Conformal/Walking Dynamics in Many-Flavor QCD

In The Light of Physics Beyond The Standard Model

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MontPellier Univ, July. 07, 2015

Motivation

- The Higgs boson with $M_{\rm H} \simeq 125$ GeV is discovered (2012, LHC-CERN)!
- The LHC second-run has started!

Why Not Investigate The Origin of EWSB?

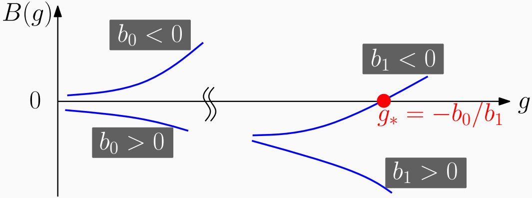
Strong Dynamics: Origin of EWSB

Questions/ Answers: New Strong Dynamics $\ni (F_i^a, G^{a\bar{a}})$

1. Physical Contents of Higgs?
   Composite Higgs $\bar{F}F$ (c.f. σ, Cooper-Pair).
2. Physics of Electroweak (EW) Symmetry Breaking?
   Chiral Symmetry Breaking.
3. Fine-Tuning Problem for $M_{\rm H} = 125$ GeV?
   Log (partly Power-Low) corrections for $M_{\rm H} = 125$ GeV.

Many Flavor, Quasi-Conformal (Walking) Dynamics

Banks-Zaks IR Fixed Point



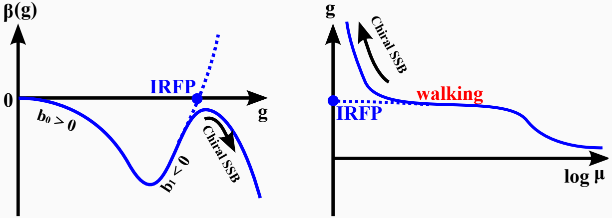
$$\beta(g) = dg(\mu)/d\mu = -b_0(N_f, N_c)g^3 - b_1(N_f, N_c)g^5 + \cdots \ , \quad (1)$$

1-Loop: $$b_0 = \frac{1}{(4\pi)^2}\left[\frac{11}{3}C_2[G] - \frac{4}{3}N_f T[r]\right] \ , \quad (2)$$

2-Loop: $$b_1 = \frac{1}{(4\pi)^4}\left[\frac{34}{3}C_2^2[G] - \left[\frac{20}{3}C_2[G] + 4C_2[r]\right]N_f T[r]\right] \ , \quad (3)$$

BZ-IRFP : $g_* = -b_0/b_1$ with $b_1 < 0$. (4)

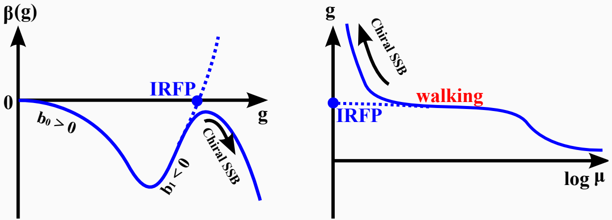
Walking Dynamics in Many-Flavor QCD



Schwinger-Dyson Equation

- If $(g_* = -b_0/b_1) > (g_{\rm cr} = N_c/(6C_2[r]))$, Chiral SSB spoils IRFP.
- Walking Dynamics in $8 \lesssim N_f \lesssim 12$, (Yamawaki et.al.('86), Holdom ('85))

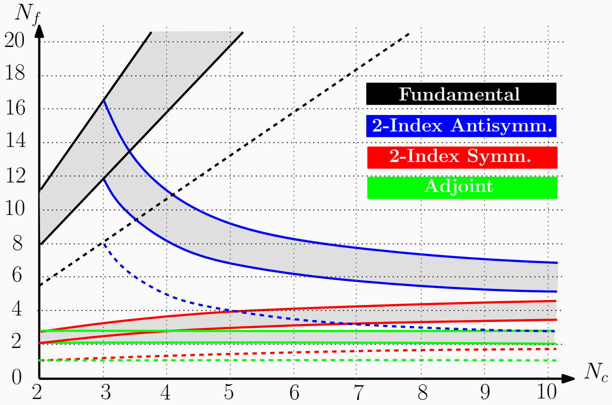
Many Flavor QCD and Walking II



Stable (light) Higgs: Techni-Dilaton $\bar{F}F$ (PNGB for Scale Sym Breaking).

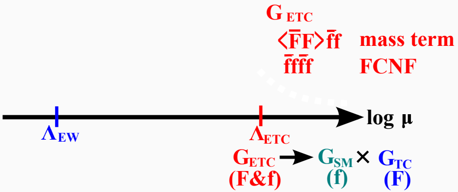
$N_f - N_c$ Phase Diagram

Ref.: Dietrich-Sannino PRD 2007.



Extended Technicolor Model

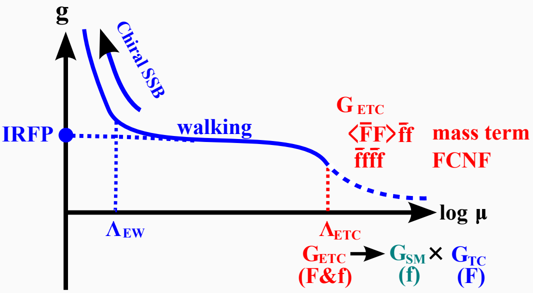
Ref.: Hill-Simmons Phys. Rept. 381 (2003).



- $\Lambda_{\rm ETC} \gtrsim 10^3$ TeV to suppress FCNC $\propto 1/\Lambda_{\rm ETC}^2$.
- SM Mass, $M_{q,l}|_{\mu=\Lambda_{\rm EW}} \propto \Lambda_{\rm EW}^3/\Lambda_{\rm ETC}^2$ gets too small.

Waling Technicolor Model

Refs.: Yamawaki et.al. ('86), Appelquist et.al. ('86).



$$M_{q,l}|_{\mu=\Lambda_{\rm EW}} \propto \frac{\Lambda_{\rm EW}^3}{\Lambda_{\rm ETC}^2} \times \left[\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm EW}}\right]^{\gamma} . \quad (5)$$

Subject of Our Study

We do not know much about Many Flavor QCD and Walking.

We Investigate...

- 8-flavor QCD $\ni (F^{a=\mathrm{rgb}}_{i=1,\cdots 8}, G^{a\bar{a}})$ with
  Lattice Gauge Theory with Monte Carlo Simulations.
  c.f. One Family Model.
- $\mathrm{SU_L}(N_f = 8) \times \mathrm{SU_R}(N_f) \to \mathrm{SU_V}(N_f)$ and Scale Symm. Breaking.
- Flavor Singlet Scalar Mass M_σ and Mass Anomalous Dimension γ.
- (The γ in the presence of UV/IR-Cuts by using SD Equation.)

Table of Contents

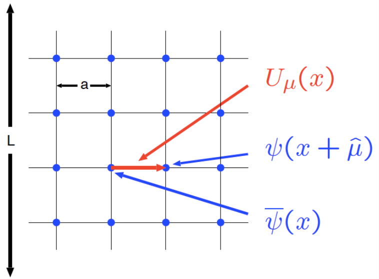
LatKMI Collaboration

Y. Aoki	(KMI, Nagoya Univ)
T. Aoyama	(KMI, Nagoya Univ)
E. Bennett	(Swansea Univ (UK))
M. Kurachi	(KEK)
T. Maskawa	(KMI, Nagoya Univ)
K. Miura	(CPT, Aix-Marseille Univ (RF) / KMI, Nagoya-Univ)
K-i. Nagai	(KMI, Nagoya Univ)
H. Ohki	(RIKEN)
T. Yamazaki	(Tsukuba Univ)
K. Yamawaki	(KMI, Nagoya Univ)
E. Rinaldi	(LLNL (US))
A. Shibata	(KEK)

Setups

Lattice Action:	$N_f = 8$ HISQ Action
	$+$ Tree-level Symanzik Gauge Action.
Argolithm:	HMC with Hasenbush pre-conditioning.
Configurations:	$\mathcal{O}(10^4)$ Configs.,
	$\beta = 3.8$, $L \in [12, 42]$, $m_f \in [0.012, 0.16]$
Observables:	F_π, M_π, M_ρ, M_{a1}, M_N, VPF, $\cdots$.
Code etc:	MILC ver.7.6.3 with some modifications, SciDac Libraly.
Computer:	KMI HPC Cluster φ,
	Nagoya-Univ-ITC CX400,
	Kyushu-Univ-RIIT CX400/HA8000.

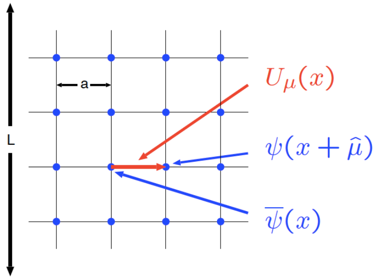
Lattice Gauge Theory I



From Talk by Yamazaki-san at KMI 2013-04-10.

- a^{-1} = UV Cutoff, L^{-1} = IR Cutoff.
- $(\psi, \bar{\psi})$: Fermions on Sites.
- $U_{\mu,x} \in \mathrm{SU}(N_c = 3)$: Gauge Fields as Shortest Wilson Lines.

Lattice Gauge Theory II



From Talk by Yamazaki-san at KMI 2013-04-10.

$$\langle O \rangle \equiv \int \mathcal{D}U\, \mathcal{P}[U] \cdot O[\psi, \bar{\psi}, U_\mu] \simeq \frac{1}{N_{\rm conf}} \sum_{n=1}^{N_{\rm conf}} O[U_n] + \mathcal{O}\left(\frac{1}{\sqrt{N_{\rm conf}}}\right) \quad (6)$$

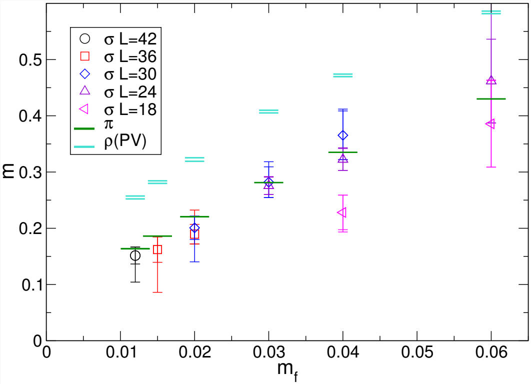
$$\mathcal{P}[U] \propto \int \mathcal{D}[\psi, \bar{\psi}]\, e^{-S_{\rm F}[\psi,\bar{\psi},U_\mu]-S_{\rm G}[U_\mu]} \,. \quad (7)$$

Lattice Gauge Theory III

$$\pi^a(\mathbf{x},t) = (\bar{\psi}\gamma_5 T^a \psi)(\mathbf{x},t).$$

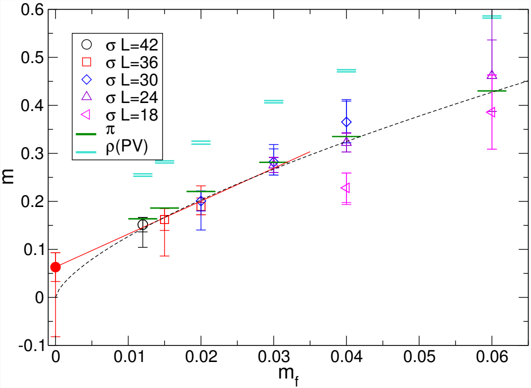
$$\begin{aligned} G_\pi(t) &= \sum_{\mathbf{x}} \langle 0|\pi(\mathbf{0},0)\pi(\mathbf{x},t)|0\rangle \\ &= \sum_{\mathbf{x}}\sum_n \int_{\mathbf{p}} \langle 0|\pi(0)|E_n(\mathbf{p})\rangle \frac{1}{2E_n(\mathbf{p})} \langle E_n(\mathbf{p})|\pi(0)|0\rangle e^{-E_n(\mathbf{p})t} \cdot e^{i\mathbf{p}\mathbf{x}} \\ &= \sum_n \frac{\left|\langle 0|\pi(0)|E_n(\mathbf{p})\rangle\right|^2}{2M_n} e^{-M_n t} + \cdots \\ &\to \frac{\left|\langle 0|\pi(0)|E_0(\mathbf{0})\rangle\right|^2}{2M_\pi} e^{-M_\pi t} + \mathcal{O}(e^{-(M_n - M_\pi)t}) \ . \end{aligned} \tag{8}$$

$N_f = 8$ Flavor Singlet Scalar σ I (Update from LatKMI PRD 2014)



Light $\sigma \sim$ Dilaton (PNGB for Broken Scale Symm.)

$N_f = 8$ Flavor Singlet Scalar σ II (Update from LatKMI PRD 2014)



Light $M_\sigma|_{m_f \to 0} \sim 0 - 780$ GeV (c.f. $F_\pi|_{m_f \to 0} = 246/\sqrt{2}$ GeV).
(c.f. LatHC Collab. ('14), Hietanen et.al. ('14), Athenodorou et.al. ('15)).

Dilaton ChPT

Ref.: Matsuzaki-Yamawaki PRL 2014. (c.f. Crewther et.al. PRD 2015).

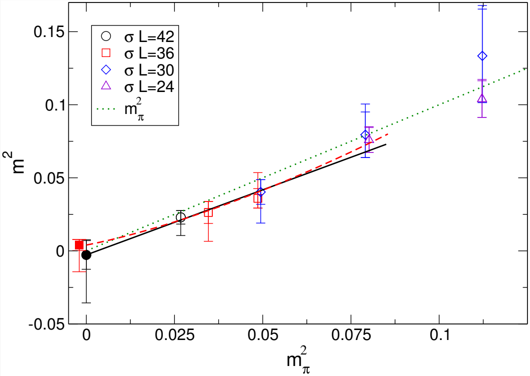
$$\begin{aligned} Z &= \int \mathcal{D}[\psi,\bar{\psi},A_\mu]\ e^{iS[\psi,\bar{\psi},A_\mu]} \\ &= \int \mathcal{D}[U,V]\ e^{iS_{\rm eff}[U,V]}\ , \\ (U,V) &= (e^{2i\pi^a(x)T^a/F_\pi}, e^{\sigma(x)/F_\sigma})\ . \end{aligned} \tag{9}$$

We expand $S_{\rm eff}$ in terms of $(\pi(x),\sigma(x))$ and read off coefficients of their quadratic terms, giving mass terms of them:

$$M_\sigma^2 = m_\sigma^2 + D\cdot M_\pi^2\ , \tag{10}$$

$$D \equiv \frac{(3-\gamma)(1+\gamma)}{4}\frac{2N_f F_\pi^2}{F_\sigma^2}\ . \tag{11}$$

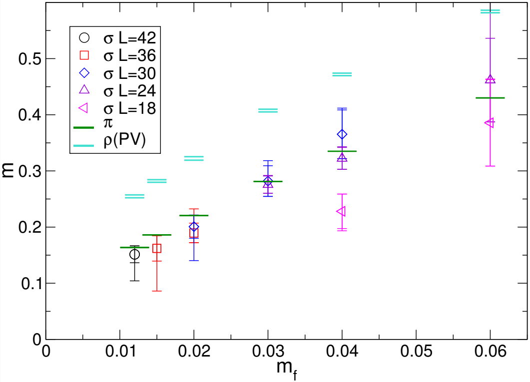
DChPT Fit



$$m_\sigma = 0 - 597 \quad \text{GeV} \ , \tag{12}$$

$$F_\sigma \simeq \sqrt{N_f} F_\pi|_{m_f \to 0} D_1 \simeq 438 \quad \text{GeV} \ . \tag{13}$$

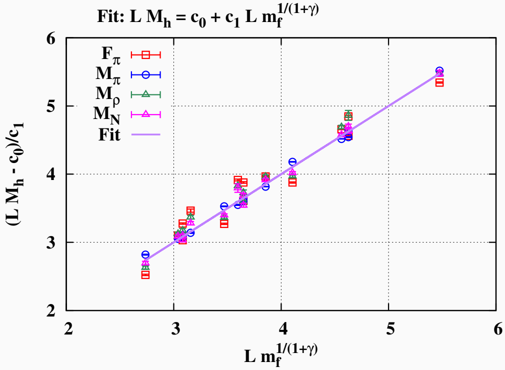
$N_f = 8$ Techni-rho Mass M_ρ (Update from LatKMI PRD 2014)



$M_\rho \sim 860 - 1930$ (GeV).

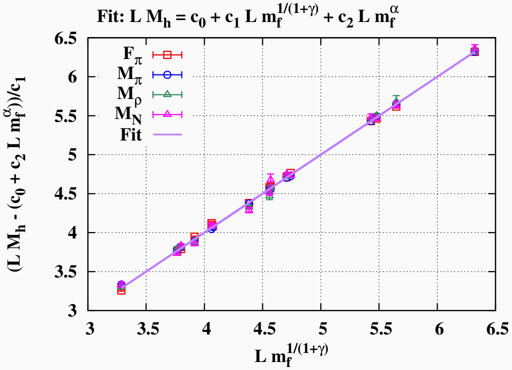
c.f. Fukano et. al. ('15).

Finite-Size Hyper-Scaling (FSHS)



$$(\gamma, \chi^2/\mathrm{dof}) = (0.709(2), 124.96).$$

FSHS with Mass Collection



$$(\gamma, \chi^2/\mathrm{dof}) = (0.893(15), 3.26) \text{ for } \alpha = 1.$$

Summary on $N_f = 8$ QCD

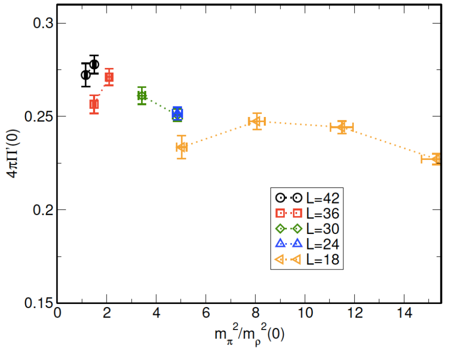
FSHS Individual	F_π	M_π	M_ρ	$M_{\rm N}$
γ	1.003(5)	0.627(2)	0.896(11)	0.810(11)
χ^2/dof	2.34	15.26	1.41	2.58

FSHS Simultaneous	γ	χ^2/dof
$LM_h = c_0 + c_1 X,\ X = Lm_f^{1/(1+\gamma)}$	0.709(2)	124.96
$LM_h = c_0 + c_1 X + c_2 Lm_f^{\alpha=1}$	0.893(15)	3.26
$LM_h = c_0 + c_1 X + c_2 Lm_f^{\alpha=2}$	0.772(5)	19.38
$LM_h = (1 + c_2 m_f^{\omega=0.35})(c_0 + c_1 X)$	1.014(35)	2.46
$LM_h = c_0 + c_1 X + c_2 L \exp(-kX)\vert_{k=0.1}$	0.617(2)	12.80

- For $m_f \in [0.012, 0.03]$, the quadratic fit (motivated by ChPT) works for all measured operators. However, $N_f(M_\pi/(4\pi F/\sqrt{2}))^2 \gtrsim 6$ and $FL \lesssim 0.85$.
- For the same m_f range, the naive hyper-scaling fit also works except few cases with somewhat larger χ^2/dof. The γ is operator dependent.

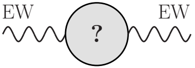
$N_f = 8$ QCD: Having Light σ, Showing Quasi-Conformal Nature with $\gamma \sim 1$.

$N_f = 8$ S-Parameter



$S \equiv 4\pi\Pi'_{V-A}(Q^2 \to 0)$

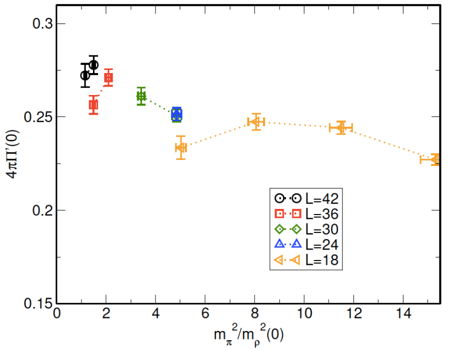
$\Pi_{V-A} \sim$

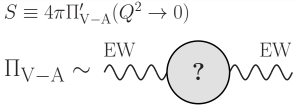


The result $S \sim 0.25 - 0.275$ is smaller than that $S_{\mathrm{QCD},N_f=2} \sim 0.43$.

For the latter, Ref.: JLQCD PRL 2008, P. Boyle et.al. PRD 2010, LSD-Collab. PRD 2014.

$N_f = 8$ S-Parameter

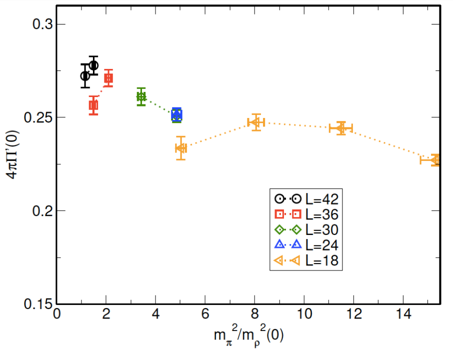




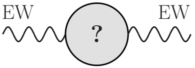
Dispersion Relation (c.f. Knecht et.al.(Large N_c '98), LSD-Collab.('14))

- $\Pi_{\rm V-A}(Q^2) = -F_\pi + \frac{Q^2}{12\pi}\int_0^\infty \frac{ds}{\pi}\frac{R_{\rm V}-R_{\rm A}}{s+Q^2}$.
- $S \propto \Pi'_{\rm V-A}(Q^2 \to 0) =$ small: Parity Doubling $R_{\rm V} \sim R_{\rm A}$.
- $(F_{\rm V}, M_{\rm V}) \simeq (F_{\rm A}, M_{\rm A})$ with $R_{\rm V/A} \simeq 12\pi^2 F_{\rm V/A}^2 \delta(s - M_{\rm V/A}^2)$

$N_f = 8$ S-Parameter



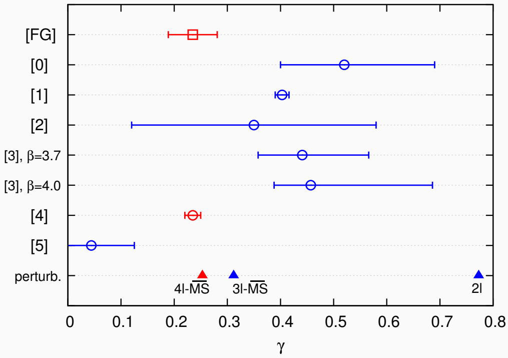
$S \equiv 4\pi\Pi'_{V-A}(Q^2 \to 0)$



Still much larger than $S_{\rm exp} = 0.03(10)$.

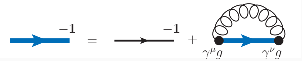
Pions: $(N_f^2 - 1) = 63 = 3 + 60$. Should Be Subtracted (Future Work).

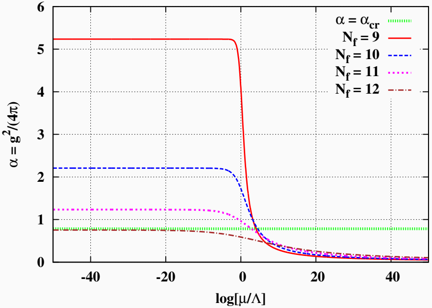
Motivation: Lattice Results on γ in Color $SU(N_c = 3)$ with $N_f = 12$



[FG] Nunes et.al. ('14), [0] Deuzemann et.al. ('09), [1] LSD-Collab ('11), (using data provided in [7]), [2] DeGrand ('11), (using data provided in [7]), [3] LatKMI ('12), [4] Cheng et.al. ('13), [5] Itou ('13), [6] Shrock ('13) [7] Fodor et.al. ('11).

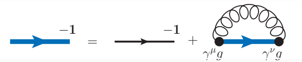
SD with Too-Loop Improved Ladder Approx.





- $N_f \geq 8.05$: Banks-Zaks (BZ) Infra-Red Fixed Point (IRFP, α_*).
- $N_f \gtrsim 12$: Conformal Window with $\alpha_* \leq \alpha_{\rm cr} = \pi/(3C_2[F])$.

SD with $\Lambda_{\mathrm{UV/IR}}$



$$iS_{\mathrm{F}}^{-1}(q) = \left[A(p^2)\not{p} - \Sigma(p^2)\right] \text{ (Fermion Invers-Prop.) },$$

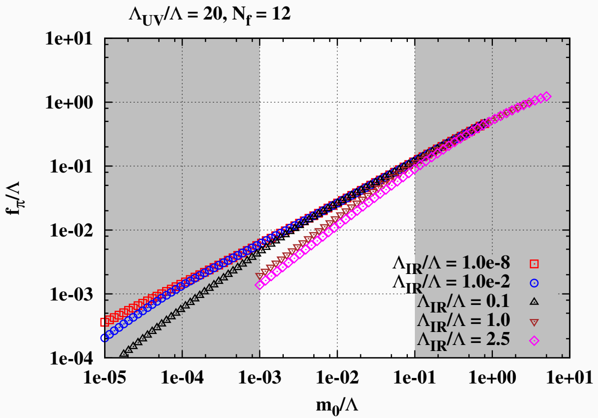
$$m_p(m_0, \Lambda_{\mathrm{UV/IR}}) = \Sigma(p^2 = m_p; m_0, \Lambda_{\mathrm{UV/IR}}) \text{ , (Pole Mass, SD-Output) },$$

$$f_\pi^2(m_0, \Lambda_{\mathrm{UV/IR}}) = \frac{N_c}{4\pi^2}\int dz\, z\, \frac{\left(1 - \frac{1}{4} z \frac{d}{dz}\right)\Sigma^2(z)}{\left(z + \Sigma^2(z)\right)^2} \text{ , (Pagels-Stoker) }.$$

State of The Arts (c.f. Previous Work (LatKMI '09))

- The full momentum dependence of the two-loop running coupling g ($\Lambda_{\mathrm{UV/IR}}$ vs scales encoded in g).
- Wide range probe in the parameter space ($\Lambda_{\mathrm{IR}}/\Lambda \in [10^{-8}, 2.5]$ and $\Lambda_{\mathrm{UV}}/\Lambda \in [1.0, 20]$ including $f_\pi/\Lambda_{\mathrm{IR}} \sim f_\pi L \sim 1$).

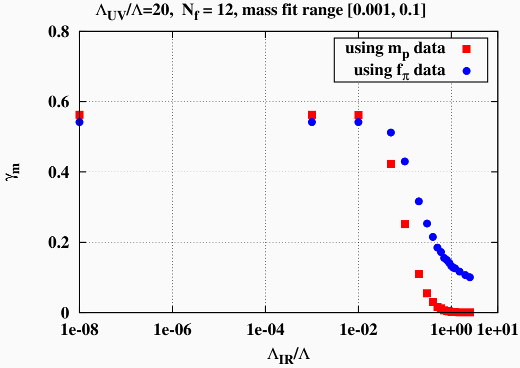
$N_f = 12$ f_π vs m_0



$$M_h = C m_0^{1/(1+\gamma_m)} \,, \quad M_h = m_p \text{ or } f_\pi \,. \tag{14}$$

$N_f = 12$ γ_m vs Λ_{IR}

$$M_h = C m_0^{1/(1+\gamma_m)} \ , \quad M_h = m_p \text{ or } f_\pi \ . \tag{15}$$



SD-Based FSHS: Formulation

$$\frac{m_0}{Z_{\rm UV}\, m_p} = \frac{A_{\omega_m}(y_{\rm IR}) D_{\omega_m}(y_\Lambda) \left(\frac{1+y_{\rm IR}}{1+y_\Lambda}\right)^{(1-\omega_m)/2} + (\omega_m \leftrightarrow -\omega_m)}{A_{\omega_m}(y_{\rm IR}) N_{\omega_m}(\max\{1,y_{\rm IR}\}) \left(\frac{1+y_{\rm IR}}{1+\max\{1,y_{\rm IR}\}}\right)^{(1-\omega_m)/2} + (\omega_m \leftrightarrow -\omega_m)} ,$$

$$A_{\omega_m}(y) = \frac{1+\omega_m}{2\omega_m} F\left[\frac{-1+\omega_m}{2}, \frac{-1+\omega_m}{2}, 1+\omega_m; \frac{1}{1+y}\right] ,$$

$$D_{\omega_m}(y) = \frac{1+\omega_m}{2} F\left[\frac{1-\omega_m}{2}, \frac{1-\omega_m}{2}, 1-\omega_m; \frac{1}{1+y}\right] ,$$

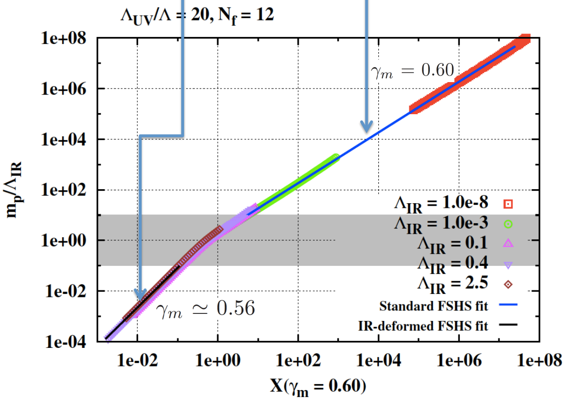
$$N_{\omega_m}(y) = F\left[\frac{1-\omega_m}{2}, \frac{3-\omega_m}{2}, 1-\omega_m; \frac{1}{1+y}\right] , \quad (16)$$

$$\omega_m = 1-\gamma_m , \quad (y_{\rm IR}, y_\Lambda) = \left(\frac{\Lambda_{\rm IR}^2}{m_p^2}, \frac{\Lambda^2}{m_p^2}\right) . \quad (17)$$

$$\frac{\hat{m}_p}{\hat{\Lambda}_{\rm IR}} = \begin{cases} C_{\rm X}(\gamma_m, Z_m^{\rm UV}) \cdot X , & X \equiv \hat{m}_0^{1/(1+\gamma_m)}/\hat{\Lambda}_{\rm IR} \quad (\Lambda_{\rm IR} \ll m_p \ll \Lambda) , \\ C_{\rm Y}(\gamma_m, Z_m^{\rm UV}) \cdot Y , & Y \equiv \hat{m}_0/\hat{\Lambda}_{\rm IR}^{-(1+\gamma_m)} \quad (m_p \ll \Lambda_{\rm IR} \ll \Lambda) . \end{cases} \quad (18)$$

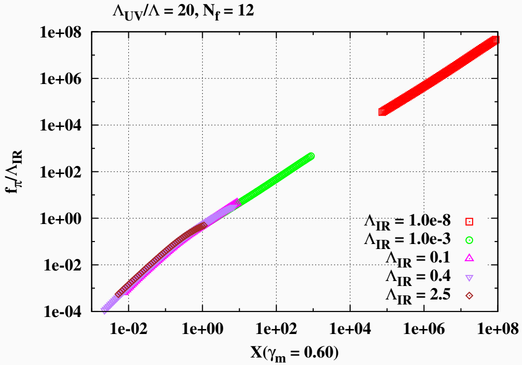
$N_f = 12$ SD-Based FSHS I

$$\frac{\hat{m}_p}{\hat{\Lambda}_{\rm IR}} = \begin{cases} C_{\rm X}(\gamma_m, Z_m^{\rm UV}) \cdot X \\ C_{\rm Y}(\gamma_m, Z_m^{\rm UV}) \cdot Y \end{cases}$$



$N_f = 12$ SD-Based FSHS II

$$\frac{\hat{m}_p}{\hat{\Lambda}_{\rm IR}} = \begin{cases} C_{\rm X}(\gamma_m, Z_m^{\rm UV}) \cdot X \ , & X \equiv \hat{m}_0^{1/(1+\gamma_m)}/\hat{\Lambda}_{\rm IR} \quad (\Lambda_{\rm IR} \ll m_p \ll \Lambda) \ , \\ C_{\rm Y}(\gamma_m, Z_m^{\rm UV}) \cdot Y \ , & Y \equiv \hat{m}_0/\hat{\Lambda}_{\rm IR}^{-(1+\gamma_m)} \quad (m_p \ll \Lambda_{\rm IR} \ll \Lambda) \ . \end{cases} \tag{19}$$



c.f. $f_\pi L$ in recent lattice results: [1] Fodor et.al. ('11), [2] Cheng et.al. ('14), [3] DeGrand ('11).

Summary

8-flavor QCD (LatKMI)

8-Flavor QCD has Light σ, and Quasi-Conformal Nature with $\gamma \sim 1$.

- A viable candidate of Walking Technicolor Model (One-Family Model).
- Theoretically interesting as a frontier of the gauge theory.

SD with $\Lambda_{\mathrm{UV/IR}}$ (Miura-Nagai-Shibata)

- The γ is strongly suppressed when the IR cutoff Λ_{IR} gets comparable to the scale (m_p, f_π).
- The SD-based FSHS allows us to avoid the suppression, explains how two slopes in FSHS result from the IR cutoffs.
- The fomulas applicable for lattice mass spectra are desirable.

Perspective

Perspective

- The scalar σ is an important subject in real-life QCD. (c.f. Crewther-Tunstall PRD2015)
- The S-Parameter (VPF) analyses are applicable for Muon $(g-2)$ physics. LABEX-OCEVU Project in CPT-Marseille

One Family Model

Farhi-Susskind model: Farhi-Susskind (1979), Dimopoulos (1980)

$$\underbrace{\begin{pmatrix} u \\ d \\ u \\ d \\ u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ c \\ s \\ c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} t \\ b \\ t \\ b \\ t \\ b \\ \tau \\ \nu_\tau \end{pmatrix}}_{\text{SM fermions}} \quad \underbrace{\begin{pmatrix} U_1 \\ D_1 \\ U_1 \\ D_1 \\ U_1 \\ D_1 \\ E_1 \\ N_1 \end{pmatrix} \cdots \begin{pmatrix} U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ E_{N_{\rm TC}} \\ N_{N_{\rm TC}} \end{pmatrix}}_{\text{Techni-fermions}}$$

Tambling

Self-breaking of the ETC gauge group