Lattice study of the Higgs-Yukawa model for BSM physics

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Outline

- Motivation.
- General consideration.
- New physics: A dim-6 operator as a representative.
- Outlook.

Motivation

- Below 750 GeV, no obvious deviation from the SM hitherto.
- Assuming that the Higgs-Yukawa sector has only a Gaussian fixed point. (discuss offline)
- The SM must be replaced by its UV completion.
- The scale for new physics is unknown.
- Triviality of the quartic coupling means higher-dim operators may play a role.

Motivation
Do we know the Higgs potential well?

- Textbook thingy

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4$$

- How about....

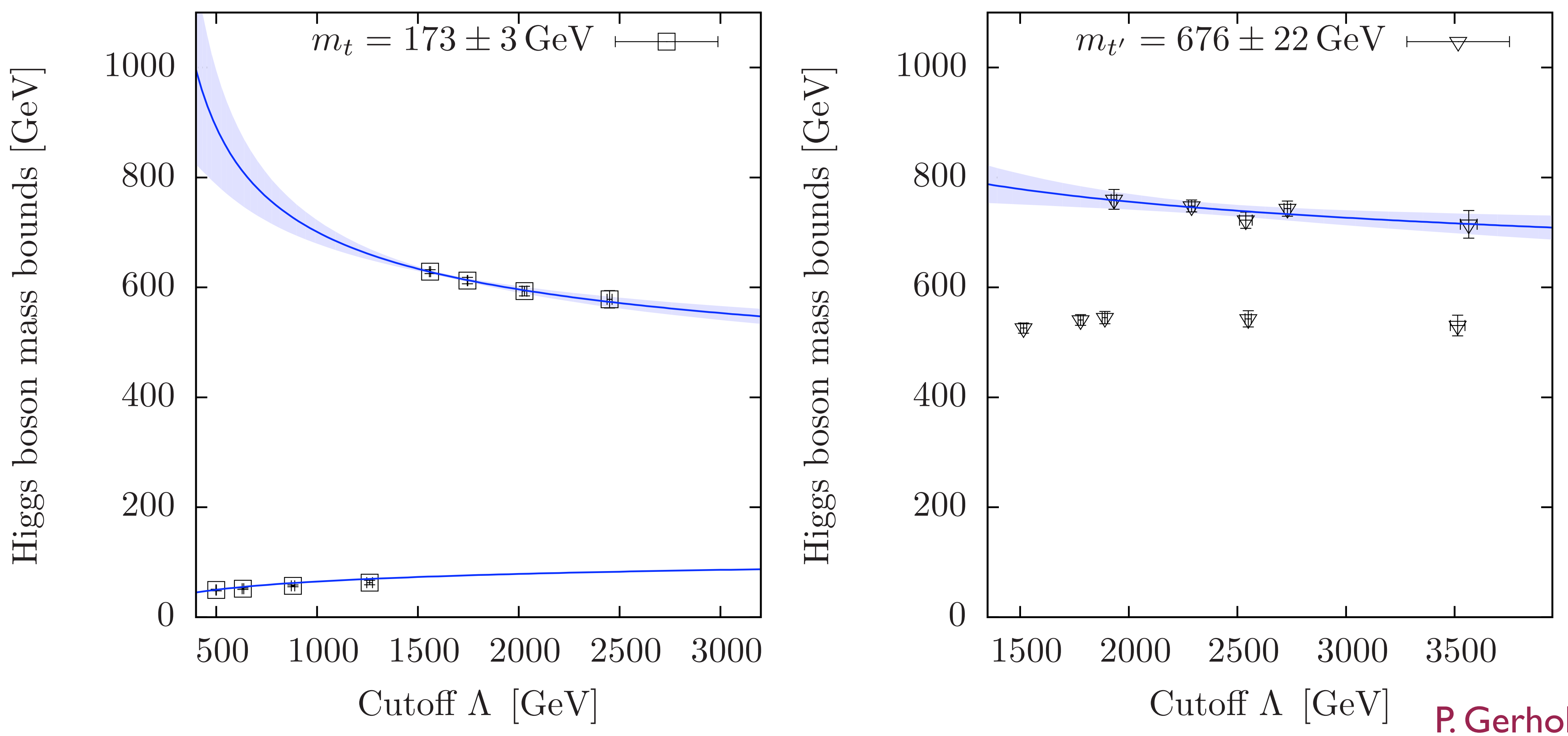
$$\tilde{V}(H) = -\lambda|H|^4 + c_6|H|^6$$

$$v^2 = \frac{4}{3}\frac{\lambda}{c_6}\,, \quad \frac{m_h^2}{v^2} = 2\lambda \quad \Longrightarrow \quad c_6 v^2 \sim 0.17$$

- Better data in the Higgsicion era.
- Lattice computation can play a role.

What the lattice did in the past…

$\lambda_6 = 0$



P. Gerhold and K. Jansen, 2011

* Constraints on the masses of extra-generation fermions from the 125 GeV scalar.

The lattice regularisation

- Discrete space-time points.
- Finite-volume.

➡ Finite number of DoF.

- Monte-Carlo method to evaluate the path integral,

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U] \, \mathrm{e}^{-S_G[U]} \, O[U] \quad \text{with} \quad Z = \int \mathcal{D}[U] \, \mathrm{e}^{-S_G[U]} \, .$$

➡ $$\langle O \rangle \approx \frac{1}{N} \sum_{\substack{U_n \text{ with} \\ \text{probability} \\ \propto \, \mathrm{e}^{-S[U_n]}}} O[U_n]$$

→ generated with a Markov chain

The continuum theory

$$S^{\mathrm{cont}}[\bar{\psi}, \psi, \varphi] = \int d^4x \left\{ \frac{1}{2} \left(\partial_\mu \varphi\right)^\dagger \left(\partial^\mu \varphi\right) + \frac{1}{2} m_0^2 \varphi^\dagger \varphi + \lambda \left(\varphi^\dagger \varphi\right)^2 + \lambda_6 \left(\varphi^\dagger \varphi\right)^3 \right\}$$
$$+ \int d^4x \left\{ \bar{t} \partial\!\!\!/ t + \bar{b} \partial\!\!\!/ b + y \left(\bar{\psi}_L \varphi \, b_R + \bar{\psi}_L \tilde{\varphi} \, t_R \right) + h.c. \right\},$$

$$\psi = \begin{pmatrix} t \\ b \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi^2 + i\varphi^1 \\ \varphi^0 - i\varphi^3 \end{pmatrix}, \quad \tilde{\varphi} = i\tau_2 \varphi^*.$$

Note: degenerate Yukawa couplings

The lattice theory

- Bosonic component:

$$S_B[\Phi] = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[\Phi_{x+\mu} + \Phi_{x-\mu}\right] + \sum_x \left(\Phi_x^\dagger \Phi_x + \hat{\lambda} \left[\Phi_x^\dagger \Phi_x - 1\right]^2 + \hat{\lambda}_6 \left[\Phi_x^\dagger \Phi_x\right]^3 \right).$$

$$a\varphi = \sqrt{2\kappa} \begin{pmatrix} \Phi^2 + i\Phi^1 \\ \Phi^0 - i\Phi^3 \end{pmatrix}, \; a^2 m_0^2 = \frac{1 - 2\hat{\lambda} - 8\kappa}{\kappa}, \; \lambda = \frac{\hat{\lambda}}{4\kappa^2}, \; a^{-2}\lambda_6 = \frac{\hat{\lambda}_6}{8\kappa^3}.$$

- Fermionic component: the overlap fermions.

→ Exact lattice chiral cymmetry.

The continuum limit

$$a \to 0 \text{ and } \Lambda \to \infty$$

- Supercomputers only know "pure numbers".
- All couplings are rescaled to be in lattice units.
- For a theory with asymptotic freedom, with symmetry "protecting" the mass, e.g., QCD:

$$g_0^2(a) \stackrel{a \to 0}{\longrightarrow} 0, \; am_0 \stackrel{a \to 0}{\longrightarrow} 0$$

while

$$g_{\mathrm{R}}^2(\mu, a) \stackrel{a \to 0}{=} \text{finite}, \; aM_{\mathrm{R}} \stackrel{a \to 0}{\longrightarrow} 0 \text{ with } M_{\mathrm{R}} = \text{finite and} \ll \Lambda.$$

➡ Keep lowering the dimensionless bare couplings.

The continuum limit

$a \to 0$ and $\Lambda \to \infty$

- A trivial theory w/o symmetry to "protect" the mass:

$$g_0^2(a) \xrightarrow{a\to 0} \text{finite},\ am_0 \xrightarrow{a\to 0} \text{finite}$$

while

$$g_{\mathrm{R}}^2(\mu, a) \stackrel{a\to 0}{=} 0,\ am_{\mathrm{R}} \xrightarrow{a\to 0} 0.$$

- In practice, we input the bare coupling:

$$g_0^2(a),\ am_0 = \text{arbitrary number}.$$

➡ Scanning in bare couplings, and keep the cut-off.

The continuum limit

$$a \to 0 \text{ and } \Lambda \to \infty$$

- The key point is the separation of the scales.
- It can be achieved at 2nd-order bulk phase transitions:

$$\xi / a \longrightarrow \infty.$$

→ non-thermal

- Condensed matter physics:

At fixed a, take $\xi \to \infty$.

- For our purpose:

At fixed ξ, take $a \to 0$.

The constraint effective potential

Fukuda and Kyriakopoulos, 1985

- Phase structure is probed using the Higgs vev,

$$\hat{v} = a\varphi_c = \langle \hat{m} \rangle = \left\langle \frac{1}{V} \left| \sum_x \Phi_x^0 \right| \right\rangle .$$

- The constraint effective potential is a useful tool,

$$\mathrm{e}^{-V U(\hat{v})} \sim \int \mathcal{D}\varphi \mathcal{D}\bar{\psi} \mathcal{D}\psi \; \delta\left(\varphi_0^0 - \varphi_c\right) \; \mathrm{e}^{-S[\varphi,\bar{\psi},\psi]},$$

$$\text{where } \varphi_0^0 = \frac{1}{V} \int d^4x \; \varphi^0 .$$

- Analytically calculated in perturbation theory.
- Numerically obtained by histograming $\hat{m}$.

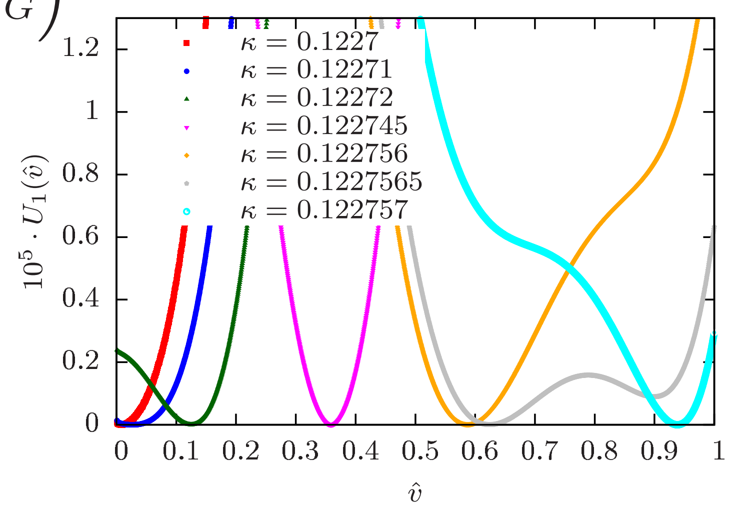
The constraint effective potential

$$U_1(\hat{v}) = U_f(\hat{v}) + \frac{m_0^2}{2}\hat{v}^2 + \lambda\hat{v}^4 + \lambda_6\hat{v}^6$$
$$+ \lambda\cdot\hat{v}^2\cdot 6(P_H + P_G) + \lambda_6\cdot\left(\hat{v}^2\cdot(45P_H^2 + 54P_GP_H + 45P_G^2) + \hat{v}^4\cdot(15P_H + 9P_G)\right).$$

$$U_2(\hat{v}) = U_f(\hat{v}) + \frac{m_0^2}{2}\hat{v}^2 + \lambda\hat{v}^4 + \lambda_6\hat{v}^6$$
$$+ \frac{1}{2V}\sum_{p\neq 0}\log\left[\left(\hat{p}^2 + m_0^2 + 12\lambda\hat{v}^2 + 30\lambda_6\hat{v}^4\right)\cdot\left(\hat{p}^2 + m_0^2 + 12\lambda\hat{v}^2 + 30\lambda_6\hat{v}^4\right)^3\right]$$
$$+ \lambda\left(3\,\tilde{P}_H^2 + 6\,\tilde{P}_H\tilde{P}_G + 15\,\tilde{P}_G^2\right) + \lambda_6\hat{v}^2\left(45\,\tilde{P}_H^2 + 54\,\tilde{P}_H\tilde{P}_G + 45\,\tilde{P}_G^2\right)$$
$$+ \lambda_6\left(15\,\tilde{P}_H^3 + 27\,\tilde{P}_H^2\tilde{P}_G + 45\,\tilde{P}_H\tilde{P}_G^2 + 105\,\tilde{P}_G^3\right),$$

where

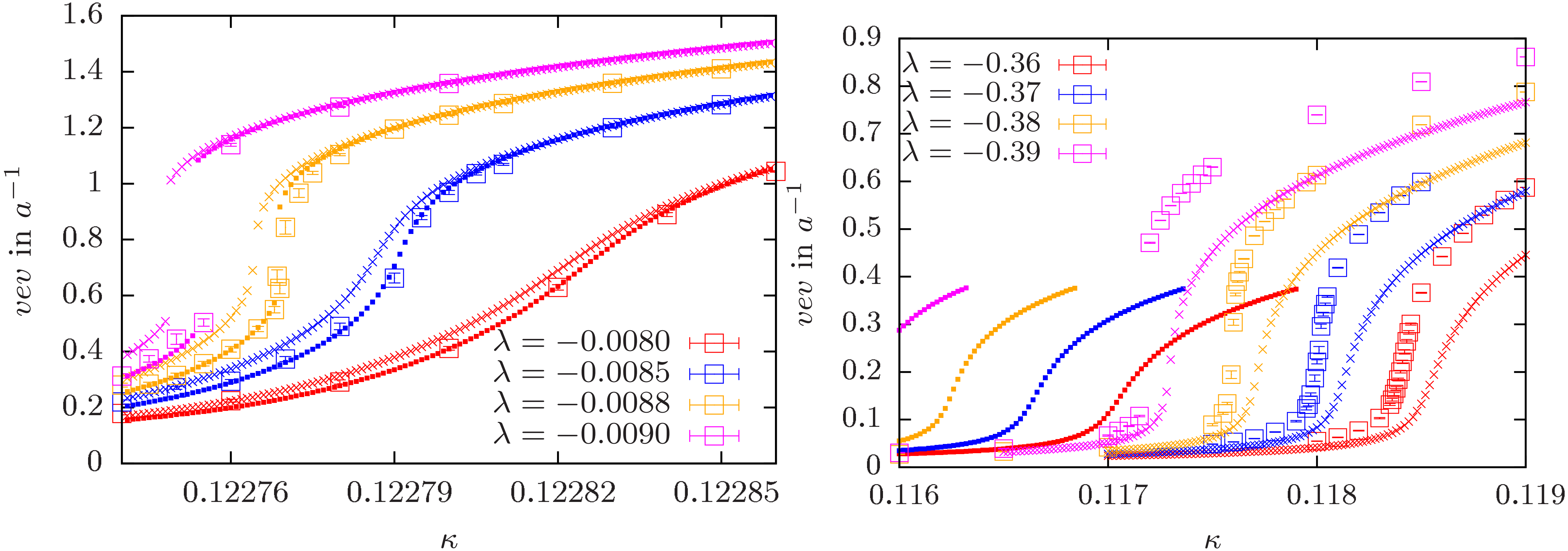
$$U_f(\hat{v}) = -\frac{4}{V}\sum_p \log\left|\nu^+(p) + y\cdot\hat{v}\cdot\left(1 - \frac{\nu^+(p)}{2\rho}\right)\right|^2$$



Investigate the non-thermal phase structure

With the dimension-6 operator

y tuned to have $m_t = 173$ GeV.

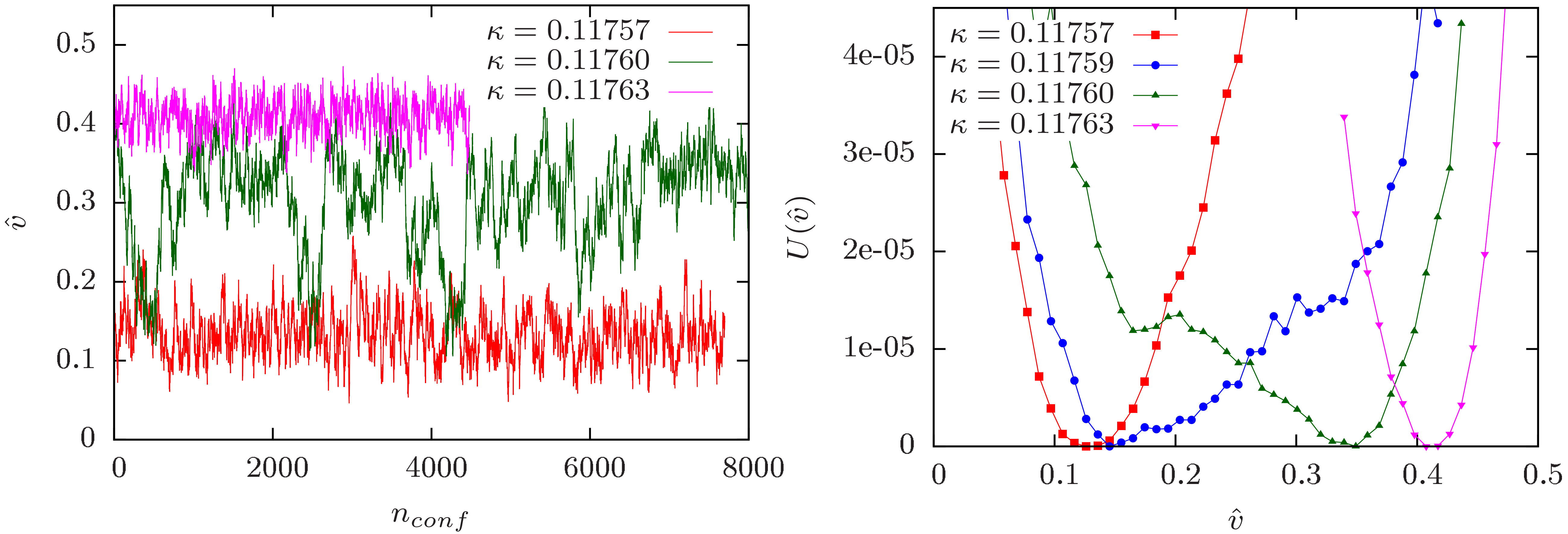


(a) $\lambda_6 = 0.001$ (b) $\lambda_6 = 0.1$

With the dimension-6 operator

y tuned to have $m_t = 173$ GeV.

$$\lambda_6 = 0.1 \text{ and } \lambda = -0.38$$

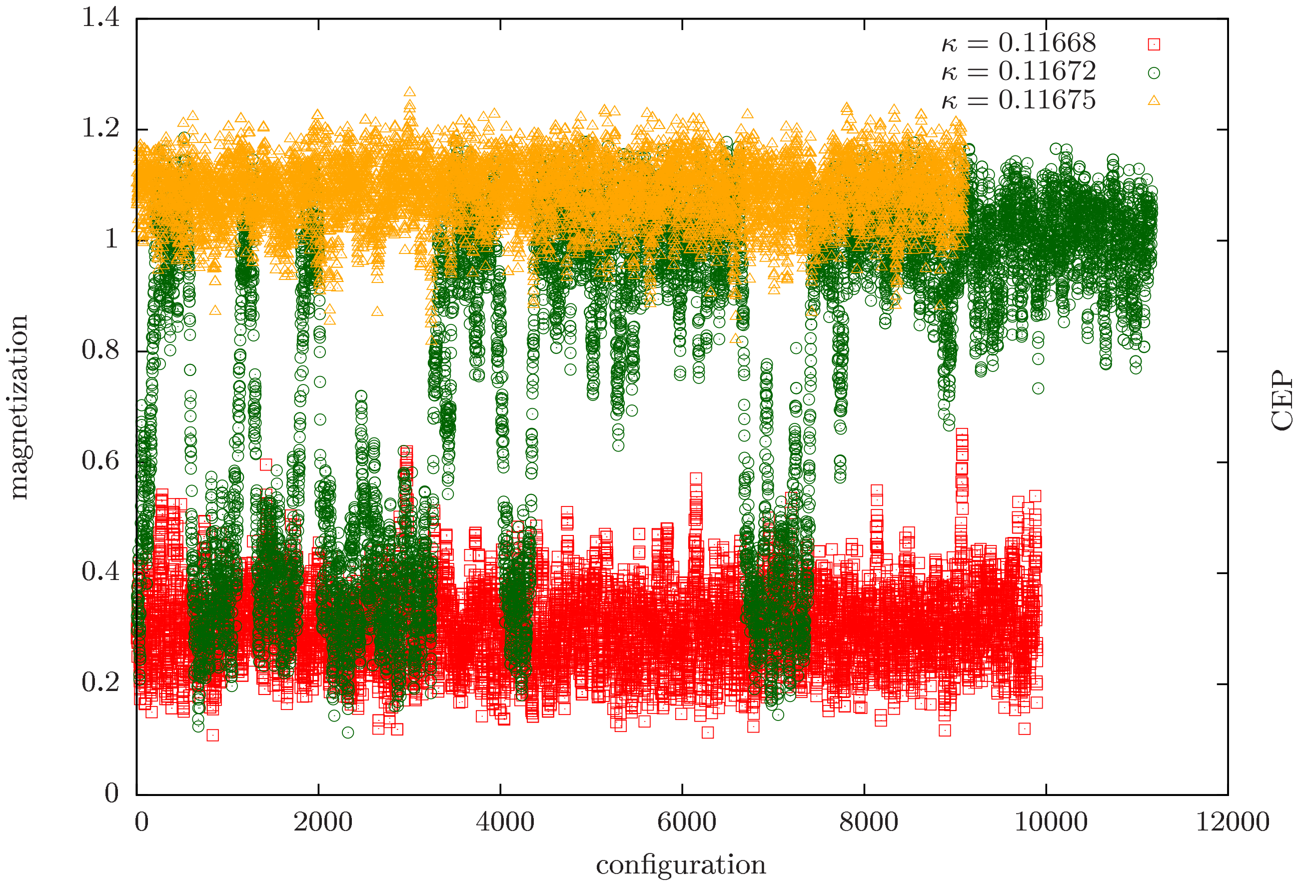


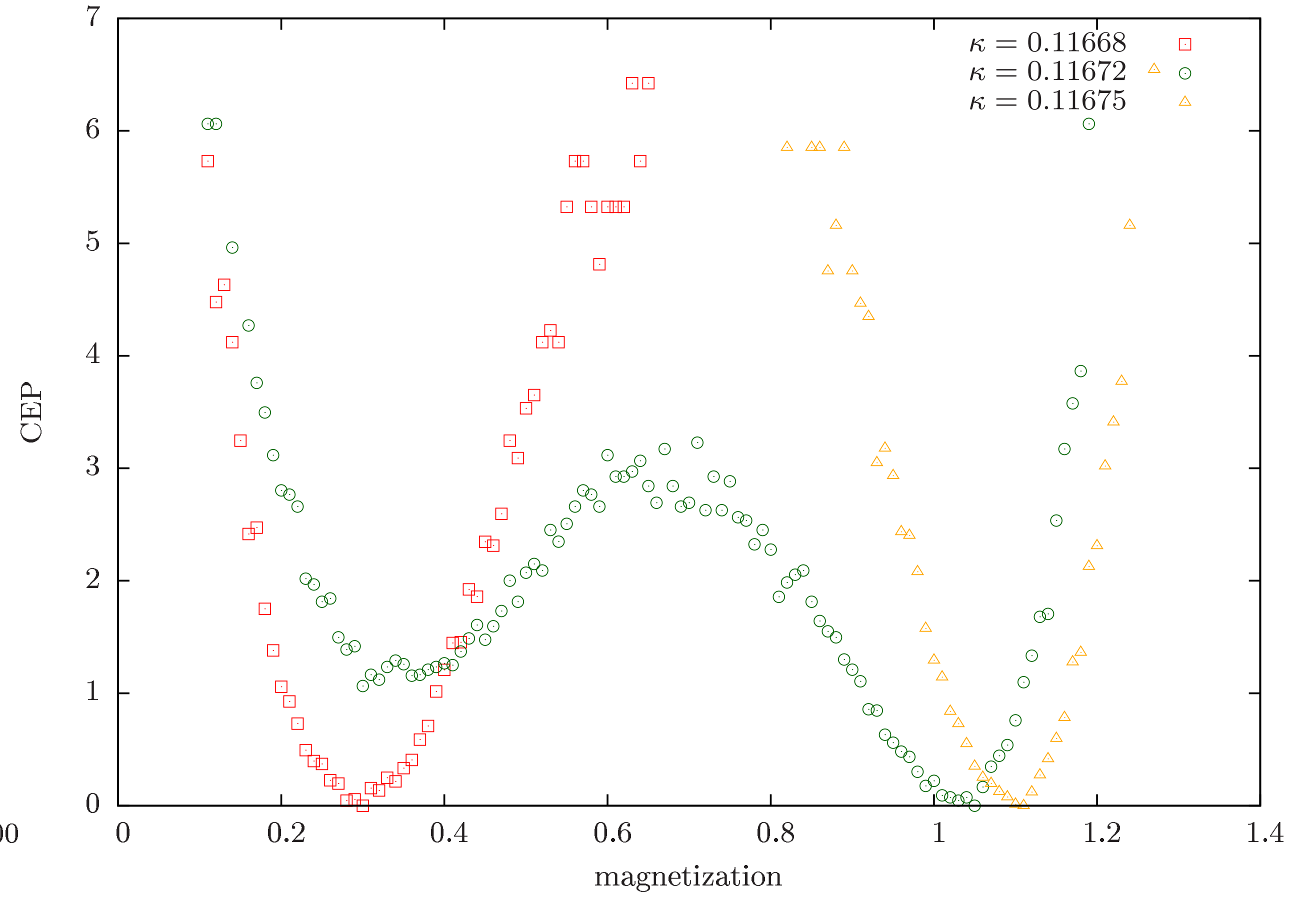
First-order phase transition expected

With the dimension-6 operator

y tuned to have $m_t = 173$ GeV.

$\lambda_6 = 0.1$ and $\lambda = -0.40$

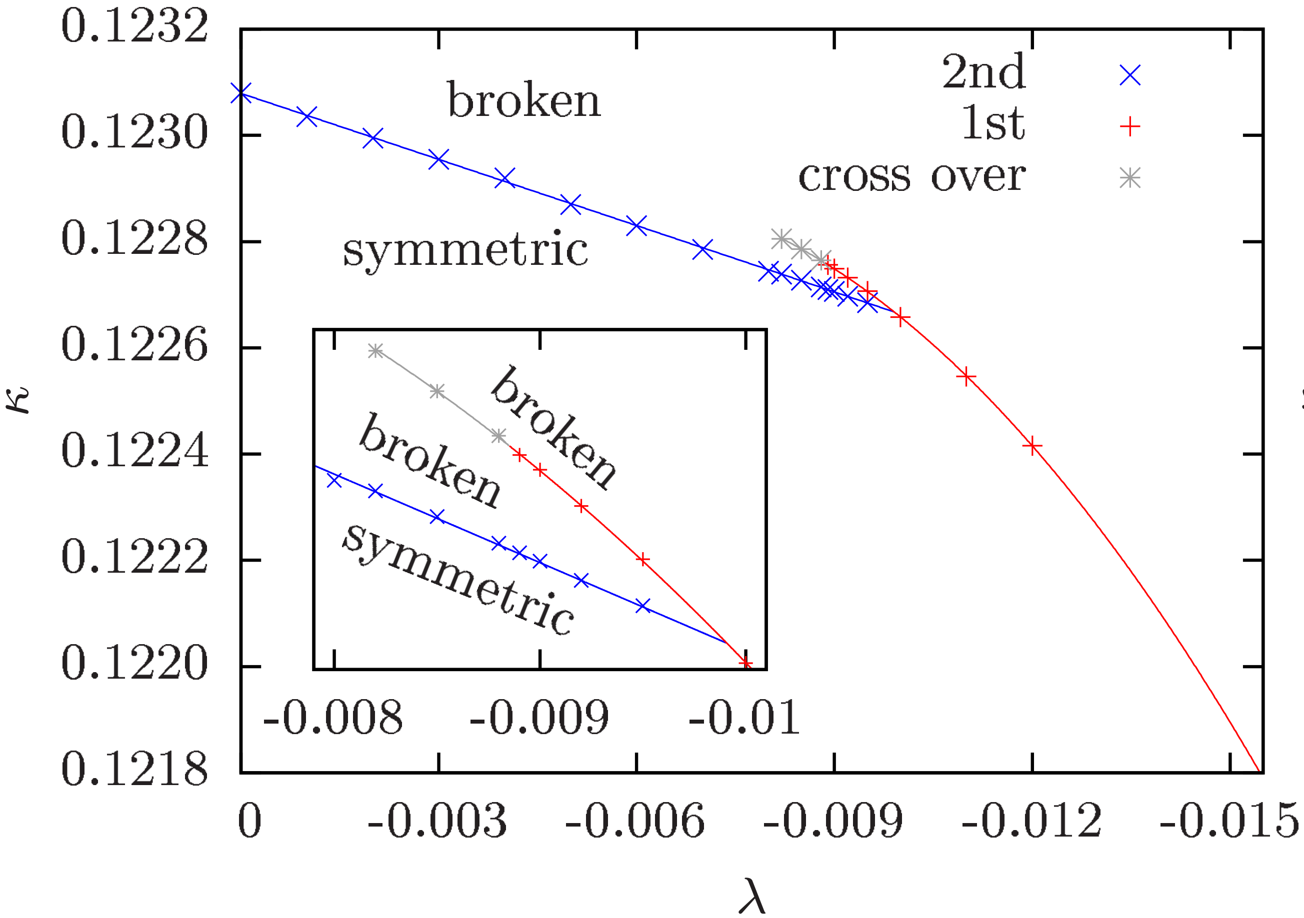


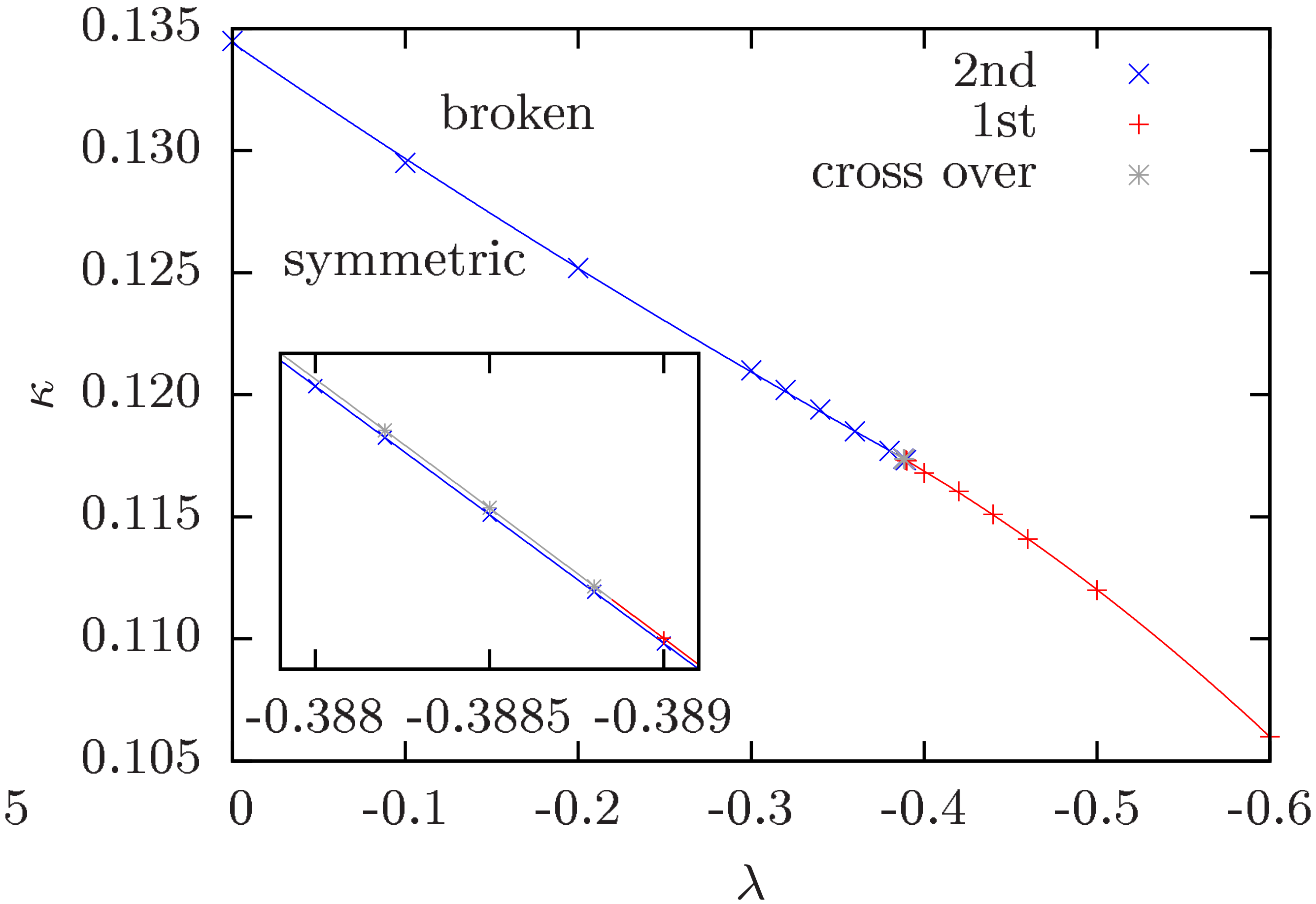


First-order phase transition expected

The phase structure

y tuned to have $m_t = 173$ GeV.

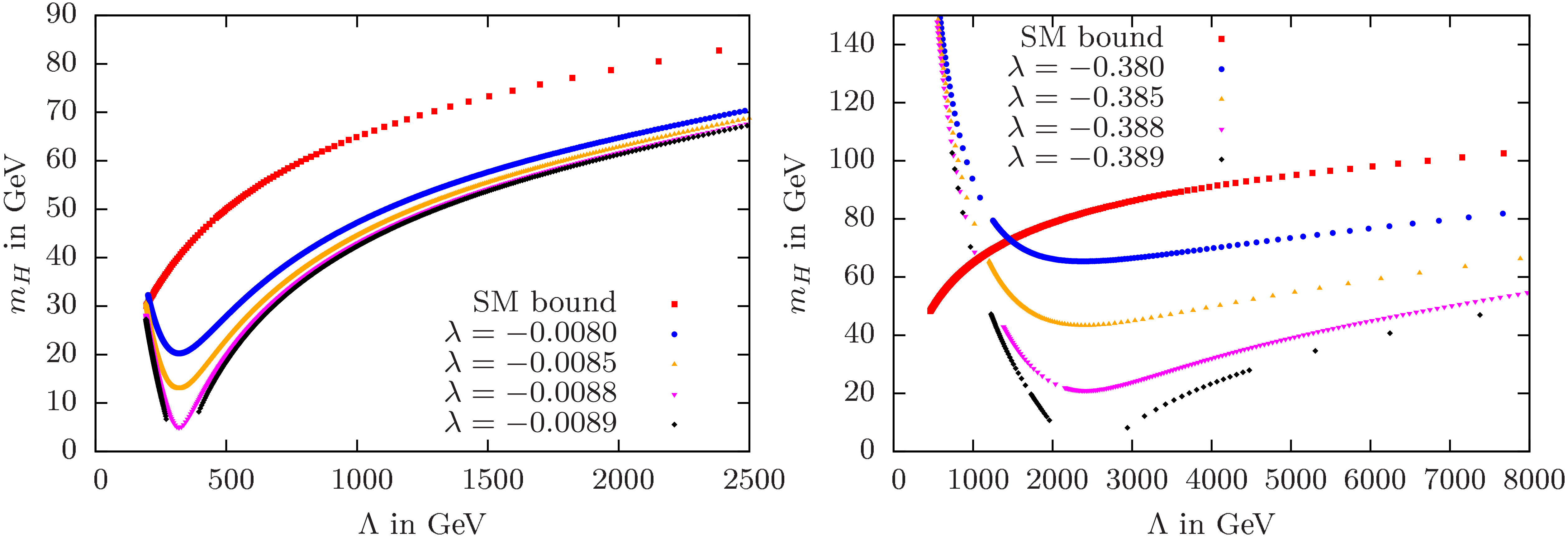




$\lambda_6 = 0.001$ $\lambda_6 = 0.1$

Effects on the Higgs boson mass

The Higgs mass lower bounds from the CEP

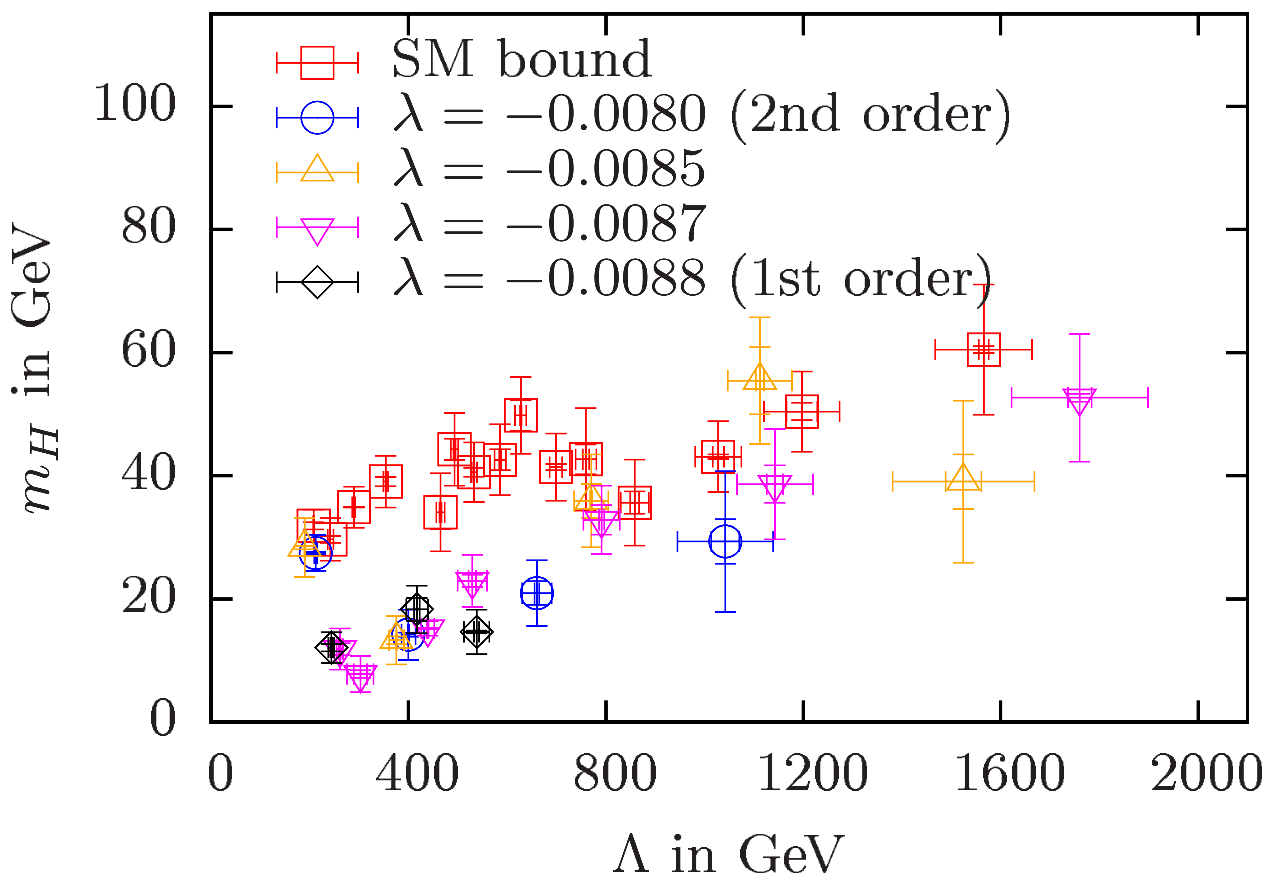


(a) $\lambda_6 = 0.001$

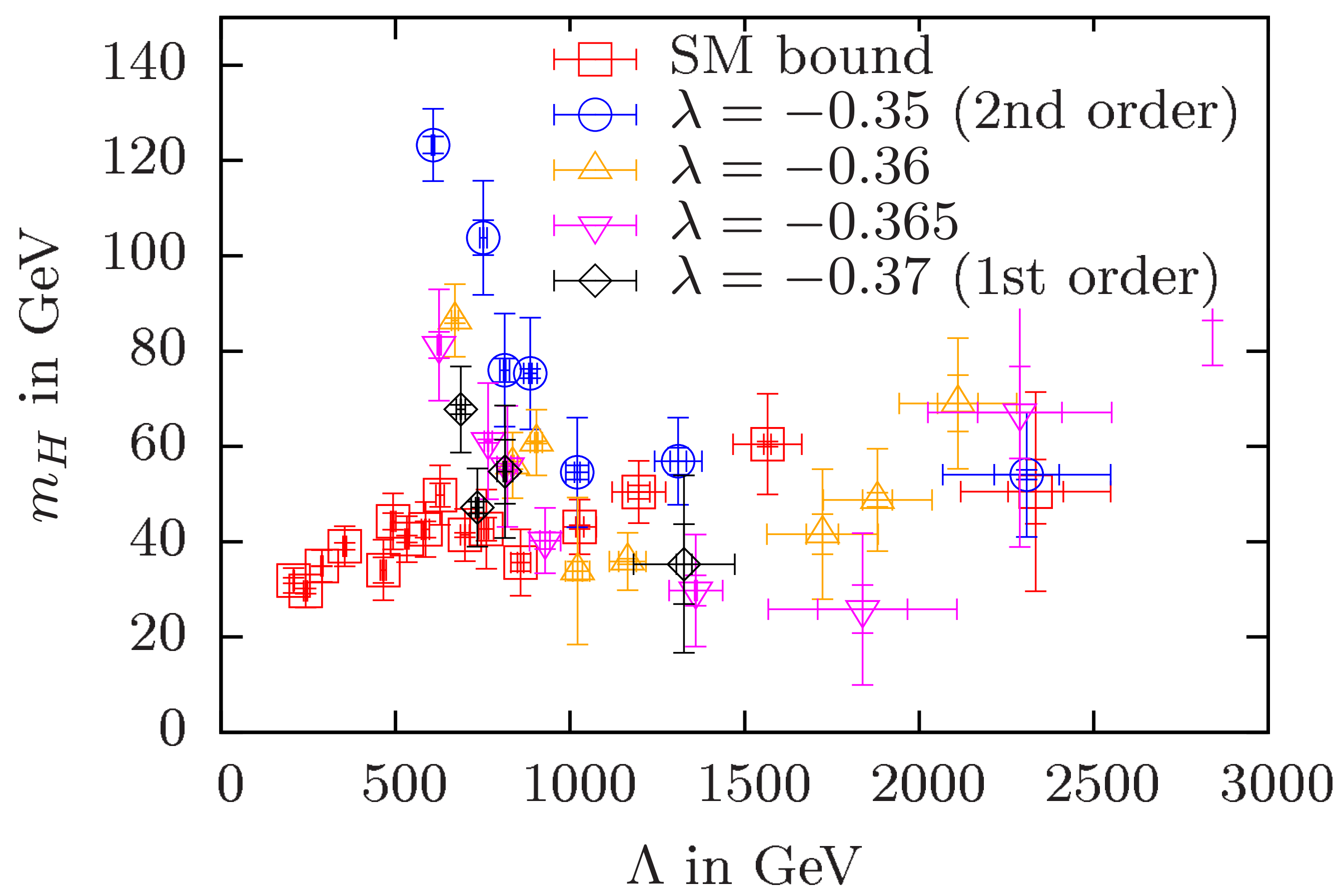
(b) $\lambda_6 = 0.1$

The Higgs mass lower bounds

y tuned to have $m_t = 173$ GeV.



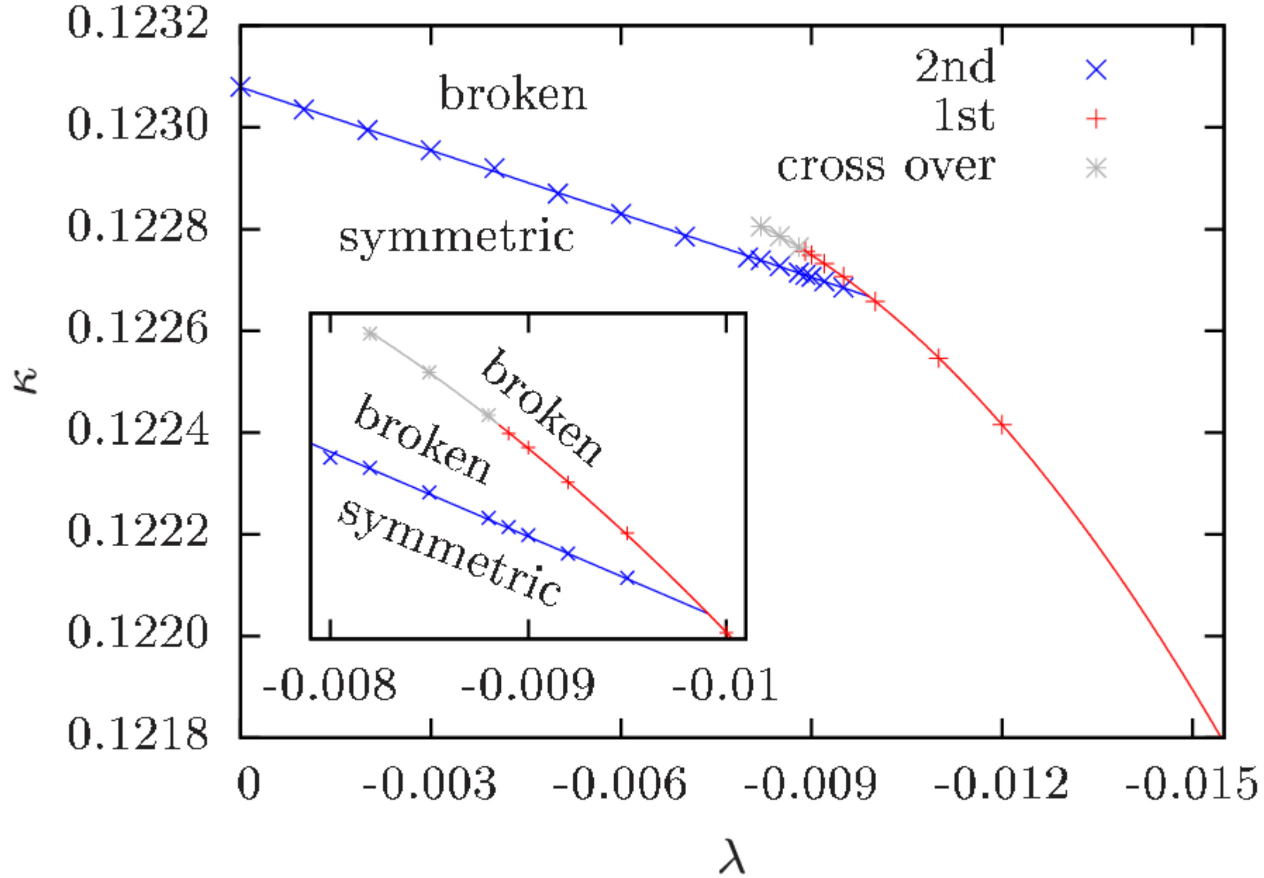
$\lambda_6 = 0.001$

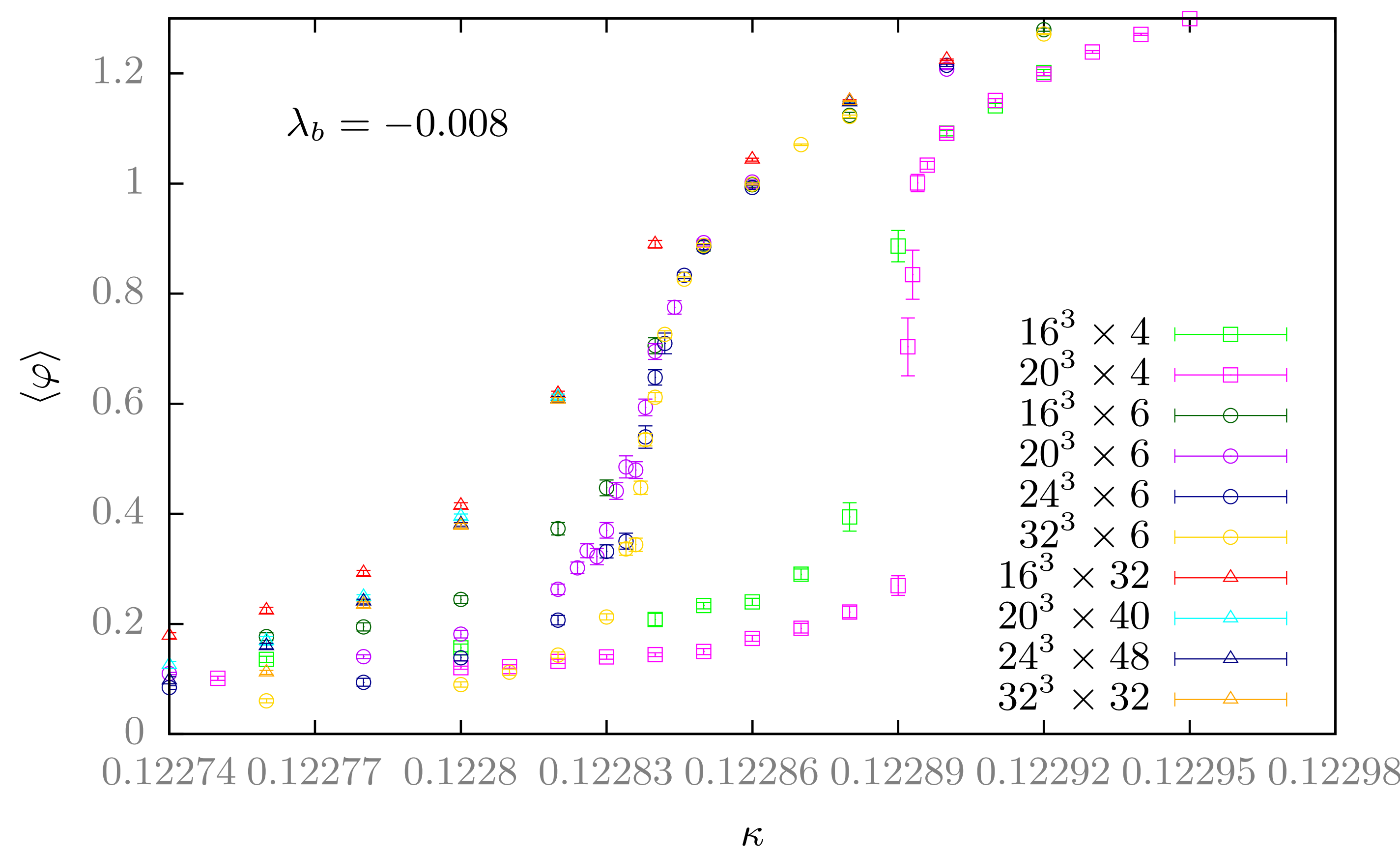


$\lambda_6 = 0.1$

Finite temperature

Non-thermal v.s. Thermal



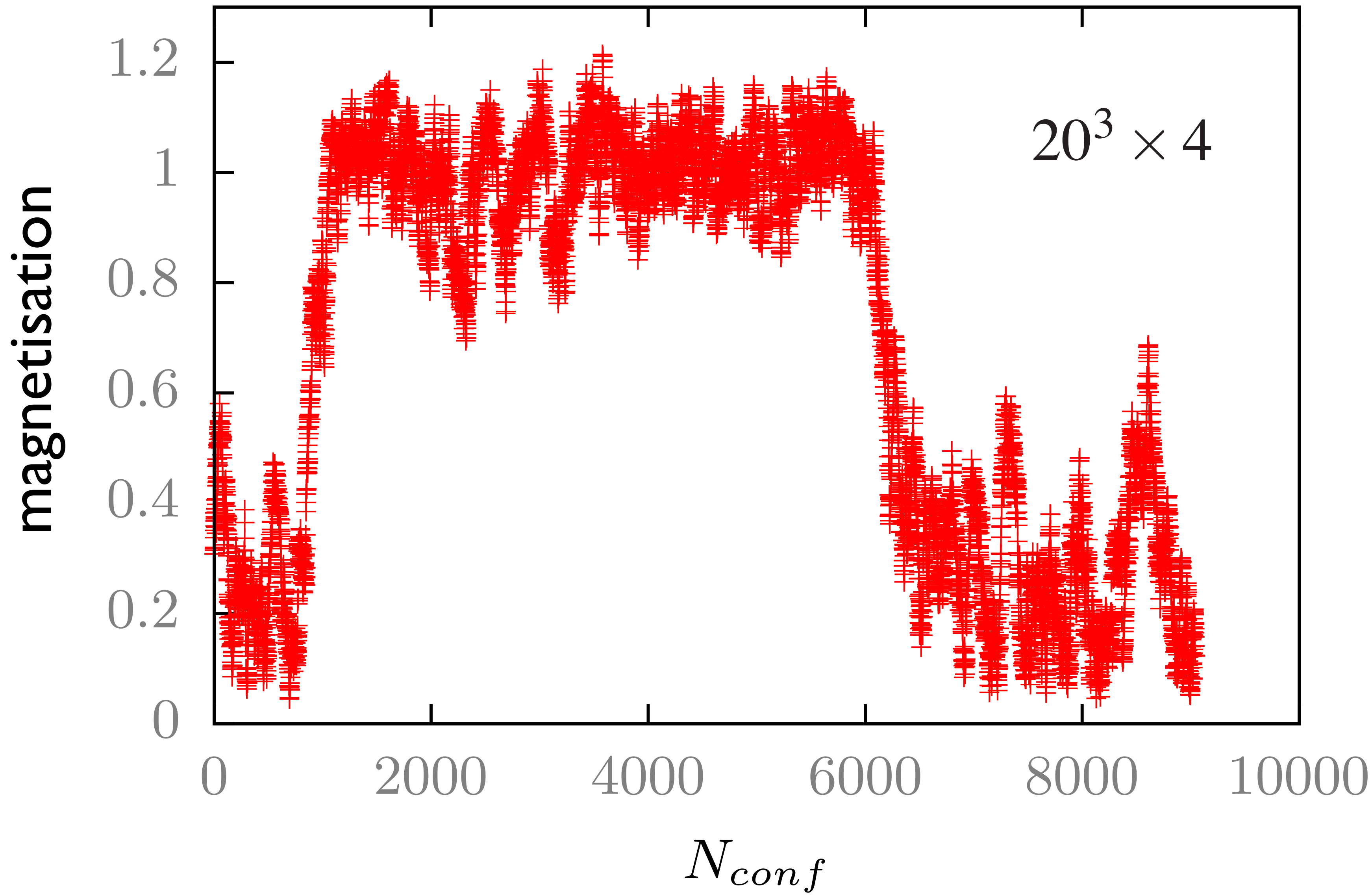


bare $\lambda_6 = 0.001$

Parameter choice at which non-thermal transition is 2nd-order, while thermal transition is 1st-order

Taking a closer look

$\lambda_b = -0.008,\ \kappa = 0.122892$



Co-existence of two states

Remarks and outlook

- The Higgs-Yukawa model and its extensions contain rich phase structure.
- Adding a dimension-6 operator can alter the spectrum significantly.
- Bounds on new physics.
- Finite-temperature.
  - ★ 1st-order transitions near the 2nd-order non-thermal transitions.
  - ★ Only observed with fermions.