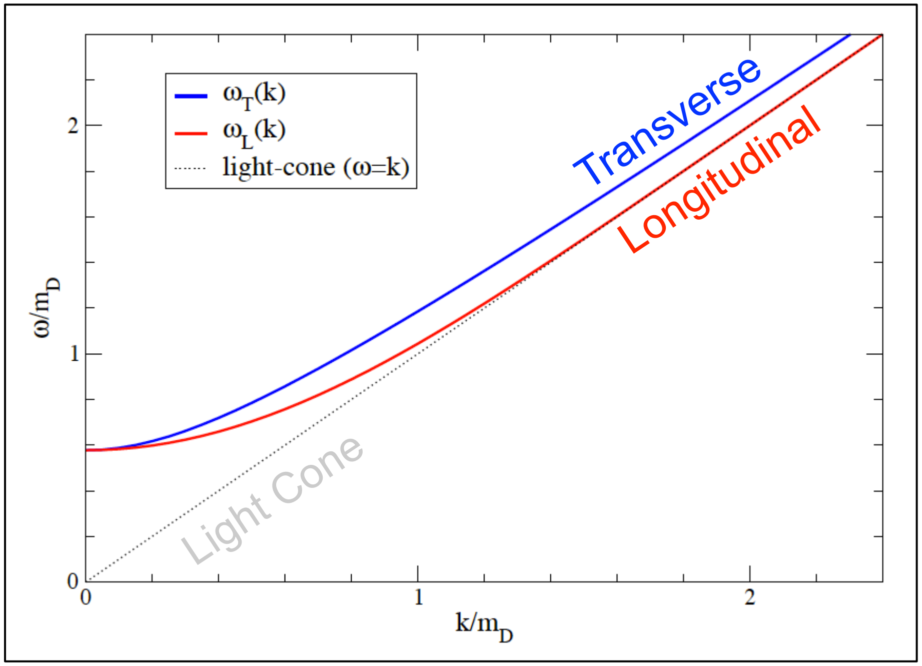
$$= -\Delta_T(p) T_p^{\mu\nu} + \Delta_L(p) n^\mu n^\nu - \xi \frac{p^\mu p^\nu}{\left(n_p^2 p^2\right)^2} \qquad \text{Coulomb}$$

$$\Delta_T(p) = \frac{1}{p^2 - \Pi_T(p)} , \qquad \Delta_L(p) = \frac{1}{-n_p^2 p^2 + \Pi_L(p)} . \qquad n_p^\mu = n^\mu - \frac{n\cdot p}{p^2} p^\mu$$

$$\Pi_T(\omega,p) = \frac{m_D^2}{2}\frac{\omega^2}{p^2}\left[1 + \frac{p^2-\omega^2}{2\omega p}\log\frac{\omega+p}{\omega-p}\right] ,$$

$$\Pi_L(\omega,p) = m_D^2\left[1 - \frac{\omega}{2p}\log\frac{\omega+p}{\omega-p}\right] .$$

$$m_D^2 = \frac{1}{3}\left(C_A + \frac{1}{2}N_f\right) g^2 T^2 .$$



HTL-Resummed Action

$$\mathcal{L}_{\rm YM} + \mathcal{L}_{\rm HTL} = \frac{1}{4} G^{\mu\nu} G_{\mu\nu} - \frac{1}{2} m_D^2 \, {\rm Tr} \left(G_{\mu\alpha} \left\langle \frac{y^\alpha y^\beta}{(y \cdot D)^2} \right\rangle_y G^\mu{}_\beta \right)$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

$$D_\mu = \partial_\mu + igA_\mu$$

- Expanding to quadratic order in A gives dressed propagator (2-point function)
- Expanding to cubic order in A gives dressed gluon three-vertex
- Expanding to quartic order in A gives dressed gluon four-vertex
- And so on . . . contains an infinite number of higher order vertices which all exactly satisfy the appropriate Slavnov-Taylor identities

Hard Thermal Loop Perturbation Theory

HTLpt : Reorganizes loop expansion around classical state of high temperature QCD which includes “hard-loop” resummed propagators and vertices. Gauge invariant by construction.

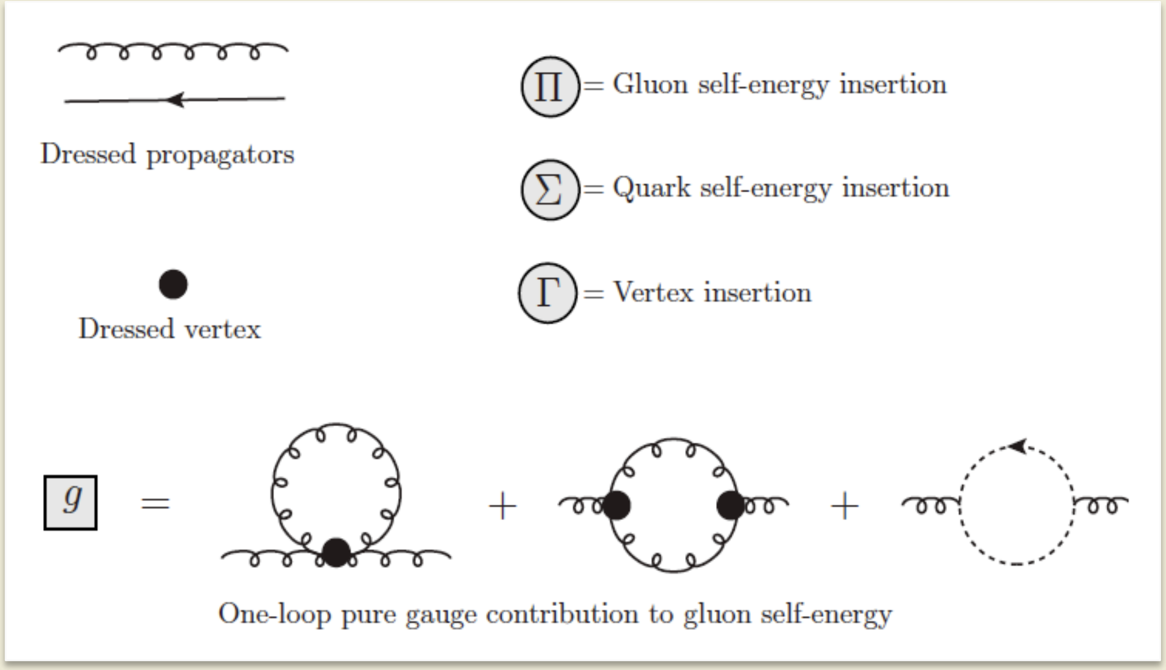
$$\mathcal{L} = \left(\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL}\right)\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL}.$$

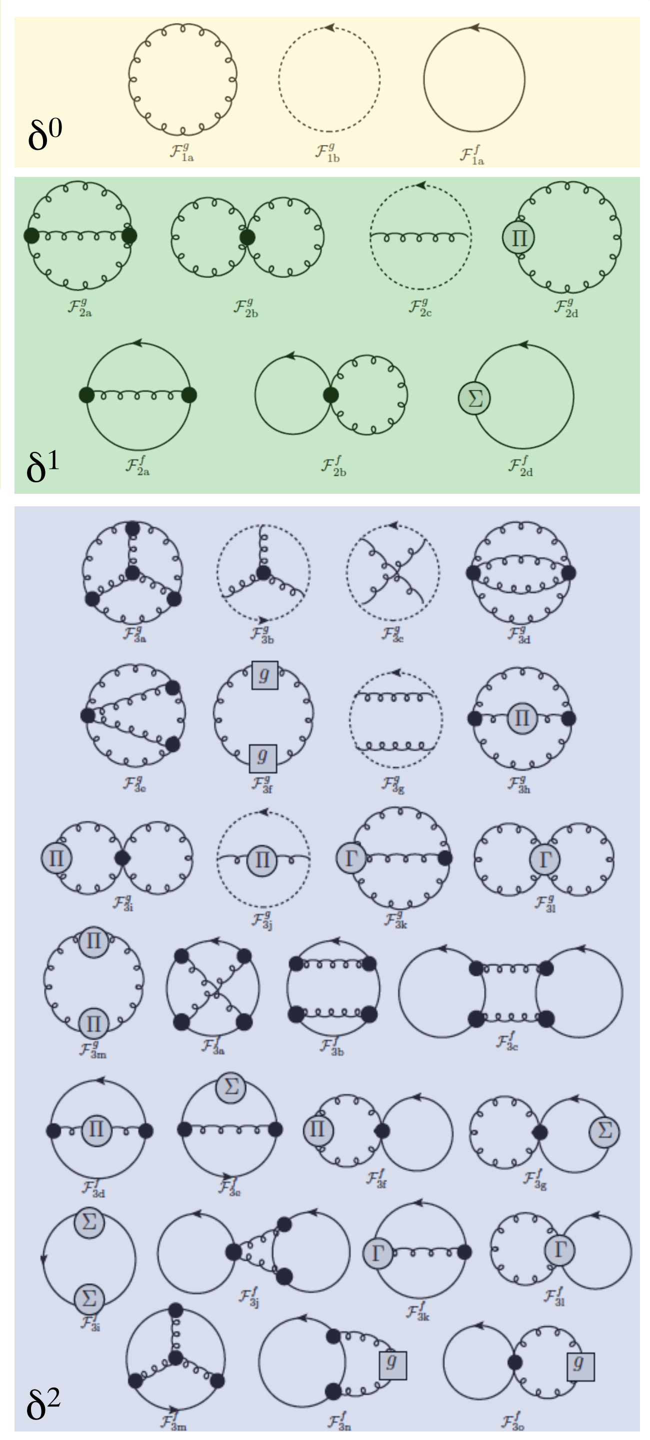
$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2 {\rm Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right) + (1-\delta)\, i m_q^2 \bar\psi\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_y \psi\,,$$

- One expands in a power series in δ, and in the end sets $\delta=1$
- The number of dressed loops generated at each order is $\delta+1$

NNLO Diagrams

Now "simply" compute all contributions up to three loops including dressed propagators and vertices





N

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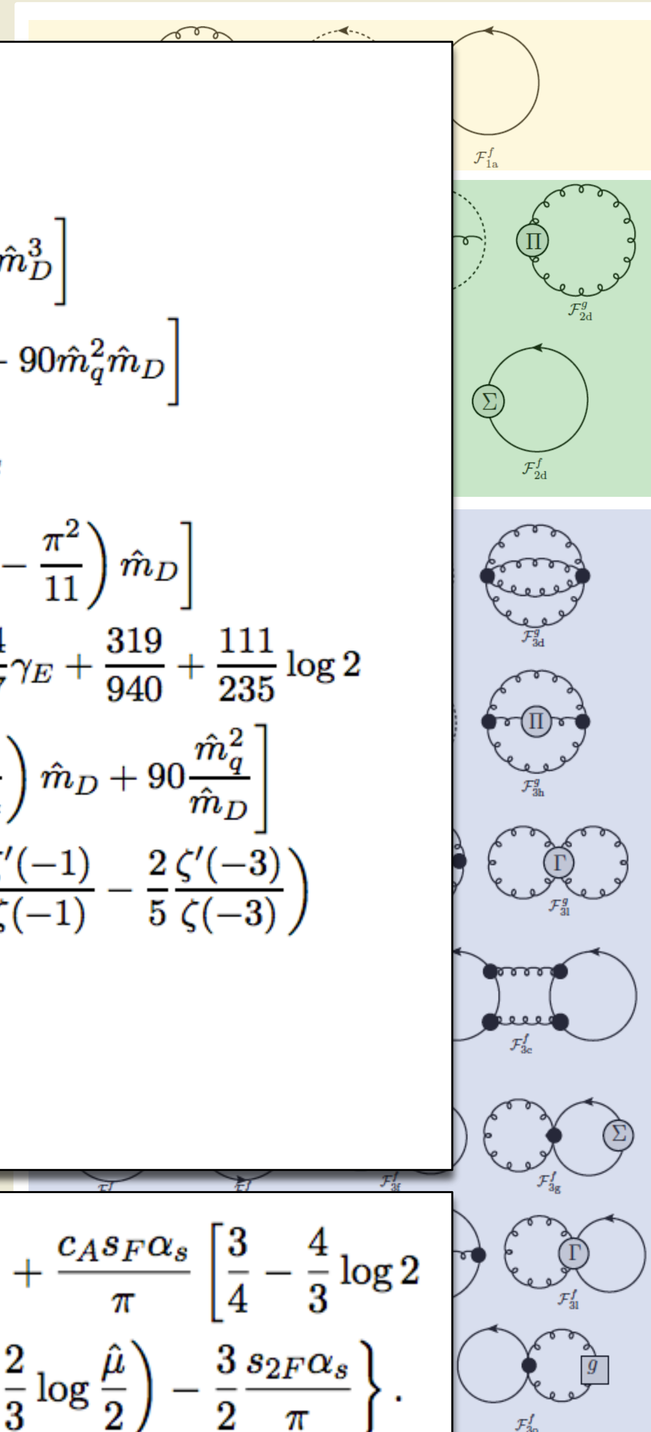
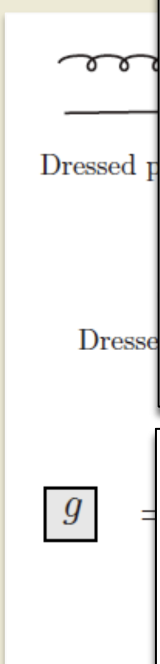
\begin{aligned}
&\Omega_{\rm NNLO} = \\
&\mathcal{F}_{\rm ideal}\Bigg\{1+\frac{7}{4}\frac{d_F}{d_A}-\frac{15}{4}\hat{m}_D^3 \\
&+\frac{c_A\alpha_s}{3\pi}\left[-\frac{15}{4}+\frac{45}{2}\hat{m}_D-\frac{135}{2}\hat{m}_D^2-\frac{495}{4}\left(\log\frac{\hat{\mu}}{2}+\frac{5}{22}+\gamma_E\right)\hat{m}_D^3\right] \\
&+\frac{s_F\alpha_s}{\pi}\left[-\frac{25}{8}+\frac{15}{2}\hat{m}_D+15\left(\log\frac{\hat{\mu}}{2}-\frac{1}{2}+\gamma_E+2\log 2\right)\hat{m}_D^3-90\hat{m}_q^2\hat{m}_D\right] \\
&+\left(\frac{c_A\alpha_s}{3\pi}\right)^2\Bigg[\frac{45}{4}\frac{1}{\hat{m}_D}-\frac{165}{8}\left(\log\frac{\hat{\mu}}{2}-\frac{72}{11}\log\hat{m}_D-\frac{84}{55}-\frac{6}{11}\gamma_E\right. \\
&\left.-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\right)+\frac{1485}{4}\left(\log\frac{\hat{\mu}}{2}-\frac{79}{44}+\gamma_E+\log 2-\frac{\pi^2}{11}\right)\hat{m}_D\Bigg] \\
&+\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi}\right)\Bigg[\frac{15}{2}\frac{1}{\hat{m}_D}-\frac{235}{16}\left(\log\frac{\hat{\mu}}{2}-\frac{144}{47}\log\hat{m}_D-\frac{24}{47}\gamma_E+\frac{319}{940}+\frac{111}{235}\log 2\right. \\
&\left.-\frac{74}{47}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{1}{47}\frac{\zeta'(-3)}{\zeta(-3)}\right)+\frac{315}{4}\left(\log\frac{\hat{\mu}}{2}-\frac{8}{7}\log 2+\gamma_E+\frac{9}{14}\right)\hat{m}_D+90\frac{\hat{m}_q^2}{\hat{m}_D}\Bigg] \\
&+\left(\frac{s_F\alpha_s}{\pi}\right)^2\Bigg[\frac{5}{4}\frac{1}{\hat{m}_D}+\frac{25}{12}\left(\log\frac{\hat{\mu}}{2}+\frac{1}{20}+\frac{3}{5}\gamma_E-\frac{66}{25}\log 2+\frac{4}{5}\frac{\zeta'(-1)}{\zeta(-1)}-\frac{2}{5}\frac{\zeta'(-3)}{\zeta(-3)}\right) \\
&-15\left(\log\frac{\hat{\mu}}{2}-\frac{1}{2}+\gamma_E+2\log 2\right)\hat{m}_D+30\frac{\hat{m}_q^2}{\hat{m}_D}\Bigg] \\
&+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{15}{64}(35-32\log 2)-\frac{45}{2}\hat{m}_D\right]\Bigg\}\,.
\end{aligned}
$$

$$
\begin{aligned}
m_D^2 = \frac{4\pi\alpha_s}{3}T^2\Bigg\{&c_A+s_F+\frac{c_A^2\alpha_s}{3\pi}\left(\frac{5}{4}+\frac{11}{2}\gamma_E+\frac{11}{2}\log\frac{\hat{\mu}}{2}\right)+\frac{c_A s_F\alpha_s}{\pi}\left[\frac{3}{4}-\frac{4}{3}\log 2\right. \\
&\left.+\frac{7}{6}\left(\gamma_E+\log\frac{\hat{\mu}}{2}\right)\right]+\frac{s_F^2\alpha_s}{\pi}\left(\frac{1}{3}-\frac{4}{3}\log 2-\frac{2}{3}\gamma_E-\frac{2}{3}\log\frac{\hat{\mu}}{2}\right)-\frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\Bigg\}\,.
\end{aligned}
$$





Technical Note - Renormalization

- All divergences are removed with four counterterms: vacuum, thermal quark and gluon masses, and coupling constant.
- Coupling constant counterterm gives canonical one-loop running
- Divergences require introduction of a perturbative renormalization scale which at finite temperature is expected to be set by the lowest finite Matsubara mode $\mu = 2\pi T$

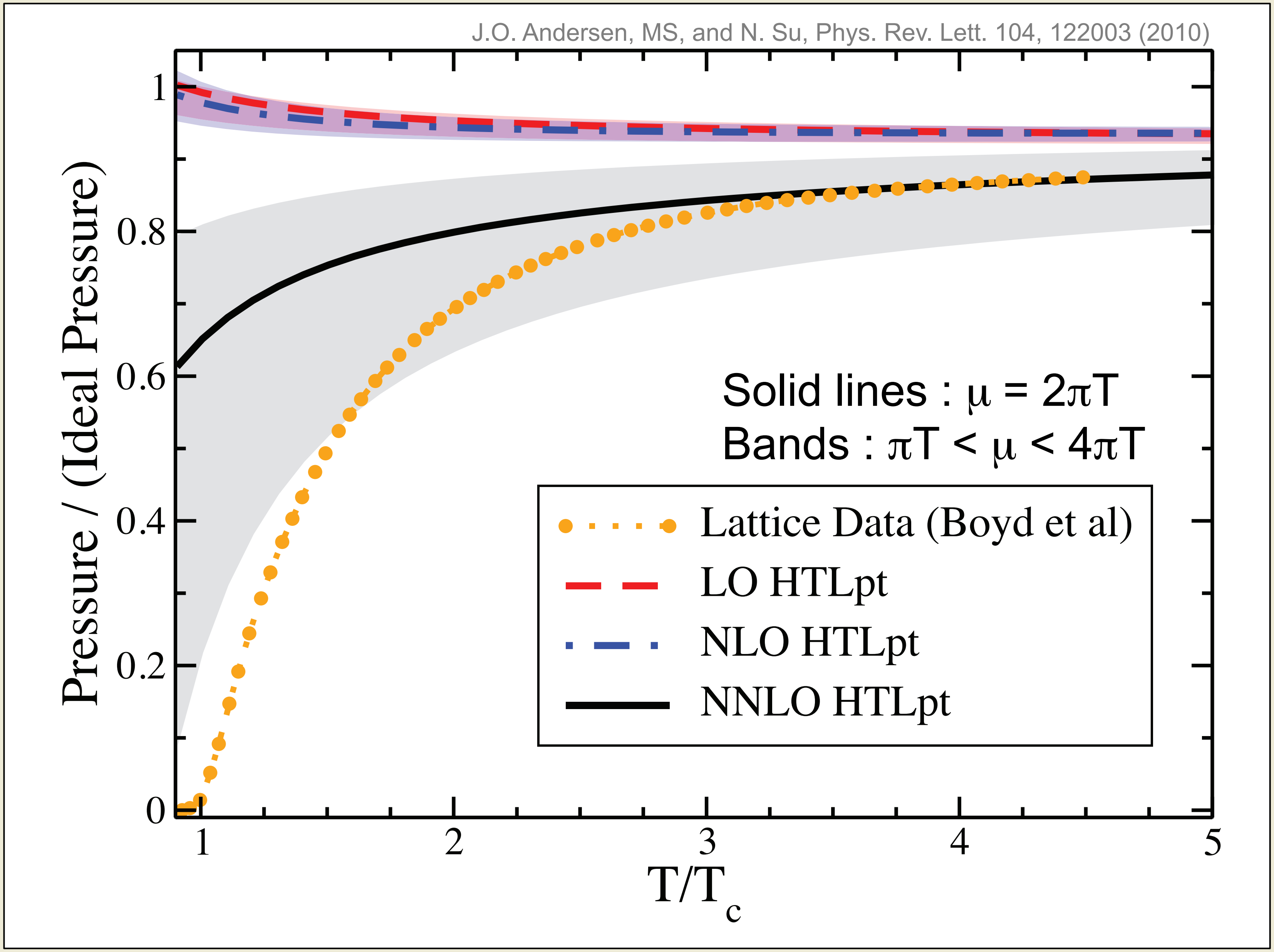
$$\Delta\mathcal{E}_0 = \left(\frac{d_A}{128\pi^2\epsilon} + \mathcal{O}(\delta\alpha_s)\right)(1-\delta)^2 m_D^4 \,,$$

$$\Delta m_D^2 = \left(-\frac{11c_A - 4s_F}{12\pi\epsilon}\alpha_s\delta + \mathcal{O}(\delta^2\alpha_s^2)\right)(1-\delta) m_D^2 \,,$$

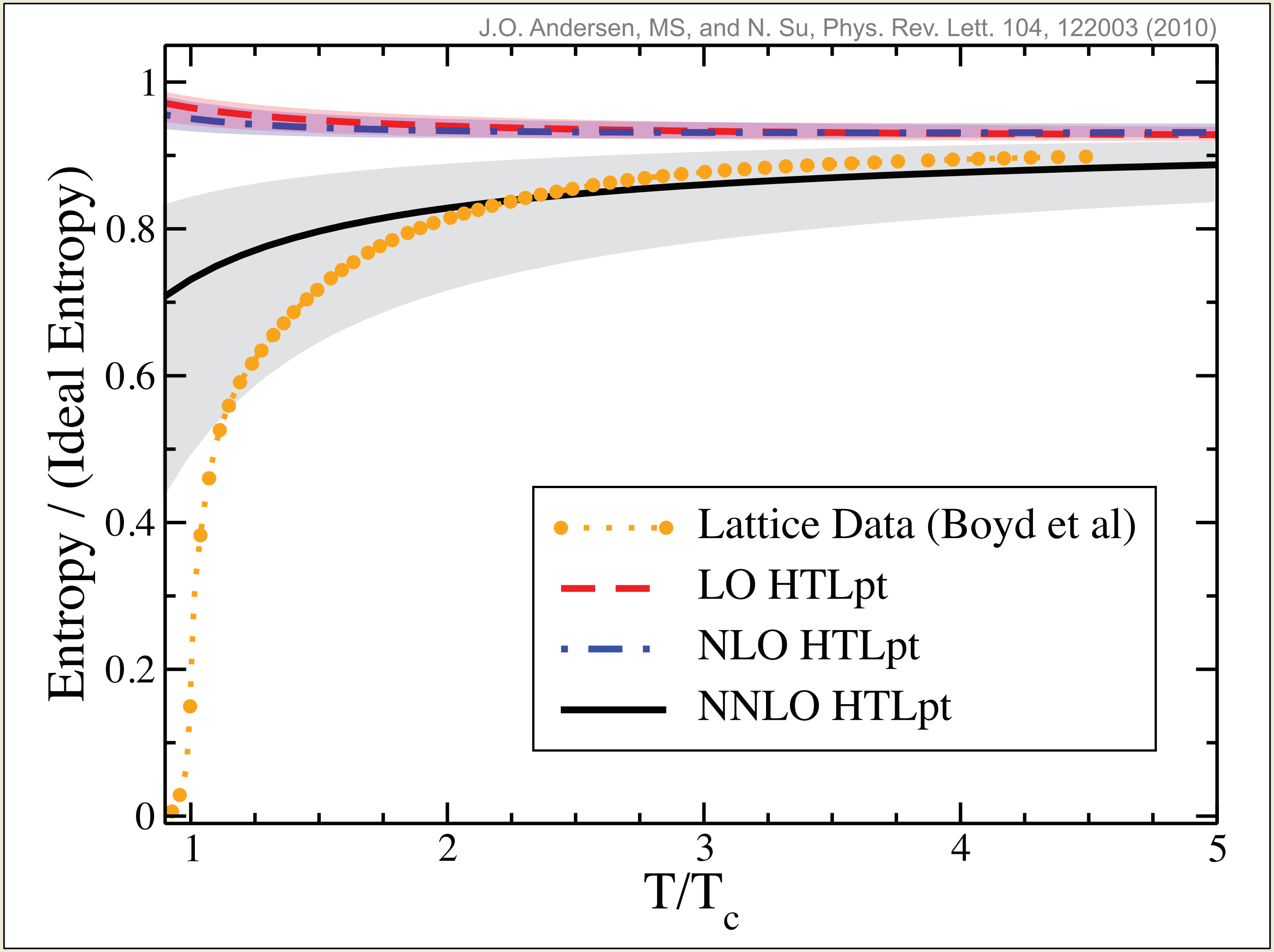
$$\Delta m_q^2 = \left(-\frac{3}{8\pi\epsilon}\frac{d_A}{c_A}\alpha_s\delta + \mathcal{O}(\delta^2\alpha_s^2)\right)(1-\delta) m_q^2 \,,$$

$$\delta\Delta\alpha_s = -\frac{11c_A - 4s_F}{12\pi\epsilon}\alpha_s^2\delta^2 + \mathcal{O}(\delta^3\alpha_s^3) \,,$$

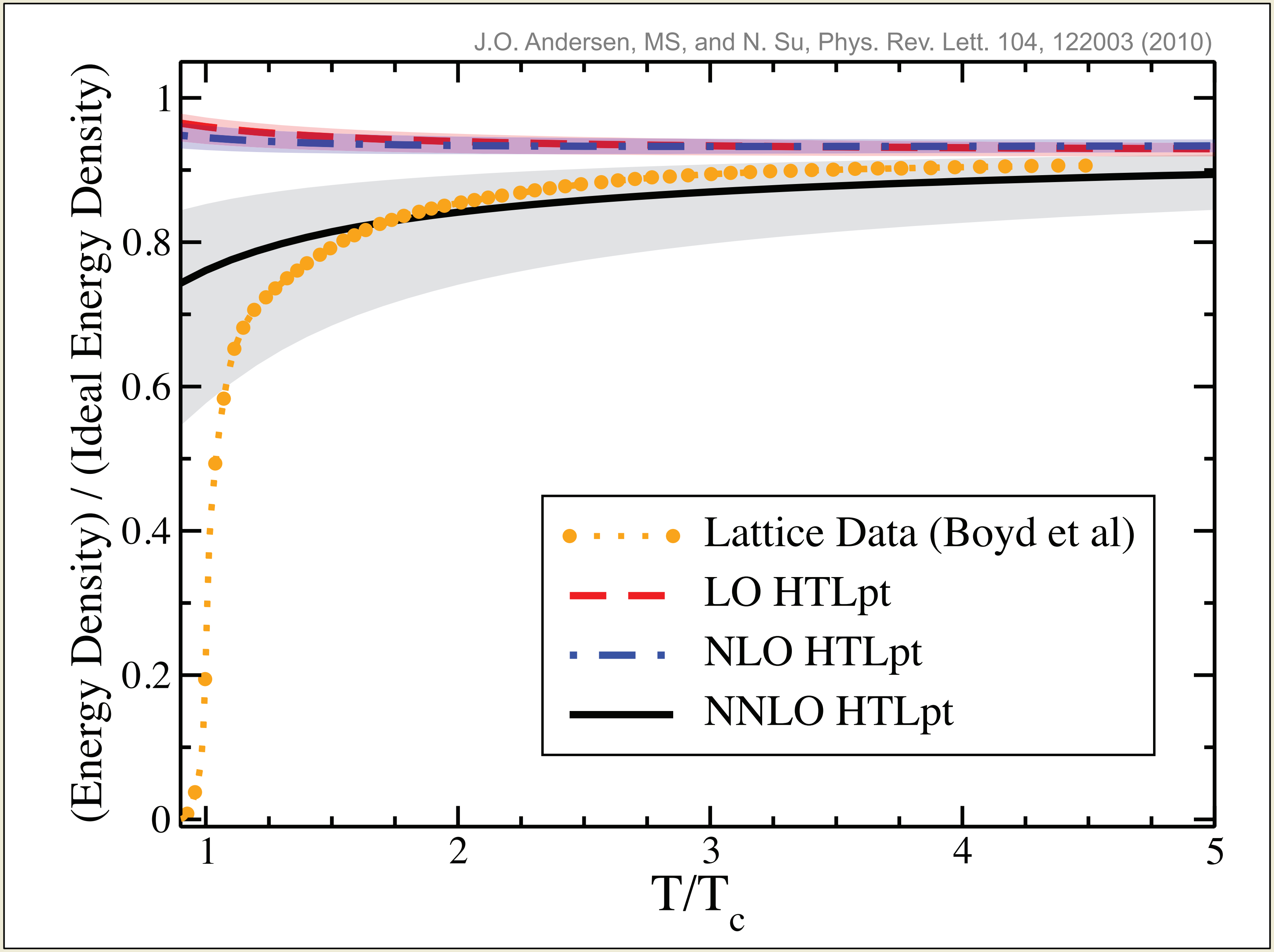
Pure Glue – Pressure

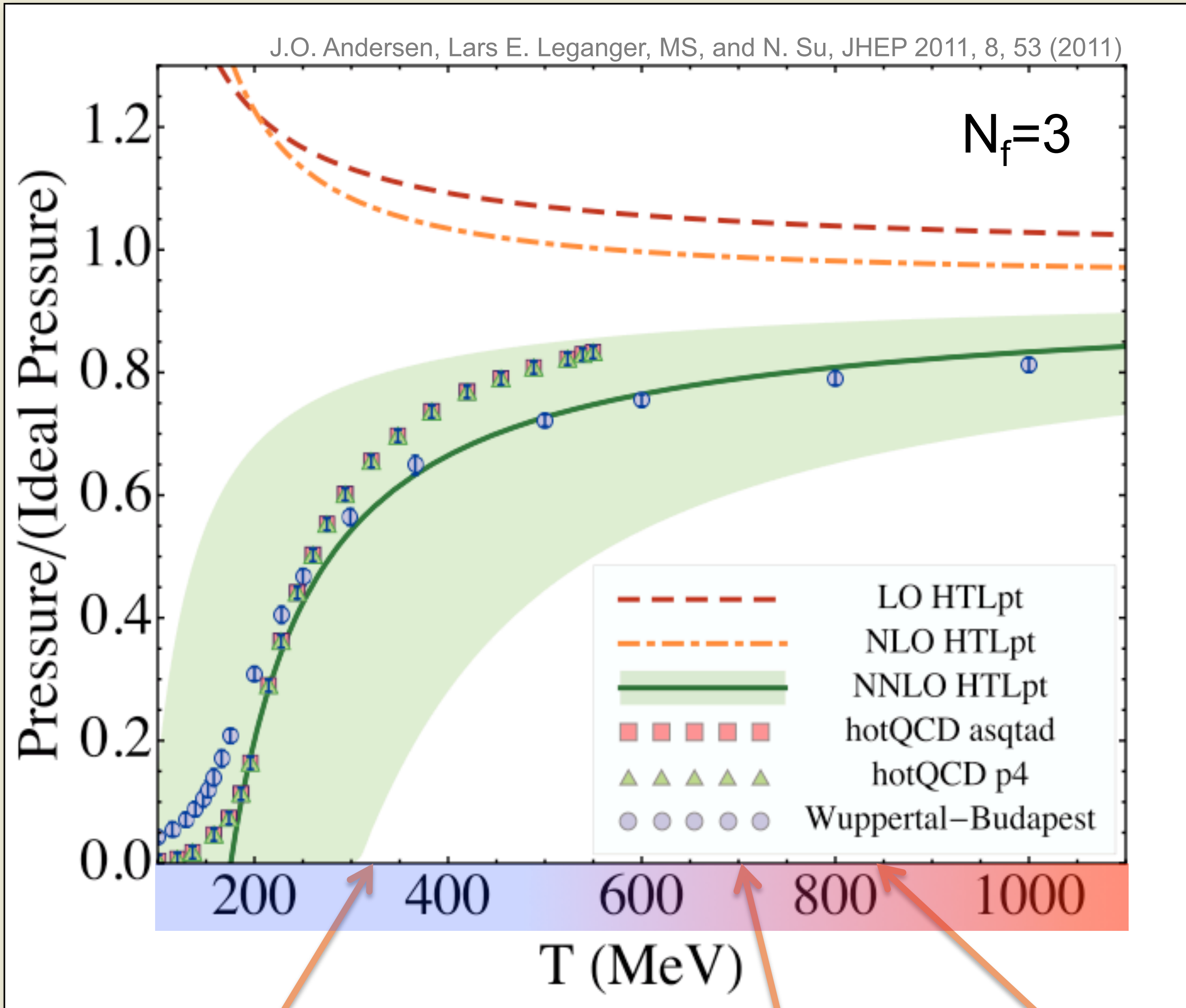


Pure Glue – Entropy



Pure Glue – Energy Density



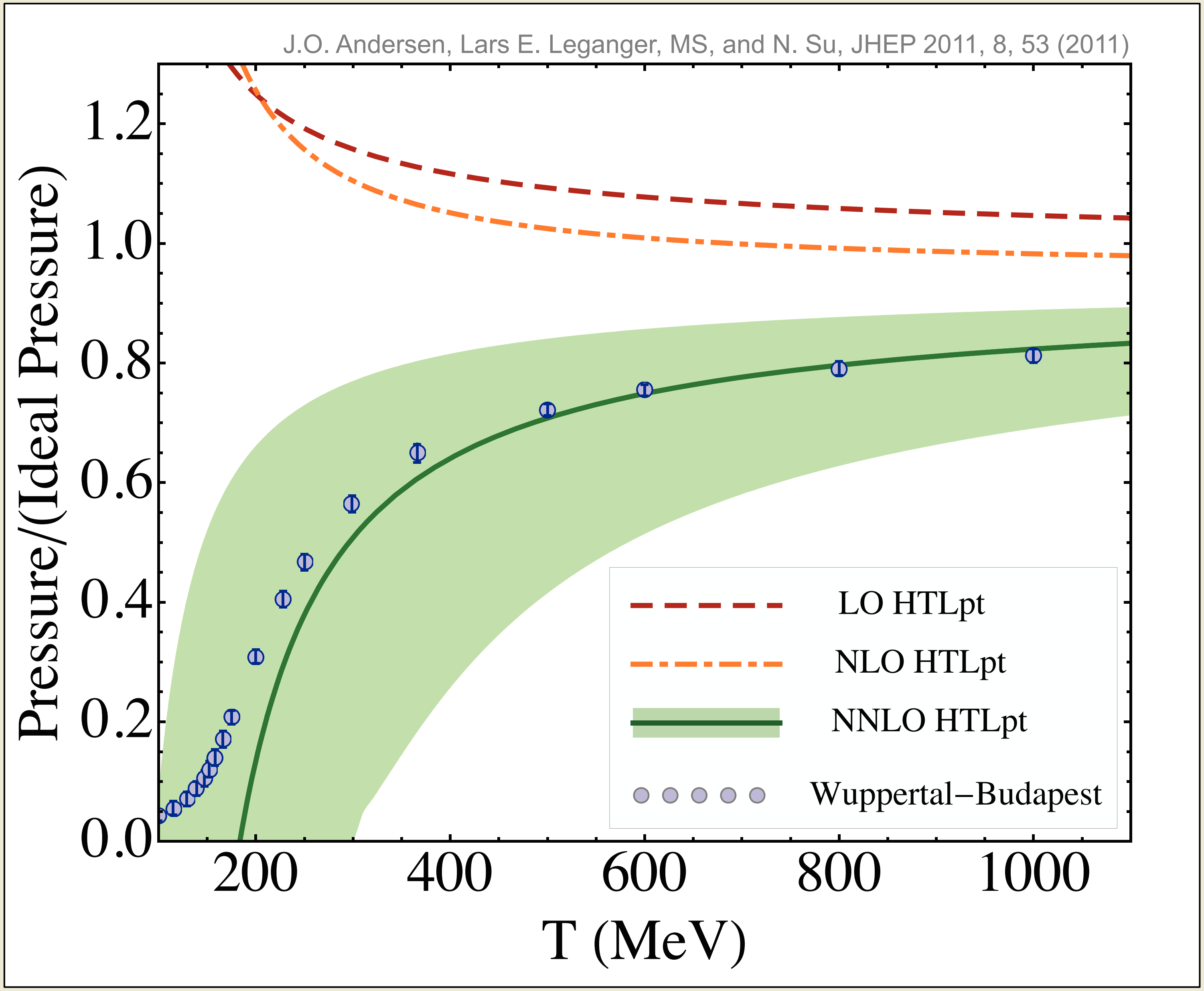


RHIC @ 200 A GeV :
T_o ~ 360 MeV ~ 2 T_c

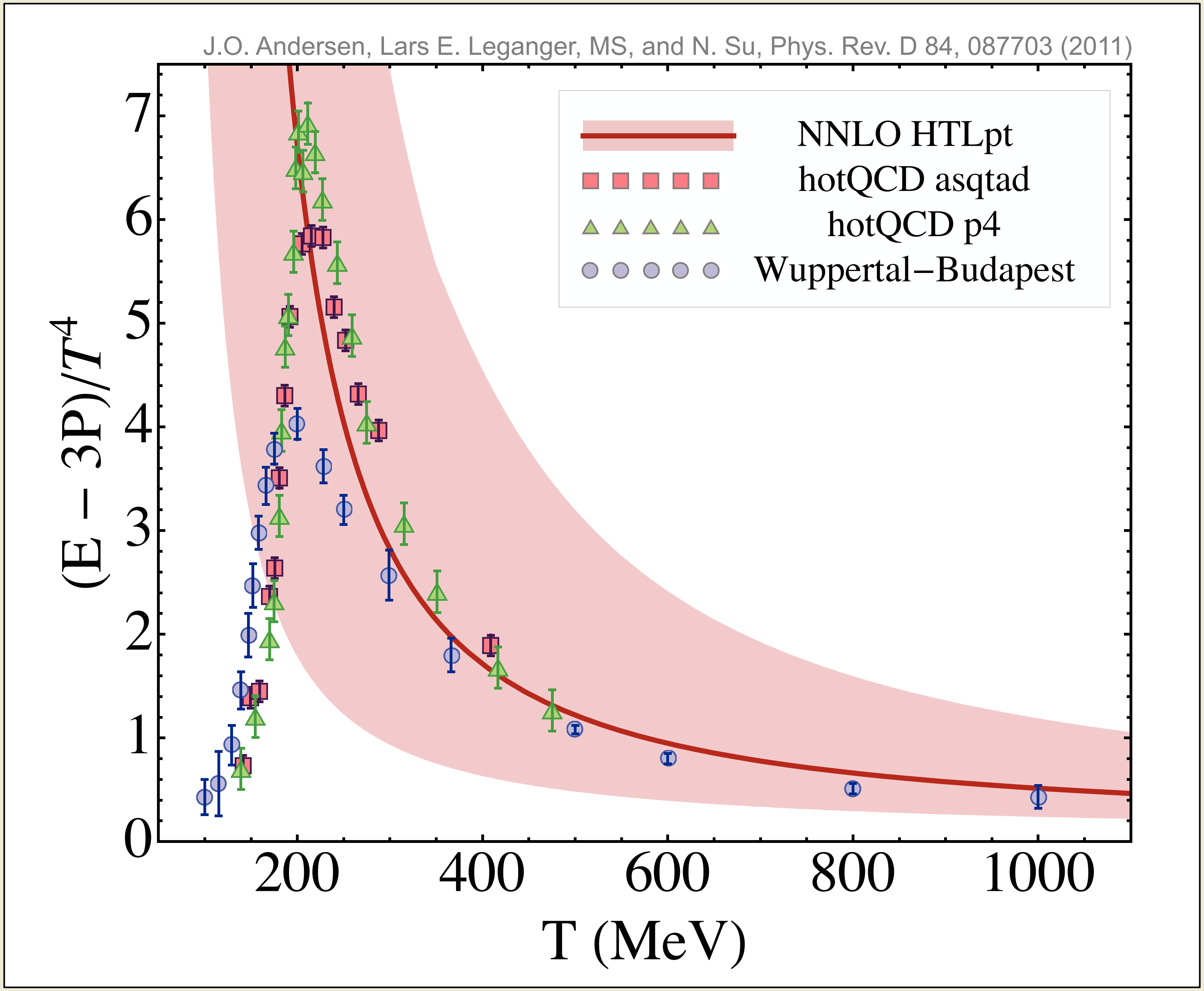
LHC @ 2.76 A TeV :
T ~ 690 MeV ~ 3.9 T_c

LHC @ 5.5 A TeV :
T_o ~ 820 MeV ~ 4.6 T_c

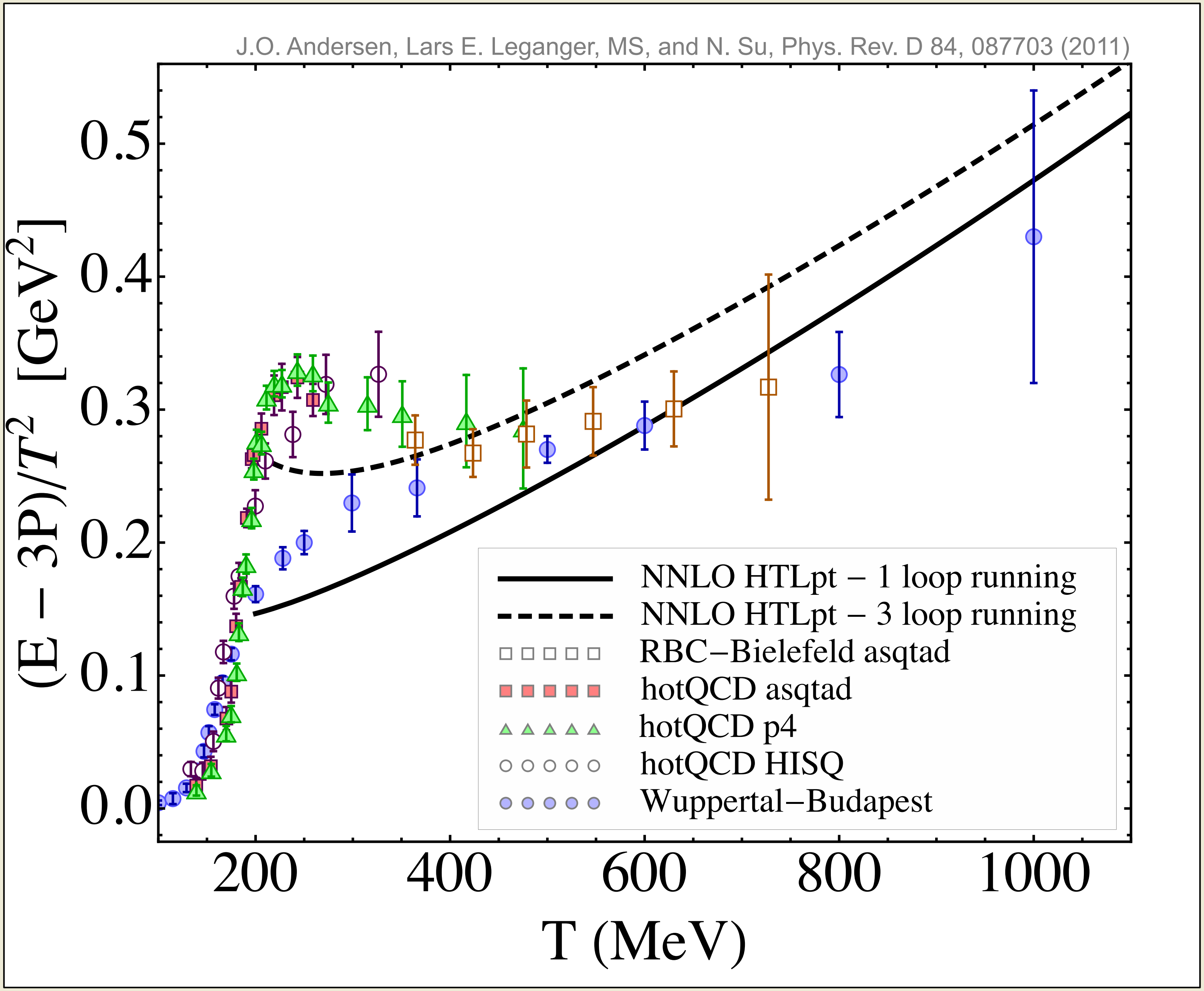
Pressure $N_f = 4$



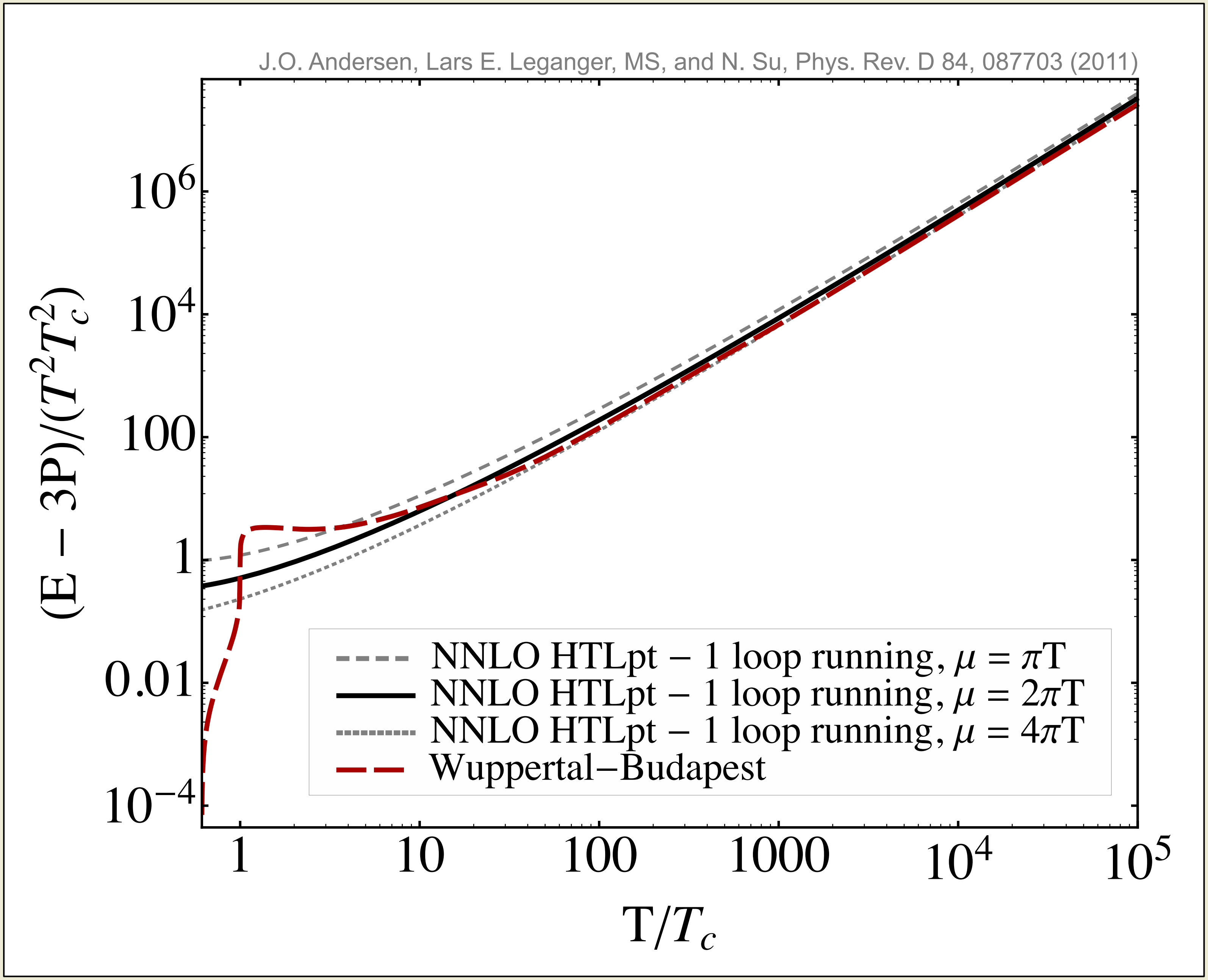
Trace Anomaly $N_f = 3$



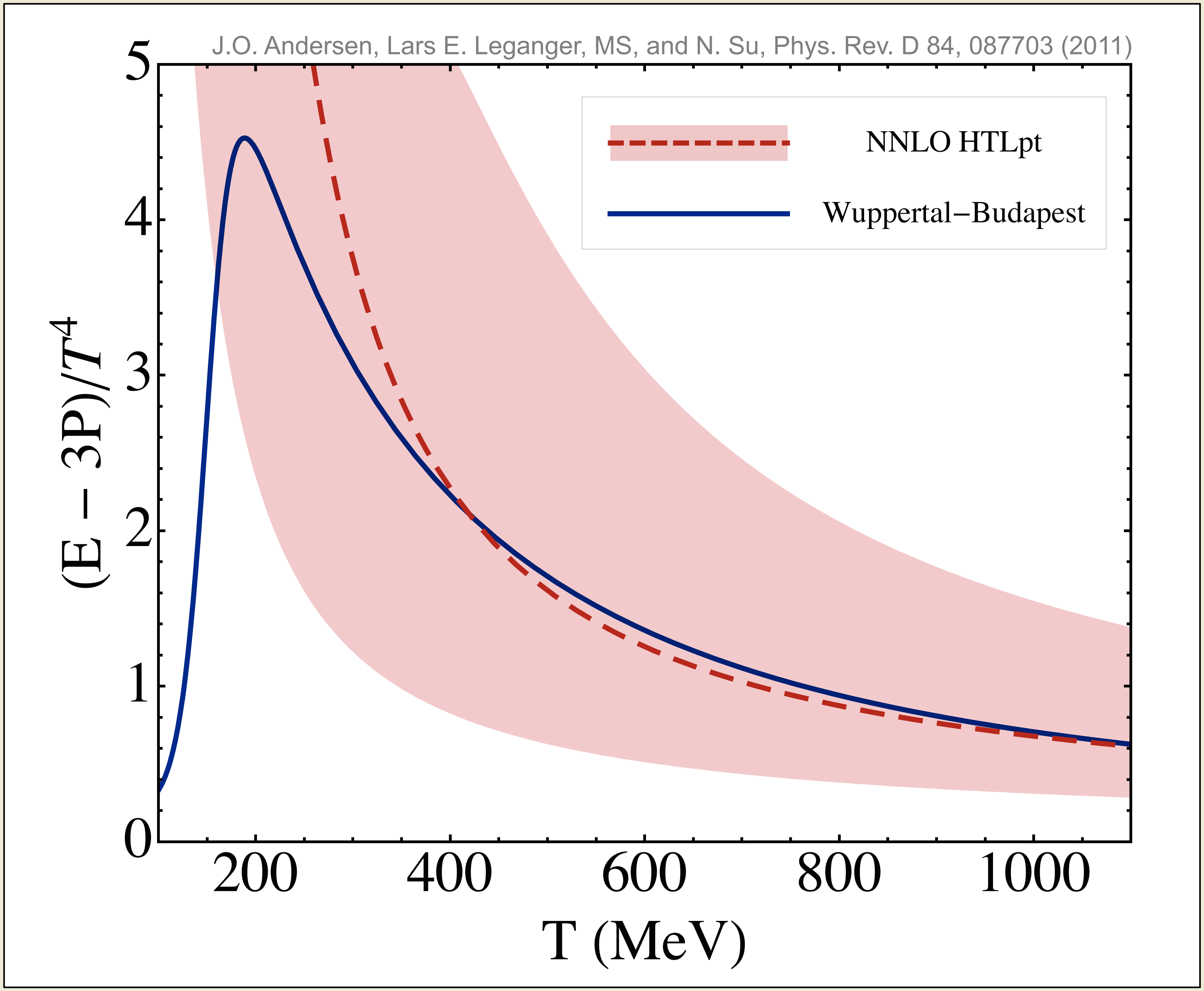
Scaled Trace Anomaly $N_f = 3$



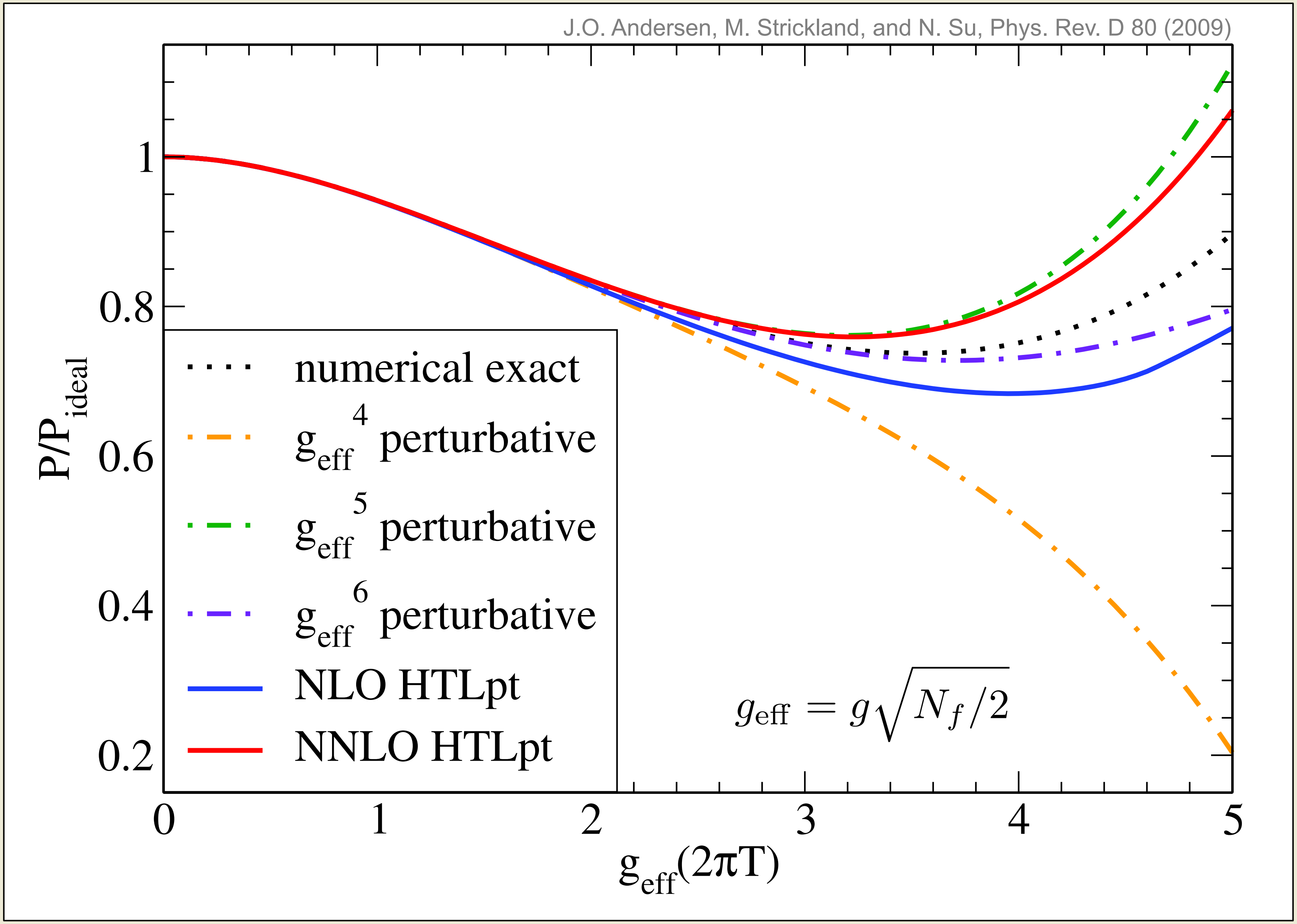
Scaled Trace Anomaly $N_f = 0$



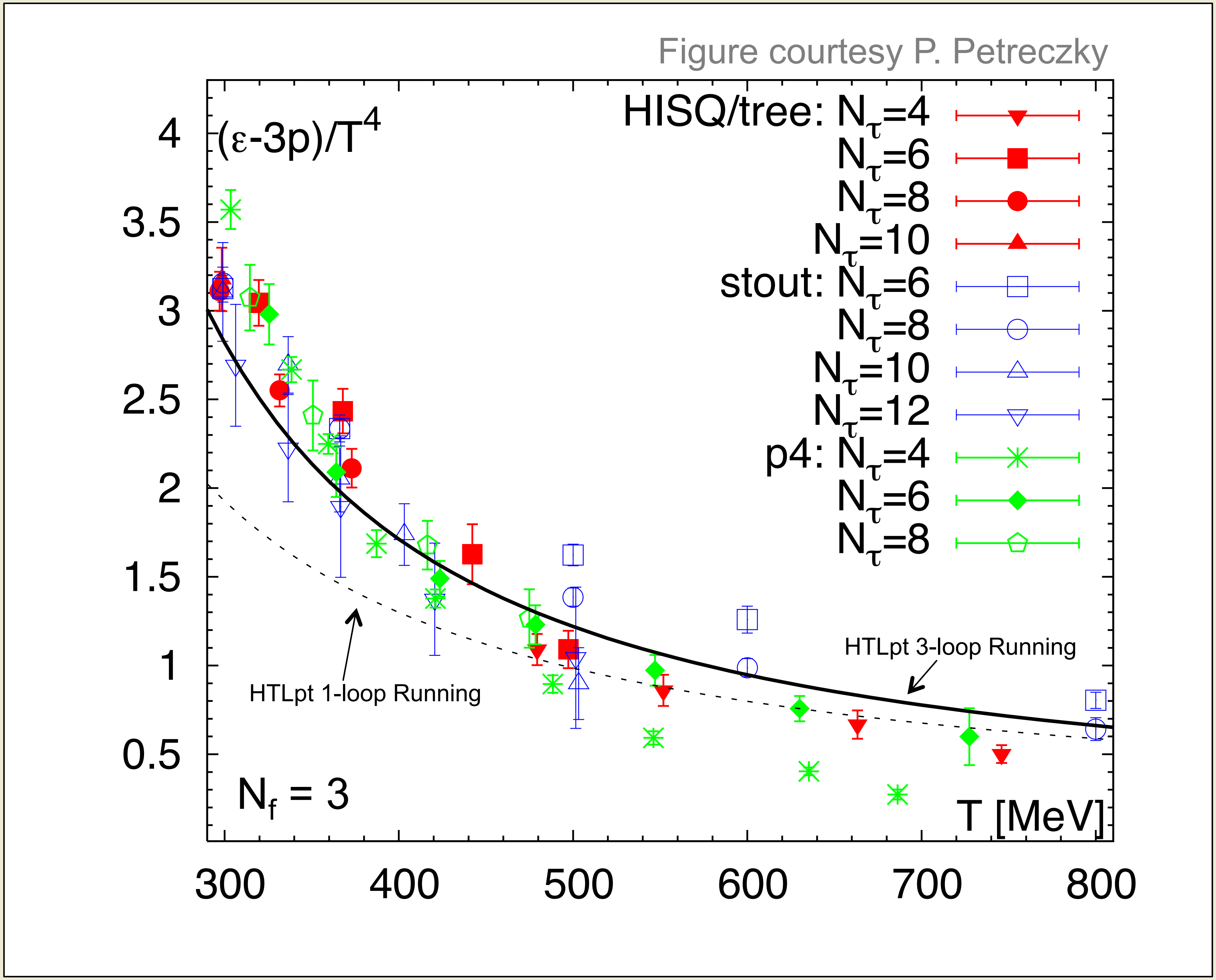
Trace Anomaly $N_f = 4$



Large N_f


J.O. Andersen, M. Strickland, and N. Su, Phys. Rev. D 80 (2009)
P/P_ideal
g_eff(2πT)
numerical exact
g_eff^4 perturbative
g_eff^5 perturbative
g_eff^6 perturbative
NLO HTLpt
NNLO HTLpt
$g_{\rm eff} = g\sqrt{N_f/2}$
0
1
2
3
4
5
0.2
0.4
0.6
0.8
1

New lattice data – Lattice 2012



Conclusions

- HTLpt works well for thermodynamics for T > 2 T_c
- Gauge invariant resummation → all order in g expressions based on high-T DOF
- Key is to properly treat high-T physics in terms of gauge-invariant quasiparticle DOF
- Coming soon: Work under way on finite chemical potential and susceptibilities – comparison with lattice looks VERY GOOD
- Hard-loop framework already applied to dynamics (real time plasma dynamics, plasma instabilities, etc)
- I came to Montpellier to begin a project to make things even better: nonlinear delta expansion and RG improvement?