Global Properties of the Growth Index of Matter Inhomogeneities in the Universe

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Phys. Rev. D 100, 083503 (2019)
arXiv:1908.00117

Submitted : Phys. Rev. D XXX (2020)
arXiv:1912.06958

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Outline

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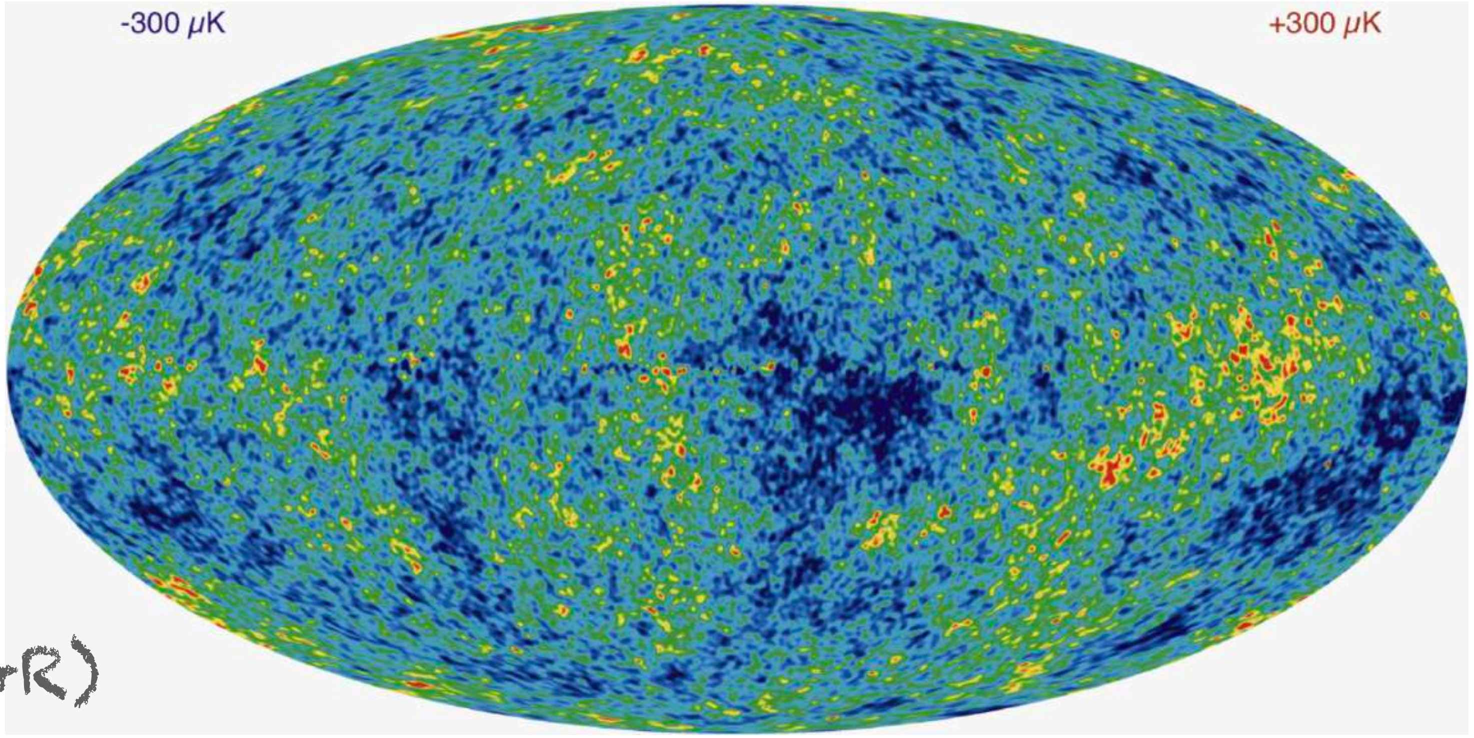
- Results
  - Dynamical System approach
  - Non-negligible decaying mode
  - Varying Equation of State $w_{DE}(a) = w_0 + w_a(1 - a)$
  - Unclustered Component Ω_x
  - Beyond GR: A bump in $G_{\rm eff}(a, k)$
  - Beyond GR: DGP Models
- Conclusion and Perspectives

Motivations

→ CMB data in perfect agreement with Gaussian, (nearly) Scale-Invariant Perturbations generated by the Inflaton ϕ



[Planck '15]

→ Tensions within the ΛCDM paradigm suggest Λ (or GR) might not be the end of the story

→ LSSF is highly-dependent on the nature of Dark Energy / Laws of gravity

One of our best probes, Discrimination between Models

→ Trace back the evolution of those seeds in time, and try to infer something about the background's expansion $H(z)$ from $\delta(z)$

→ Next generation of data will further constrain γ (Euclid & WFIRST)

Assumptions

- We place ourselves long after the RD Epoch ($z \ll z_{eq}$)

  \+ Flat Universe ($\Omega_k \sim 0$) → Insured by Inflation!

$$h^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0}) \exp\left[3\int_0^z dz' \frac{1+w_{DE}(z')}{1+z'}\right]$$

- Newtonian Framework valid for dust, non-relativistic matter (DM & Baryons) deep inside the Hubble Radius ($k \gg aH$)

  → GR treatment includes corrections $\mathcal{O}\left(\frac{a'}{a}\right)^2 \sim a^2H^2$ (dilation terms) which are negligible inside the Horizon

- We only consider models where DE does not cluster! $\delta\rho_{DE} \sim 0$ (Valid for Λ, Quintessence and non-interacting DE)

- As usual $\delta \ll 1$, as soon as $\delta\rho_m \sim \bar{\rho}_m$ the scale becomes non-linear and our formalism breaks down.

Formalism: Linear PT

"Density Contrast"

$$\delta_i \equiv \frac{\delta\rho_i}{\bar{\rho}_i}$$

Euler's Equation

$$D_t\mathbf{u} \equiv (\partial_t + \mathbf{u}\cdot\nabla_\mathbf{r})\mathbf{u} = -\frac{\nabla_\mathbf{r}P}{\rho} - \nabla_\mathbf{r}\Phi$$

Continuity Equation

$$\partial_t\rho = -\nabla_\mathbf{r}\cdot(\rho\mathbf{u})$$

Poisson's Equation

$$\nabla_\mathbf{r}{}^2\Phi = 4\pi G\rho$$

Perturbed Quantities

$$\begin{aligned}\rho(\mathbf{r},t) &= \bar{\rho}(t) + \delta\rho(\mathbf{r},t)\\ &= \bar{\rho}(t)(1+\delta(\mathbf{r},t))\end{aligned}$$

Do the same for $\Phi, P, \mathbf{u}$

⟶ Linearise & drop product of fluctuations

+ Adiabatic Assumption

$$\nabla P = \left(\frac{\partial\bar{P}}{\partial\bar{\rho}}\right)\nabla\rho$$

Insured by Inflation!

Yields the master equation governing the evolution of perturbations

$$\ddot{\delta} + \underbrace{2H\dot{\delta}}_{\text{Hubble Friction}} = \underbrace{4\pi G\rho\delta}_{\text{Driving Force}} + \frac{c_s^2}{a^2}\nabla_x^2\delta \xrightarrow[\delta]{\text{FT}} \ddot{\delta}_{\cancel{k}} + 2H\dot{\delta}_{\cancel{k}} = 4\pi G\rho\delta_{\cancel{k}} - \cancelto{0}{c_s^2\frac{k^2}{a^2}\delta_k}$$

$$\delta\rho(\mathbf{r},t) = \int\frac{d^3k}{(2\pi)^3}e^{-i\,\mathbf{k}\cdot\mathbf{r}}\delta\rho_k(t)$$

We'll drop this...

-> Valid for each k

For Dust, P=0

Linear Matter Perturbations

$$\frac{d}{dt} = H\frac{d}{d\ln a}$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m = \boxed{4\pi G\rho_m\delta_m}$$

$$\frac{3}{2}H^2\Omega_m\delta_m$$

$$\delta_m \equiv \frac{\delta\rho_m}{\bar{\rho}_m}$$

"Density Contrast"

Rewritten in terms of a(t):

$$\frac{df}{d\ln a} + f^2 + \left(2+\frac{\dot{H}}{H^2}\right)f = \frac{3}{2}g\,\Omega_m\,,$$

$g(a,k) = G_{\mathrm{eff}}/G_N$

$$f \equiv \frac{d\ln\delta_m}{d\ln a} = \Omega_m(z)^{\gamma(z)}$$

The Growth Function f

The Growth Index γ

$\gamma(z=0) \simeq 0.55$

In agreement with ΛCDM

Solving for γ (Ω_m):

Encodes the modification of gravity

$$6w_{DE}\Omega_{DE}\ln\Omega_m\frac{d\gamma}{d\ln\Omega_m} + 3w_{DE}(\Omega_m)\Omega_{DE}(2\gamma-1) + 1 + 2\Omega_m^{\gamma} - 3\,g\,\Omega_m^{1-\gamma} = 0$$

Evaluated @ Present Time

Using Redshift Space Distortions (RSDs) we can constrain γ via the robust observable

r.m.s density fluctuation

$$f\sigma_8(z) \equiv f(z)\cdot\sigma_8(z) = -(1+z)\frac{\sigma_{8,0}}{\delta_0}\delta_m'(z)$$

@scales k=8h⁻¹.Mpc

The Growth Index $\gamma^{\Lambda CDM}$

Setting

$w_{DE} = -1$ & $g = 1$

$$-6\Omega_{DE} \ln \Omega_m \frac{d\gamma}{d \ln \Omega_m} - 3\Omega_{DE}(2\gamma - 1) + 1 + 2\Omega_m^{\gamma} - 3\Omega_m^{1-\gamma} = 0$$

$$\boxed{\Omega_{DE} = 1 - \Omega_m}$$

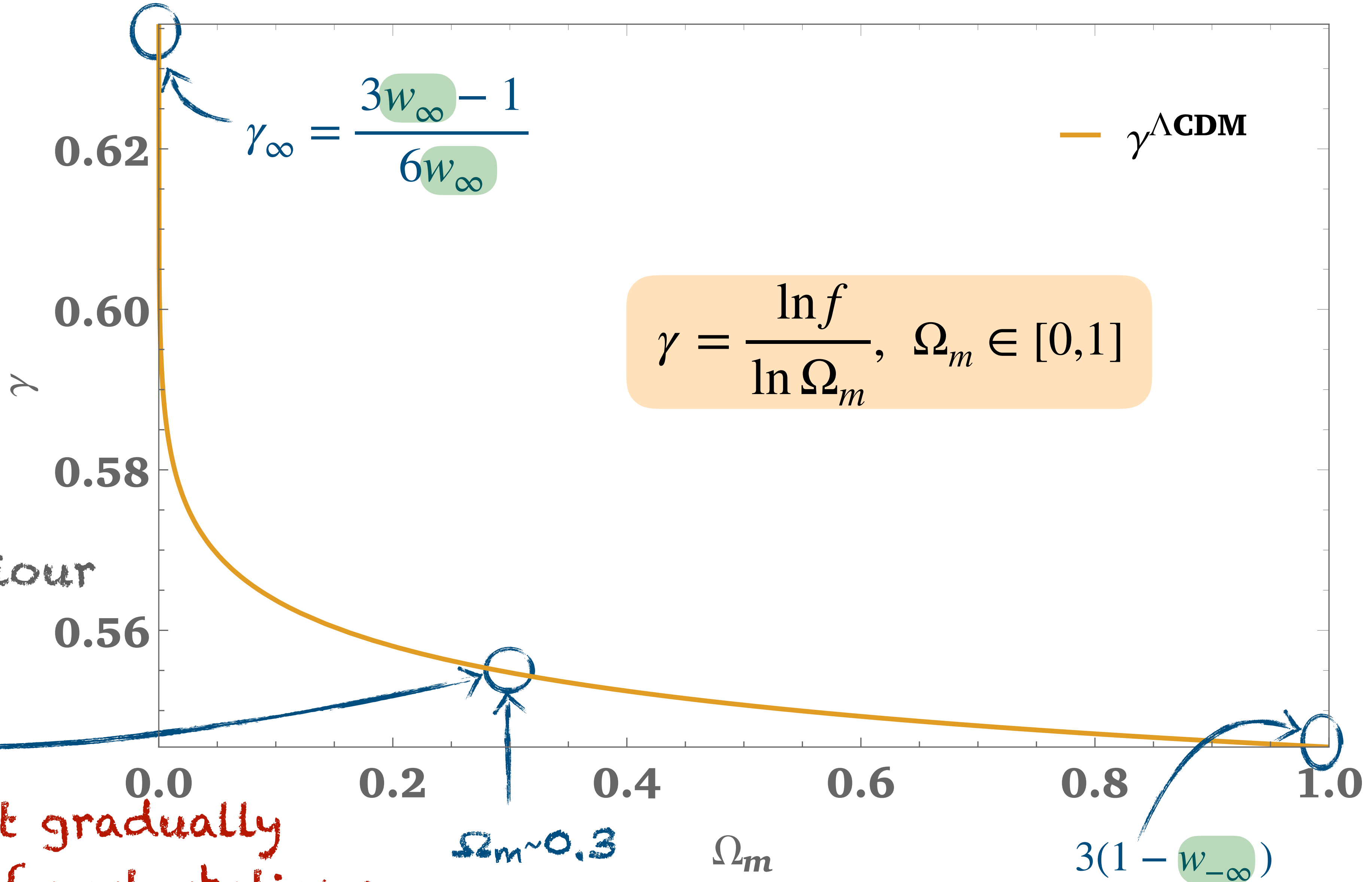
Starting at $\Omega_m = 1$

$$\gamma_{-\infty}^{\Lambda CDM} = \frac{6}{11} \simeq 0.5454$$

Can solve numerically for $\gamma^{\Lambda CDM}$

↪ Nearly constant behaviour

$$\gamma_0^{\Lambda CDM} \simeq 0.55$$



$$\gamma_{\infty} = \frac{3w_{\infty} - 1}{6w_{\infty}}$$

$$\gamma = \frac{\ln f}{\ln \Omega_m}, \quad \Omega_m \in [0,1]$$

Ω_m~0.3

$$\gamma_{-\infty} = \frac{3(1 - w_{-\infty})}{5 - 6w_{-\infty}}$$

As the DE component gradually catches up, the growth of perturbations slows and γ increases rapidly towards the asymptotic value $\gamma_{\infty}^{\Lambda CDM} = \frac{2}{3}$

Notice the Dependence on w_{DE}

Dynamical System Approach

Look for critical points such that

$$\frac{d\Omega_m}{dN} = \frac{d\gamma}{dN} = 0$$

$$\frac{d\Omega_m}{dN} = \boxed{\alpha}(1 - \Omega_m)\Omega_m$$

$$\frac{d\gamma}{dN} = -\frac{\alpha(2\gamma - 1)(1 - \Omega_m) + \boxed{\tilde{F}(\Omega_m;\gamma)}}{2\ln(\Omega_m)}$$

"Autonomous System"

Fixed Points

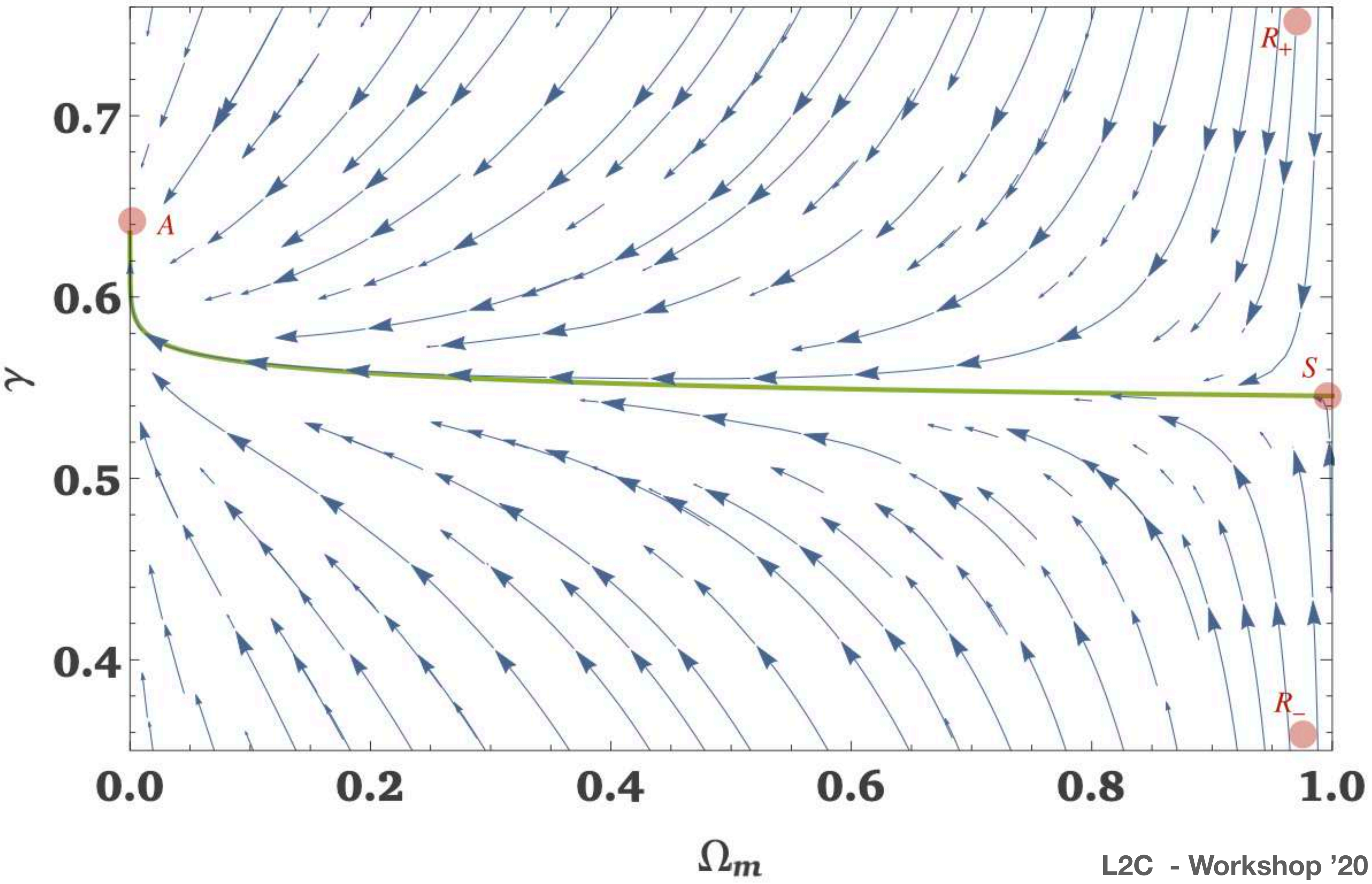
A =

$$\left(\Omega_m = 0\,, \gamma_\infty = \frac{3w_\infty - 1}{6w_\infty}\right)$$

S =

$$\left(\Omega_m = 1\,, \gamma_{-\infty} = \frac{3(1 - w_{-\infty})}{5 - 6w_{-\infty}}\right)$$

$R_\pm$ is a reppeler at ± ∞



The Growth Index $\gamma^{\Lambda CDM}$

Setting

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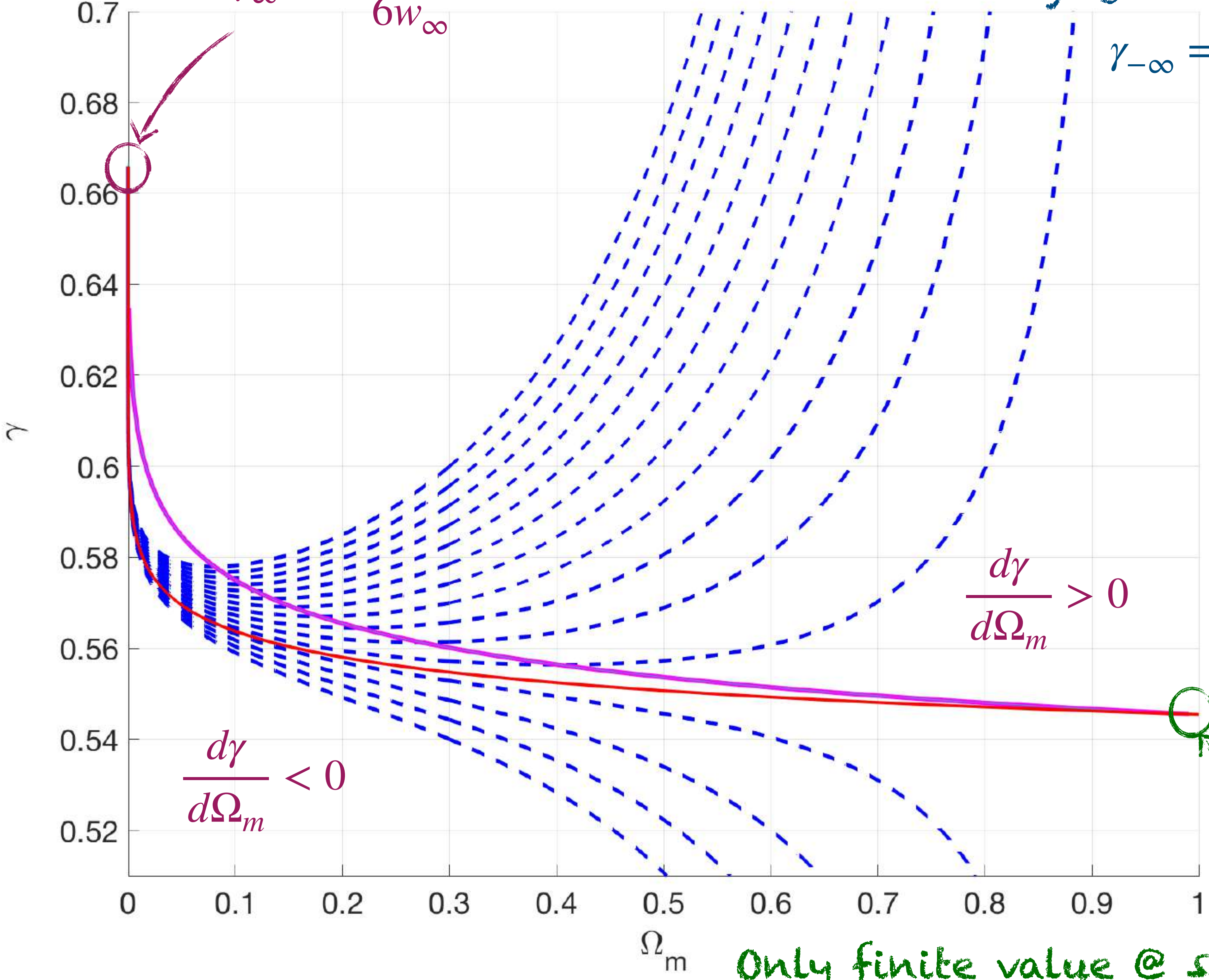


$$\gamma_\infty = \frac{3w_\infty - 1}{6w_\infty}$$

Non-negligible Decaying Mode

$\gamma_{-\infty} = \pm\infty$

$$\frac{d\gamma}{d\Omega_m} > 0$$

$$\frac{d\gamma}{d\Omega_m} < 0$$

Only finite value @ $\Omega_m = 1$

Purely growing mode!

$$f = \frac{\delta_1}{\delta} f_1 + \frac{\delta_2}{\delta} f_2$$

In the future

$$f \sim C\, a^{\frac{1}{2}(3w_\infty - 1)} \to 0$$

$$\Omega_m \sim a^{3w_\infty}$$

$$\boxed{\gamma = \frac{\ln f}{\ln \Omega_m}}$$

Function in (Ω_m, γ)-plane

Such that:

$$\frac{d\gamma}{d\Omega_m} = 0$$

Varying EoS for DE

EoS Parameterized by :

$$w_{DE}(a) = w_0 + w_a(1-a)$$

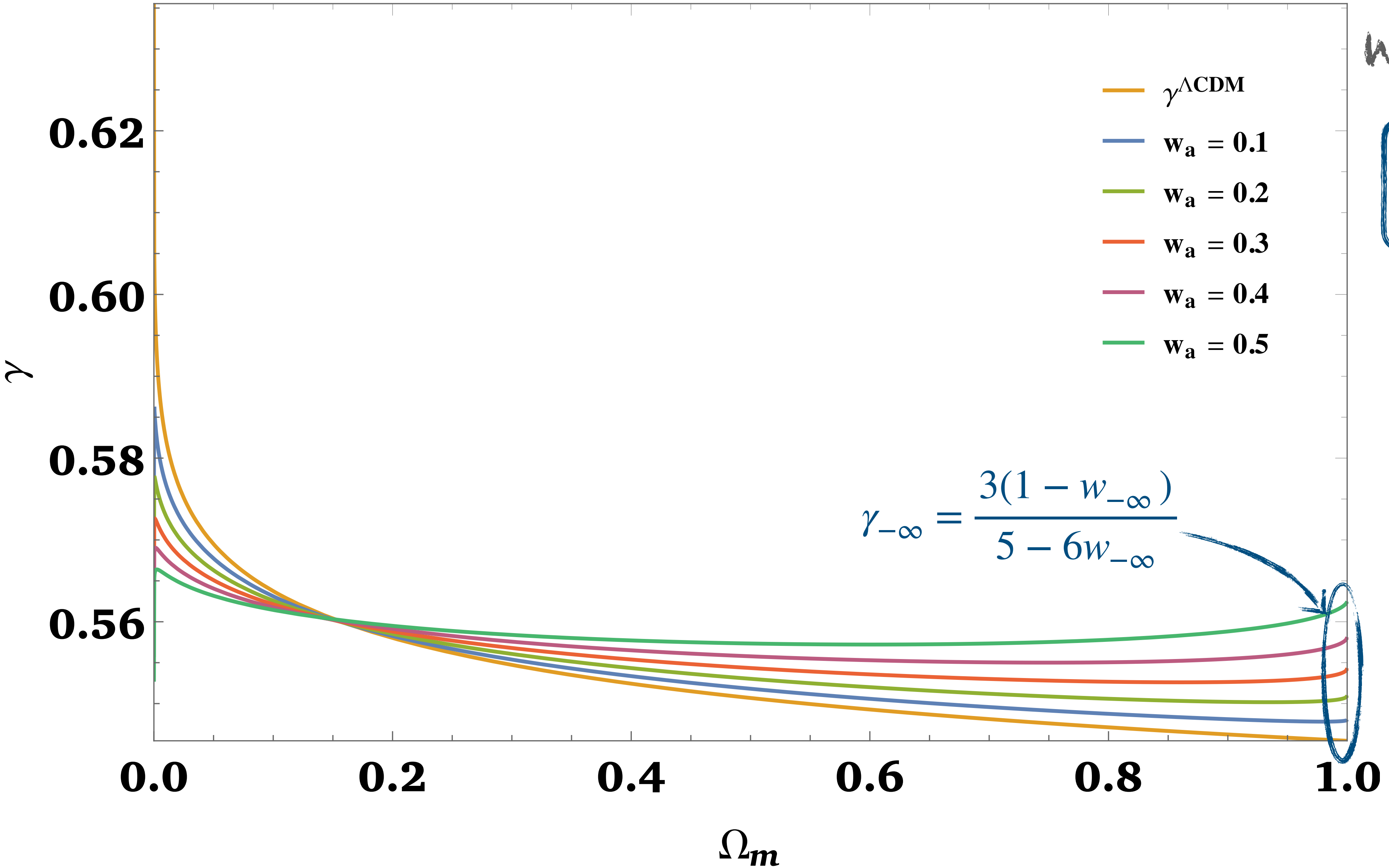
$$6w_{DE}(\Omega_m)\ln\Omega_m\frac{d\gamma}{d\ln\Omega_m} + 3w_{DE}(\Omega_m)\Omega_{DE}(2\gamma-1) + 1 + 2\Omega_m^{\gamma} - 3\,g\,\Omega_m^{1-\gamma} = 0$$

Using the useful relation:

$$w_{DE} = \frac{1}{3(1-\Omega_m)}\frac{d\ln\Omega_m}{d\ln a}$$

We can solve for a!

$$a(\Omega_m) \to w_{DE}(\Omega_m) \to \gamma(\Omega_m)$$



$$\gamma_{-\infty} = \frac{3(1-w_{-\infty})}{5-6w_{-\infty}}$$

We restrict ourselves to

$$\boxed{w_{-\infty} = w_0 + w_a < 0}$$

To ensure MD

&

$$\boxed{w_a > 0}$$

To ensure DE Domination

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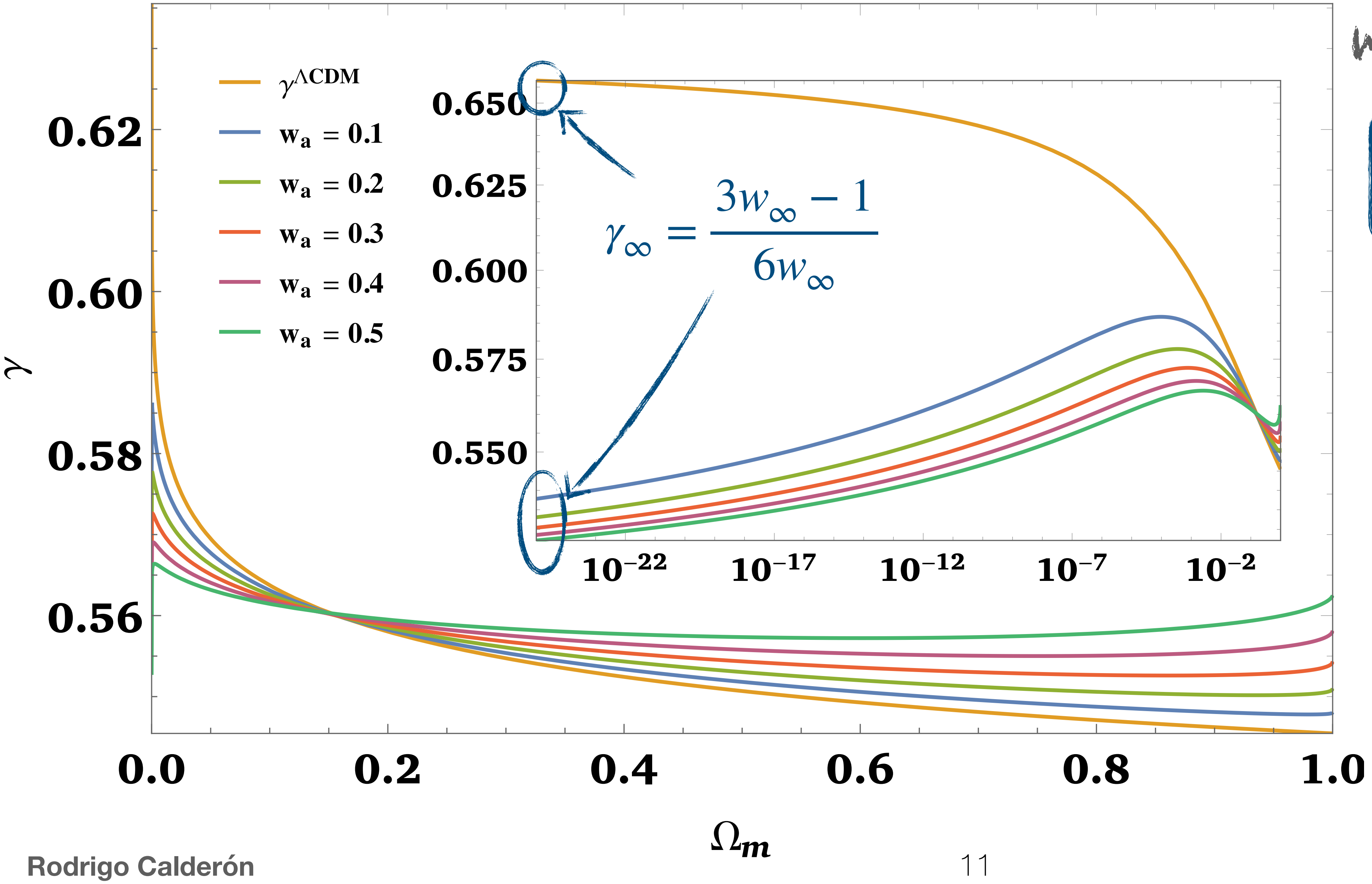

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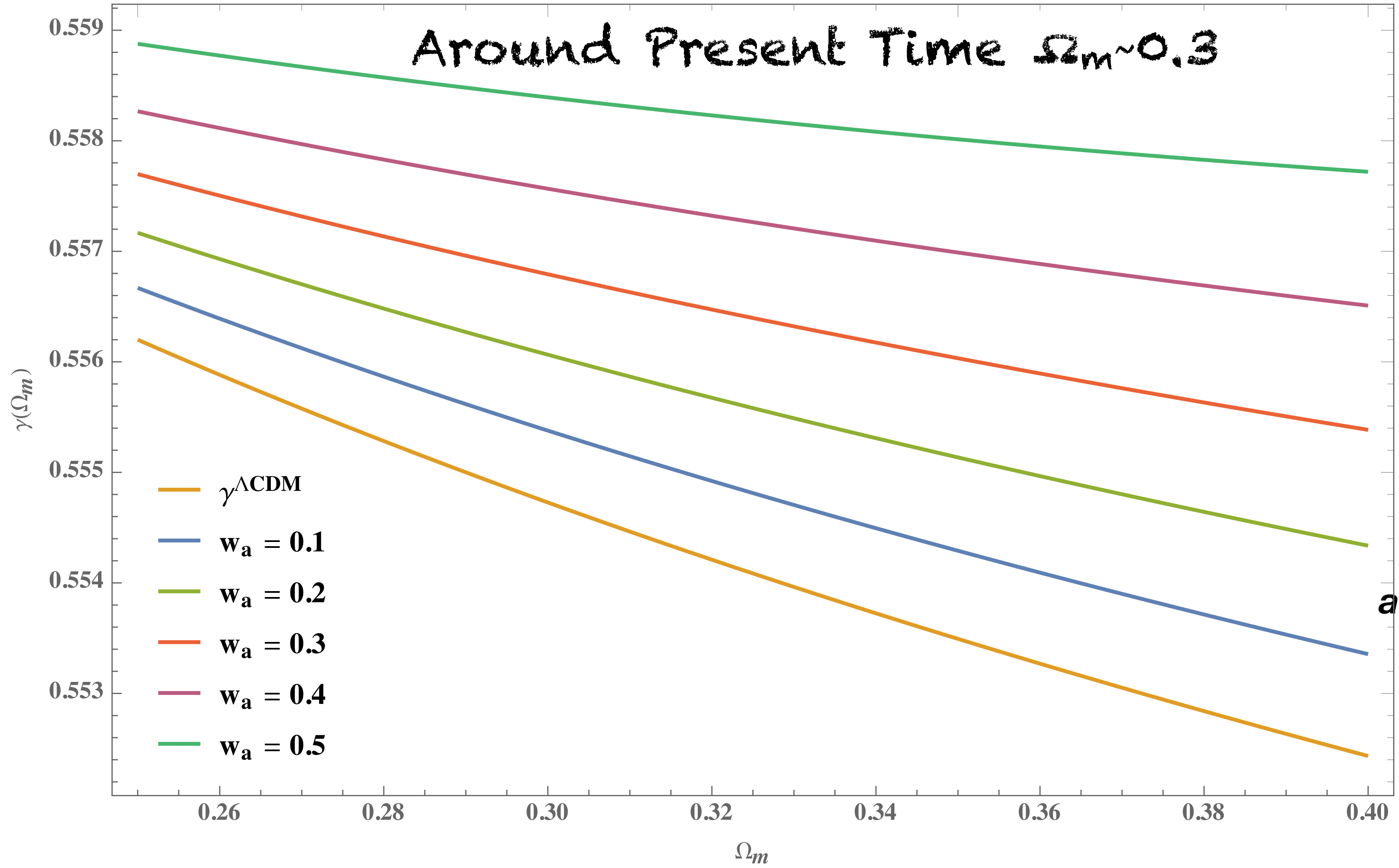
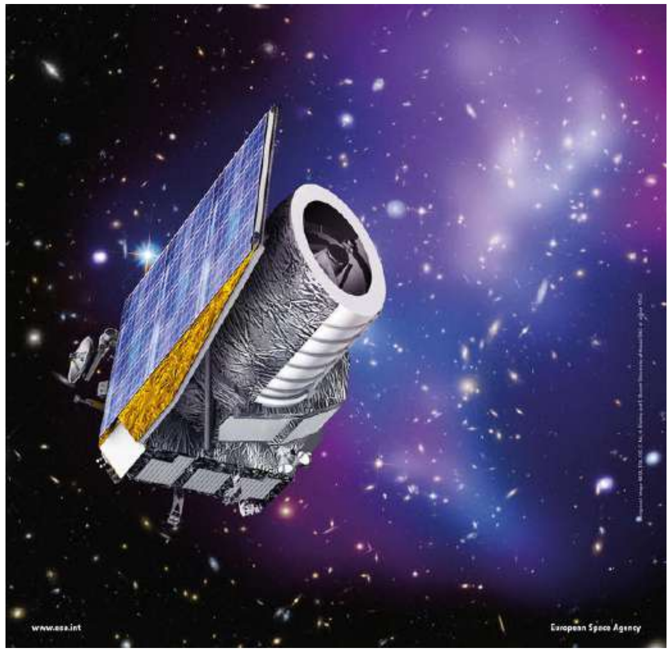


We are indirectly probing $w_{DE}(a)$!

[Credit: euclid-ec.org]



"Next generation of data (in Particular Euclid but also WFIRST) are expected to constrain γ_1 to the percent level"

$$(\gamma_0, \gamma_1) \simeq (0.55, -0.02)$$

$$\frac{d\gamma}{dz}\bigg|_{z=0}$$

Unclustered Component Ω_x

$$\Omega_{DE} = 1 - \Omega_m^{tot}$$
$$\Omega_m = (1-\varepsilon)\Omega_m^{tot}$$
$$\Omega_x = \varepsilon\Omega_m^{tot}$$

In the neighbourhood

$\Omega_m^{tot} \simeq 1$

$$\cancel{-6w_{-\infty}\ln(1-\varepsilon)\frac{d\gamma^\varepsilon}{d\ln(1-\Omega_m^{tot})}} + \tilde{F}(1-\varepsilon;\gamma^\varepsilon) = 0$$

Any finite $\gamma^\varepsilon_{-\infty}$

$$\boxed{\tilde{F}(1-\varepsilon;\gamma^\varepsilon) = 0} \leftrightarrow \gamma^\varepsilon_{-\infty} = \frac{3}{5}(1+\frac{\varepsilon}{25})$$

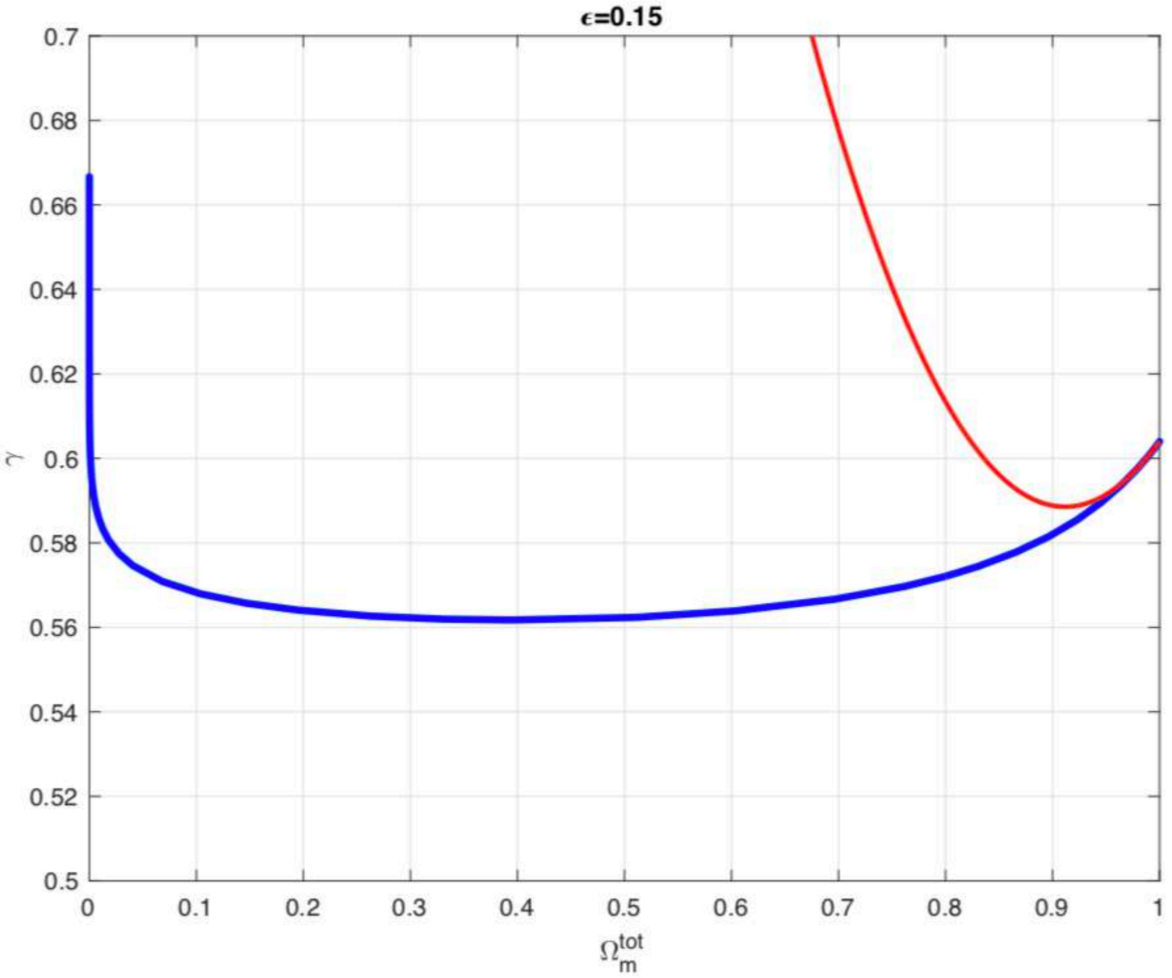
Family of tracking DE solutions with constant γ found in

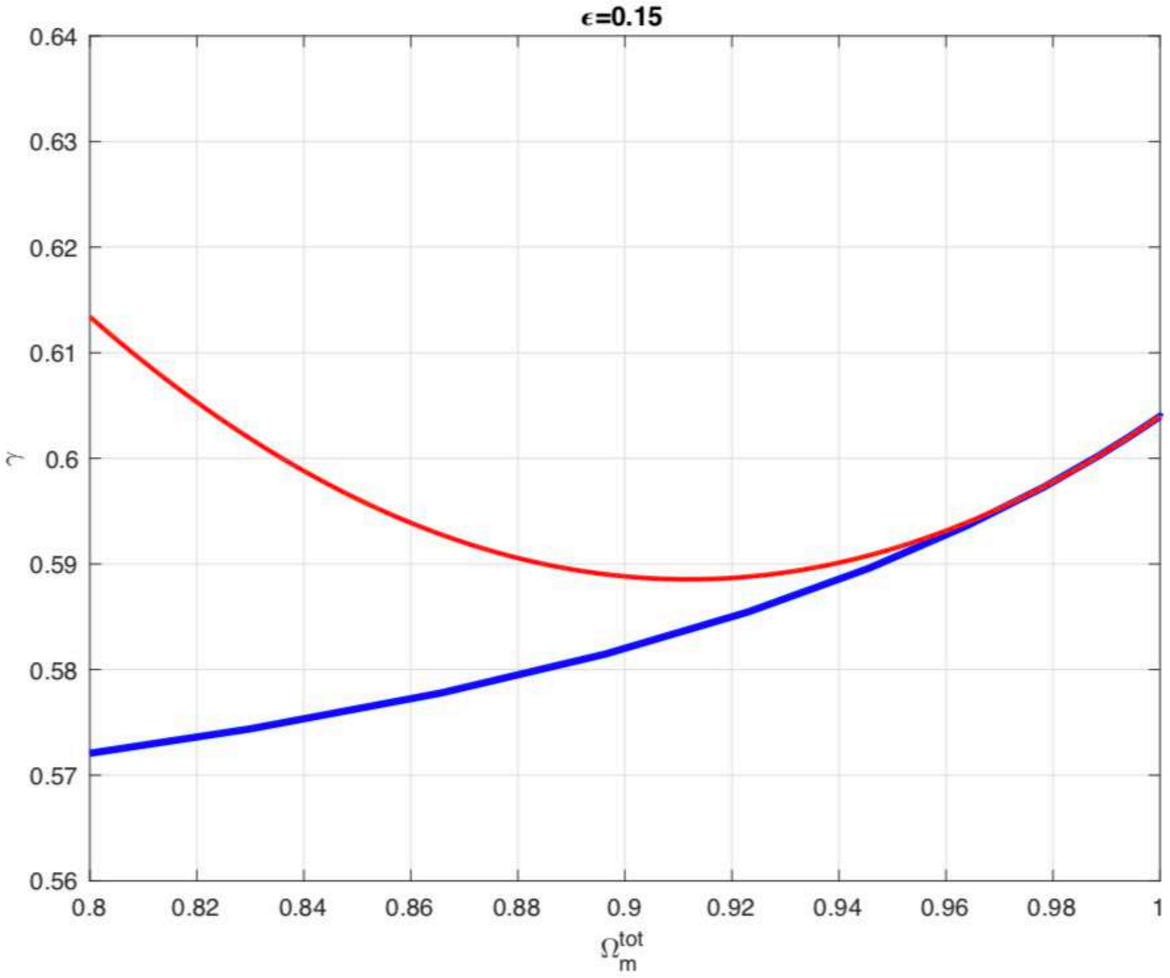
[arXiv: 1610.00363v4]

Does not depend on any non-zero $w_{-\infty}$

System not continuous in $(\Omega_m^{tot} = 1, \varepsilon = 0)$

$$\lim_{\varepsilon\to 0}\gamma^\varepsilon_{-\infty} \equiv \lim_{\varepsilon\to 0}\gamma^\varepsilon(\Omega_m^{tot} \to 1) \neq \lim_{\Omega_m \to 1}\gamma(\Omega_m) \equiv \gamma_{-\infty}$$





Beyond General Relativity

MG enters through the source term in the equation

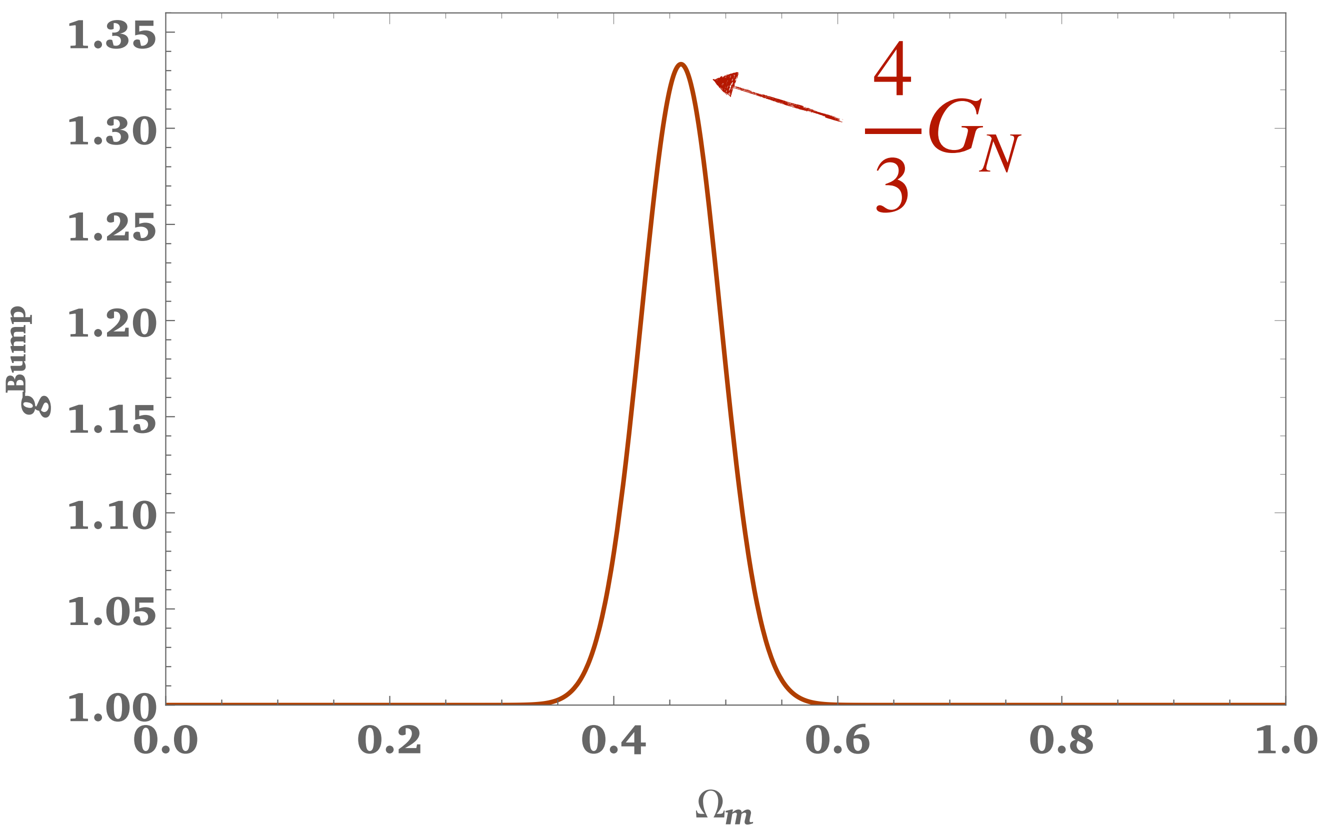
$$\frac{df}{d\ln a} + f^2 + \left(2 + \frac{\dot{H}}{H^2}\right) f = \frac{3}{2} g \Omega_m ,$$

Notice how γ is more sensitive to MG at early times! ($\Omega_m \sim 1$)

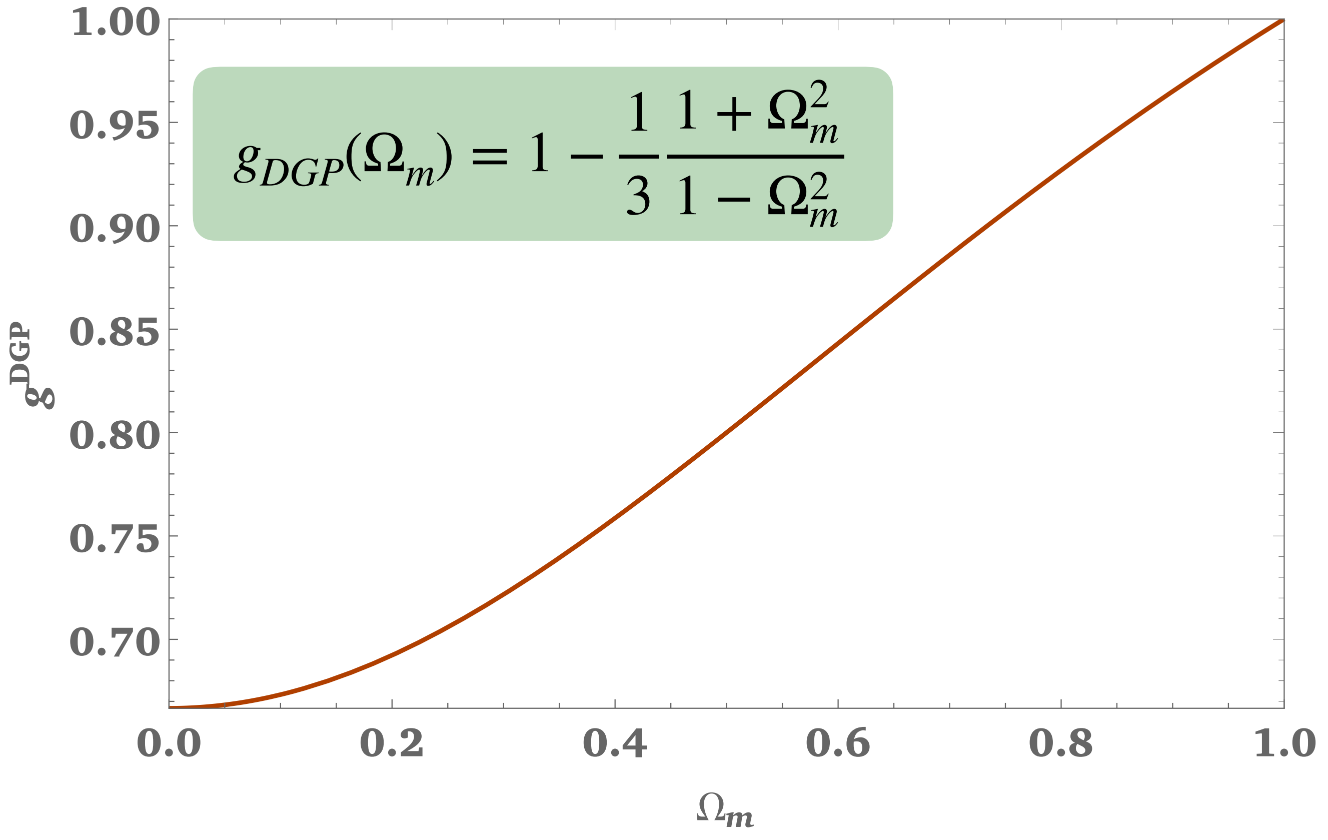
Modified Poisson equation $-k^2 \Phi = 4\pi G_{\rm eff}\, \rho$

$$G \to G_{\rm eff}(a, k)$$

→ A Bump/Dip in $G_{\rm eff}$



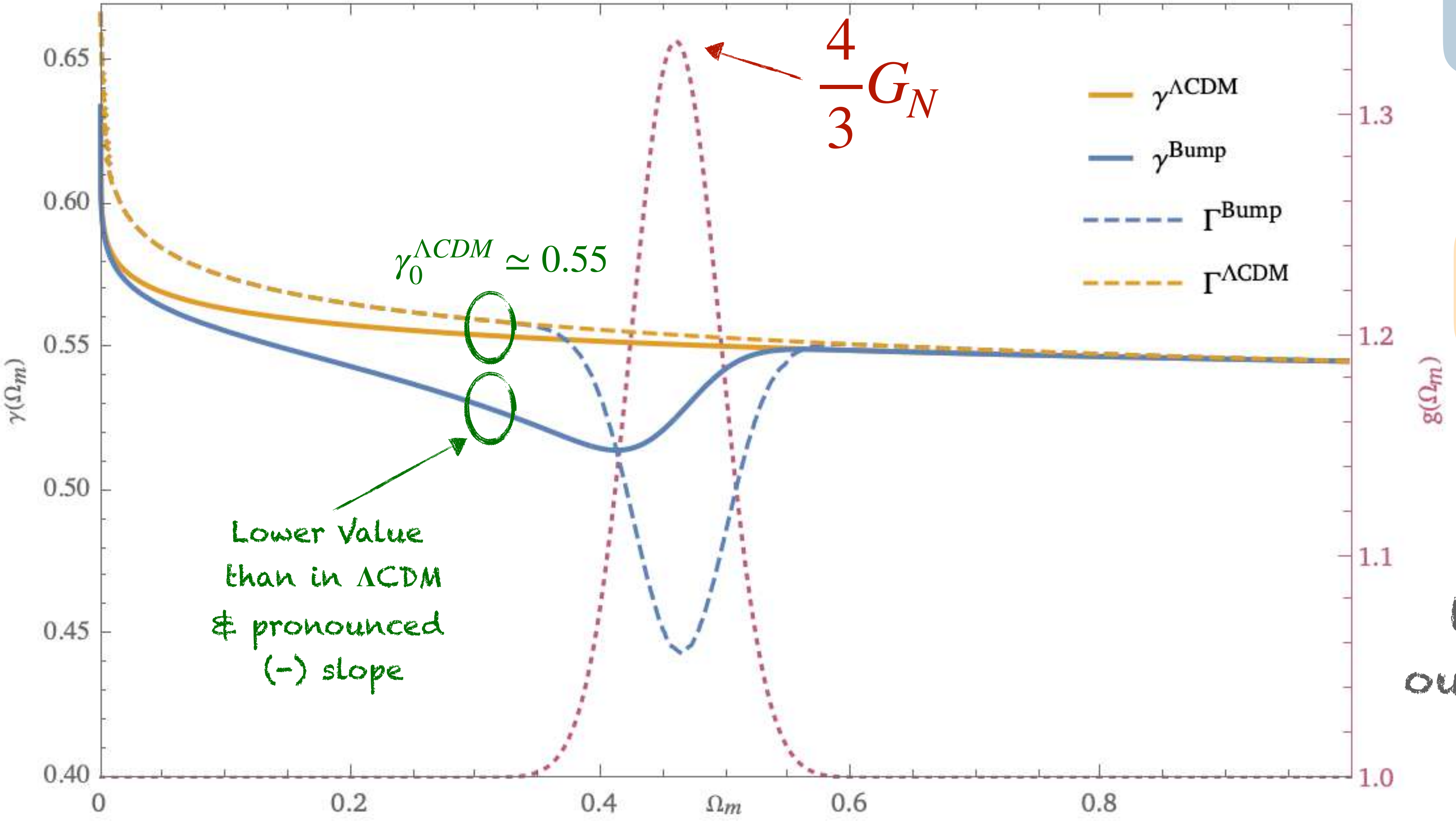
→ Braneworld Gravity (DGP)



$$g_{DGP}(\Omega_m) = 1 - \frac{1}{3}\frac{1+\Omega_m^2}{1-\Omega_m^2}$$

Beyond GR: A bump in $G_{\rm eff}$

→ Behaviour previously found in many f(R) theories...



$\frac{4}{3}G_N$

$\gamma_0^{\Lambda CDM} \simeq 0.55$

Lower Value than in ΛCDM & pronounced (-) slope

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\rm eff}\rho_m\delta_m$$

As $G_{\rm eff}$ ↗, f also ↗

$$\gamma = \frac{\ln f}{\ln \Omega_m}, \ \Omega_m \in [0,1]$$

→ γ decreases!

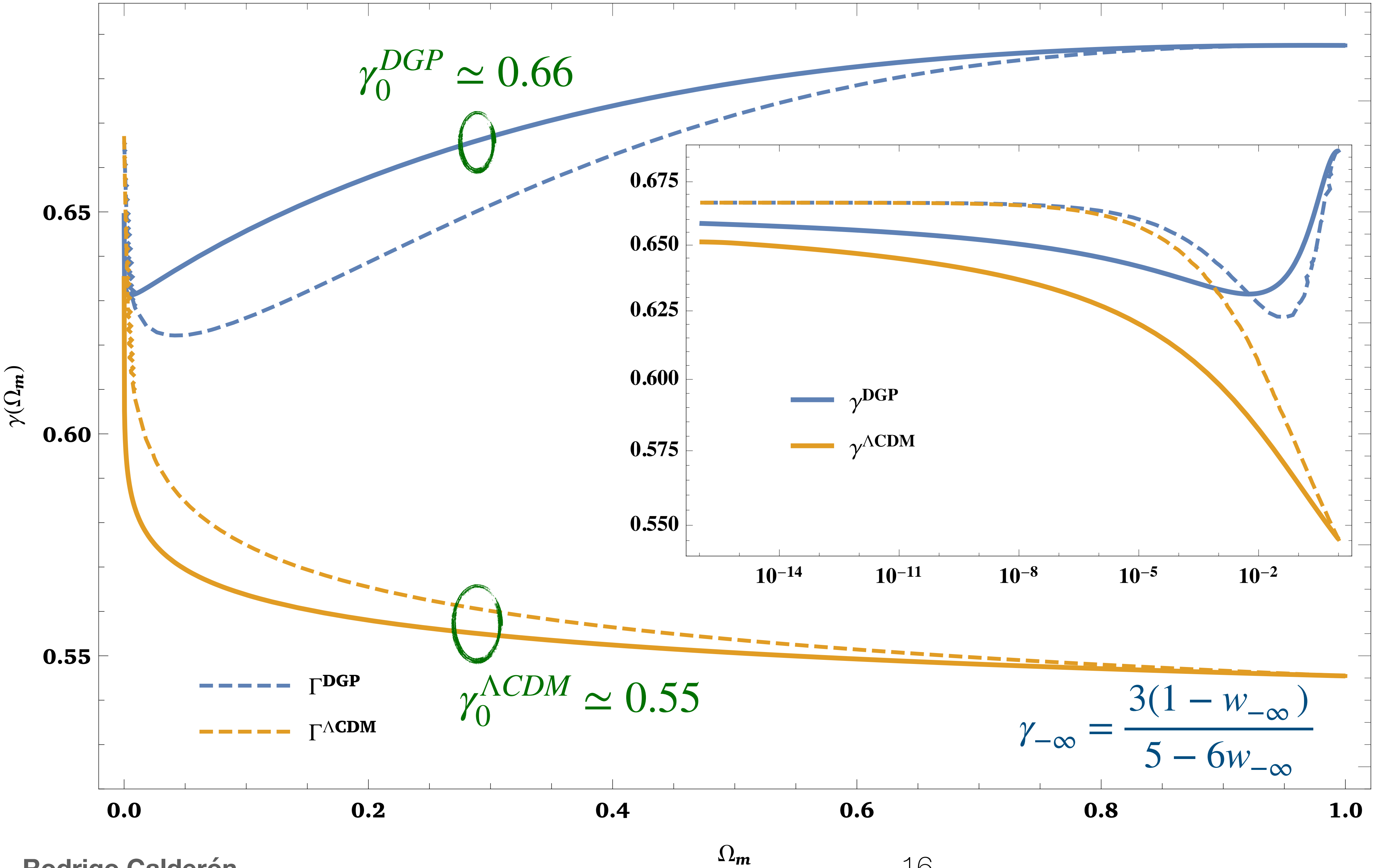
We could even infer our position with respect to the bump!

Beyond GR: DGP Models γ^{DGP}

Dvali-Gabadadze-Porrati : Braneworld Gravity

$$w_{DGP}(\Omega_m) = -(1+\Omega_m)^{-1}$$

$$g_{DGP}(\Omega_m) = 1 - \frac{1}{3}\frac{1+\Omega_m^2}{1-\Omega_m^2}$$



$\gamma_0^{DGP} \simeq 0.66$

$\gamma_0^{\Lambda CDM} \simeq 0.55$

$$\gamma_{-\infty} = \frac{3(1-w_{-\infty})}{5-6w_{-\infty}}$$

In these models:

$$g_{-\infty}^{DGP} = 1$$

$$w_{-\infty}^{DGP} = -\frac{1}{2}$$

$$\gamma_{-\infty}^{GR}(w = -\frac{1}{2}) = \frac{9}{16}$$

$$\gamma_{-\infty} = \frac{3(1-w_{-\infty}-d)}{5-6w_{-\infty}}$$

$$\left(\frac{dg}{d\Omega_m}\right)_{-\infty} = \frac{1}{3} \neq 0$$

$$\gamma_{-\infty}^{DGP} = \frac{11}{16}$$

Conclusions

- A global analysis of the behaviour of $\gamma(\Omega_m)$ yields some interesting (mathematical) properties of the underlying model
- Variable EoS $w_{DE}(a) = w_0 + w_a(1-a)$
  - Follows essentially the same phenomenology as $\gamma^{\Lambda CDM}$ on low-Z, with (very) different behaviours in the past & future
- Unclustered Component Ω_x
  - The limits $\varepsilon \to 0$ & $\Omega_m^{\text{tot}} \to 1$ do not commute. Singularity at $\varepsilon = 0$
- Beyond GR: Bump/ DGP Models
  - A bump in G_{eff} yields Lower values for $\gamma_0 \equiv \gamma(z=0)$ than ΛCDM (-) Slope!
  - Explicit origin of $\gamma_{-\infty}^{DGP} = \frac{11}{16}$ coming from $\left(\frac{dg}{d\Omega_m}\right)_{-\infty} = \frac{1}{3} \neq 0$
  - Valid for any MG that does not satisfy $g'(\Omega_m)|_{-\infty} = 0$
- An (accurate enough) estimation of $\gamma(z)$ & $\gamma'(z)$ could indicate a departure from ΛCDM (&/or GR)