Primordial non-gaussianities or relativistic effects in Large Scale Structures?

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Based on 1811.05452, 1912.13034

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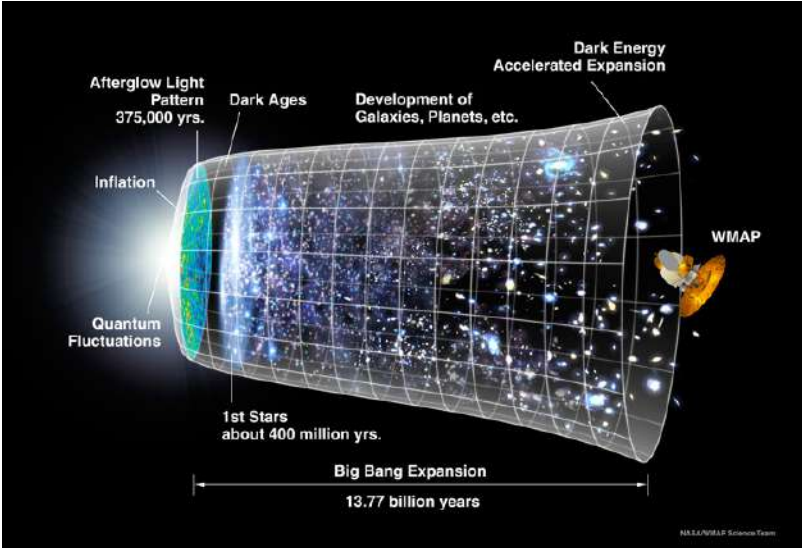
Large Scale Structures (LSS) formation



Cosmological structures formation

Fluids mechanics in an expanding universe.

Large Scale Structures (LSS) formation

In LSS, split between large scales *background* (expanding universe, well defined mean density) and intermediate scales *perturbations* (density differs little from background).

Cosmic structures grow out of tiny initial fluctuations and are studied through perturbation theory.

Newtonian structure formation

- Study of LSS on scales smaller than the Hubble scale (3000 h^{-1} Mpc).
- typically $v \sim 10^{-2}$, $\phi \sim 10^{-5}$.
- Linear fluids mechanics in an expanding universe: success story (cf. CMB).

The current challenge for LSS: to handle the non-linear regime.
Future surveys (Euclid, LSST, SKA): probe a fraction of the Hubble scale.
$\rightarrow$ need for a precise (1 % accuracy) understanding of LSS.

Epic Battle: Newton vs Einstein

For CDM (non-relativistic matter):

- On background level (FLRW): Newton and Einstein agree.
- For linear (scalar) perturbations: Newton and Einstein agree.
- In the non-linear regime: small scales: *Newton and Einstein agree*.

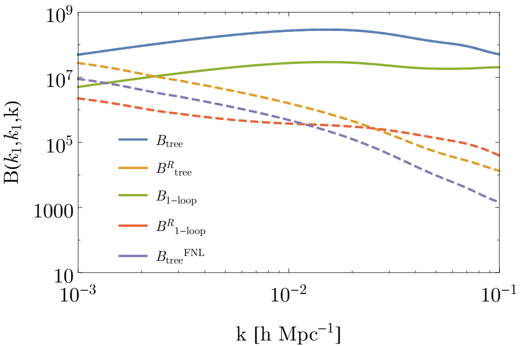
A case for Einstein

Relativistic structure formation

- Relativistic matter content of the universe (neutrinos, cosmic strings, DDE).
- Gravity has 6 degrees of freedom (2 scalars, 2 vectors and 2 tensors)
- Backreaction: how non-linear evolution impacts means quantities.
- Observations are made on the relativistic perturbed light cone.

I argue (1811.05452)

The bispectrum in the squeezed limit at 1-loop receives relativistic corrections due to the dynamics of the CDM field of the same order than Newtonian results.



Bispectrum: generalities

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The bispectrum in the squeezed limit at 1-loop receives relativistic corrections due to the dynamics of the CDM field of the same order than Newtonian results.

Power spectrum vs Bispectrum

$$\langle \delta(\boldsymbol{k}_1, t)\delta(\boldsymbol{k}_2, t)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2) P(k_1, t)\,, \quad (1)$$

$$\langle \delta(\boldsymbol{k}_1, t)\delta(\boldsymbol{k}_2, t)\delta(\boldsymbol{k}_3, t)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3, t)\,. \quad (2)$$

Note that the bispectrum couples the scales !!

Planck 2018 constraints

$f_{\mathrm{NL}}^{\mathrm{loc}} = -0.9 \pm 5.1$, $f_{\mathrm{NL}}^{\mathrm{equil}} = -26 \pm 47$, $f_{\mathrm{NL}}^{\mathrm{ortho}} = -38 \pm 24$.
Could LSS improve those constraints?

In principle: 'yes'!

LSS: $N_{\mathrm{modes}}^{\mathrm{LSS}} \sim V k_{max}^3 \sim 10^{10}$:
$V = (10^4 \mathrm{Mpc}/h)^3$; $k_{max} = 0.5h.\mathrm{Mpc}^{-1}$.
CMB: $N_{\mathrm{modes}}^{\mathrm{CMB}} \sim S k_{max}^2 \sim 10^7$.

Bispectrum: generalities

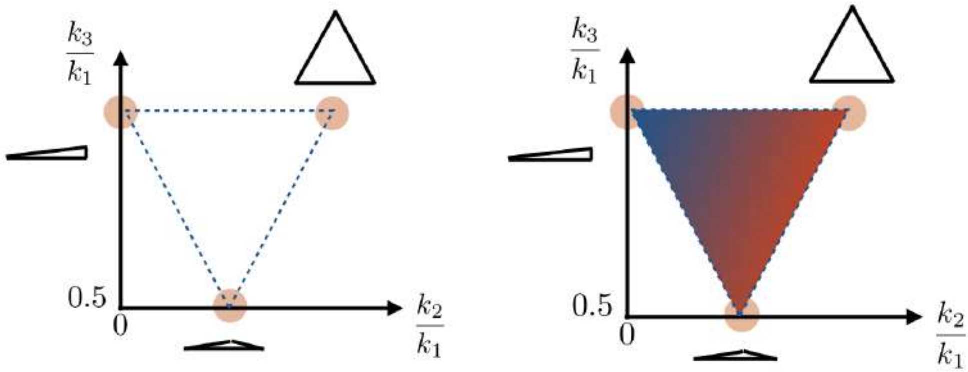

Image credit: J. Noreña

The red zone is degenerated with non-linear growth, biasing and astrophysics.

The blue zone, the the squeezed limit is believed to be much more solid.

Bispectrum for Fundamental physics

The squeezed limit contains model independent information about the physics during inflation.

Theorem: (Consistency relations), Maldacena 0210603

If only one light scalar field is active during inflation, the behavior of the three-point correlation function, in the squeezed limit, is entirely fixed by the two-point correlation function.

Physically, long wavelength gravitational potential is locally unobservable for an universe evolving with a single degree of freedom.

Way out of the theorem:

- Several fields active during inflation Sugiyama 1101.3636
- higher spin Arkani-Hamed 1503.08043
- 'modified' gravity Tahara 1805.00186
- anisotropic inflation Emaml 1511.01683
- presence of an electric field Chua 1810.09815

These theorems also apply to the late universe (Creminelli 1309.3557)
$\rightarrow$ probe the early universe with LSS observables.

Interlude (advertisement): Schwinger effect in the early universe

Schwinger effect

Above a critical value for an electric field: particle production occurs 'Schwinger effect' (Sauter 1930, Schwinger 1954). Not detected today.
Cosmology: if during inflation, a strong electric field is present: particle production Fröb 1401.4137, Kobayashi 1408.4141.
Their behavior depends on their spin (**Stahl** 1507.01686), on the spatial dimensions (**Bavarsad** 1602.06556) and changes if one adds a magnetic field (**Bavarsad** 1707.03975).

Produce non-gaussianities

'Cosmic-collider': to couple the pairs created to the inflaton leads to a unique signal Chua 1810.09815.

Impact primordial gravitational waves

Only for higher spin like SU(2) fields Lozanov 1805.09318.

Interlude (advertisement): Schwinger effect in the early universe

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Stability of de Sitter space

backreaction to the inflation dynamics (**Bavarsad** 1602.06556)

Primordial magnetogenesis

Triggering particle production may help to generate the seed for the large scale magnetic field observed today **Stahl** 1603.07166, **Stahl** 1806.06692, Sobol 1807.09851.

Conclusions

Motivations

- While most of LSS do not need relativity, the bispectrum couples scales, its non-linear evolution has to be calculated within GR.
- The bispectrum in the squeezed limit is 'protected' from astrophysical effects (equivalence principle)
- In LSS, the bispectrum can be used to probe early universe physics.
- The next generation of LSS experiments should be able to measure $f_{\rm NL} = \mathcal{O}(1)$.

General Relativity: diffeomorphism invariance

Perturbations around a FLRW universe

$$ds^2 = -(1+2\phi)dt^2 + 2\omega_i dx^i dt + a(t)^2 \left[(1-2\psi)\delta_{ij} + h_{ij}\right] dx^i dx^j . \qquad (3)$$

Poisson gauge

- $\delta^{ij}\omega_{i,j} = \delta^{ij}h_{ij} = \delta^{jk}h_{ij,k} = 0$.
- Velocity of the fluid:

$$u^\mu = \left(1 - \phi + \frac{a^2 v^2}{2}, v^i\right) .$$

- Physical interpretation simple.
- Gauge used for relativistic N-body simulations `gevolution` (Adamek 1604.06065).

Synchronous-Comoving gauge

- $\delta^{ij}h_{ij} = \delta^{jk}h_{ij,k} = 0$ and $u^0 = 1$.
- Velocity of the fluid:

$$u^\mu = \left(1, -\frac{(1+2\psi)\partial_i\omega + w_i}{a^2(t)}\right) ,$$

where $\omega_i \equiv \partial_i\omega + w_i$.

- Gauge relevant when it comes for observation: use the time measured by a local observer.

Weak Field Approximation

Typically $v \sim 10^{-2}$, $\phi \sim 10^{-5}$, but: $\delta = \frac{2}{3(aH)^2} k^2 \phi \sim \frac{0.1\text{Mpc}^{-1}}{10^{-6}\text{Mpc}^{-1}} \phi \sim 1$.
$\rightarrow$ Work perturbatively in v and ϕ but full non-linear in δ.

Weak field approximation

Perturbations around a FLRW universe

$$ds^2 = -(1+2\phi)dt^2 + 2\omega_i dx^i dt + a(t)^2\left[(1-2\psi)\delta_{ij} + h_{ij}\right]dx^i dx^j. \qquad (4)$$

Let $\epsilon = 10^{-2} = \frac{aH}{k}$, the weak field approximation consists in

variable	order in Poisson gauge	order in comoving gauge
∂_i/H	$\mathcal{O}(\epsilon^{-1})$	$\mathcal{O}(\epsilon^{-1})$
∂_t/H	$\mathcal{O}(1)$	$\mathcal{O}(1)$
ϕ,ψ	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^2)$
$\chi\equiv\phi-\psi$	$\mathcal{O}(\epsilon^4)$	-
w_i	$\mathcal{O}(\epsilon^3)$	$\mathcal{O}(\epsilon^3)$
ω	-	$\mathcal{O}(\epsilon^2)$
h_{ij}	$\mathcal{O}(\epsilon^4)$	$\mathcal{O}(\epsilon^4)$
δ	$\mathcal{O}(1)$	$\mathcal{O}(1)$
v^i	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon)$

We will present results at $\mathcal{O}(\epsilon^2)$, but one can in principle calculate at any order.

Equation of motion

Conservation of the energy momentum tensor + Einstein equation

$$\nabla_\mu(\rho u^\mu) = 0, u^\mu \nabla_\mu u^\nu = 0, G_{\mu\nu} = T_{\mu\nu}. \tag{5}$$

Full non-linear equations: Euler + conservation of mass

$$\dot{\delta} + \theta = -\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}_{12}) \alpha(\boldsymbol{k}_1, \boldsymbol{k}_2) \theta(\boldsymbol{k}_1) \delta(\boldsymbol{k}_2) + \mathcal{S}_\delta[\delta, \theta],$$

$$\dot{\theta} + 2H\theta + \frac{3H^2}{2}\delta = -2\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}_{12}) \beta(\boldsymbol{k}_1, \boldsymbol{k}_2) \theta(\boldsymbol{k}_1) \theta(\boldsymbol{k}_2) + \mathcal{S}_\theta[\delta, \theta].$$

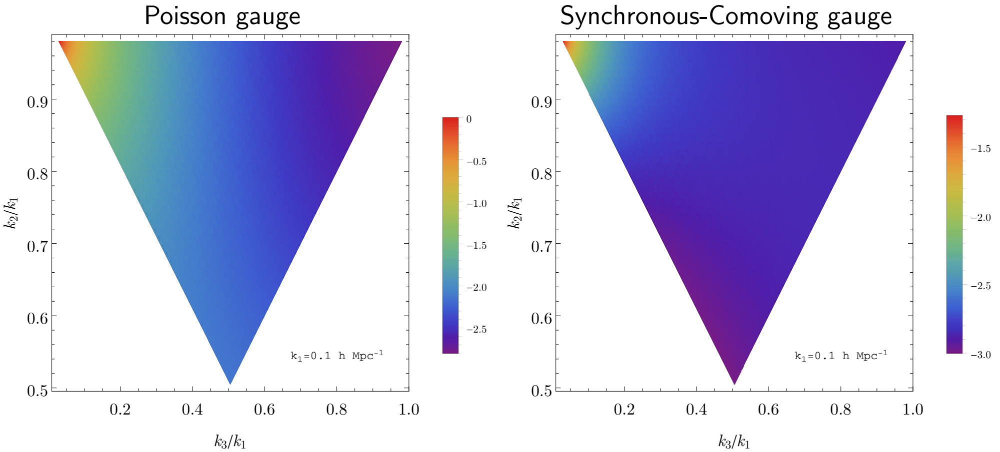
$\theta \equiv \partial_i v^i$. Use G^0_i to include frame dragging effects (ω_i) and G^0_0 for potentials ϕ, ψ. $S_{\delta/\theta}$ are the relativistic corrections: $\mathcal{O}(\epsilon^2)$, eg. $\sim \dot{\delta}\delta/k^2$.

Perturbation theory: take $\delta \ll 1$

$$\delta(\boldsymbol{k}, t) = \sum_{n=1}^{\infty} a^n(t) \int_{\boldsymbol{k}_1..\boldsymbol{k}_n} \Big[F_n(\boldsymbol{k}_1, .., \boldsymbol{k}_n) + a^2(t) H^2(t) F_n^R(\boldsymbol{k}_1, .., \boldsymbol{k}_n) \Big] \delta_l(\boldsymbol{k}_1)..\delta_l(\boldsymbol{k}_n) \, .$$

The end !

We plot $\frac{B^R(k_1,k_2,k_3)}{B(k_1,k_2,k_3)}$ (1-loop=stopping at $n = 4$ in perturbation theory).



Some differences from SPT

- Initial conditions
- Renormalization of the background

Initial conditions (IC)

We want to solve:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3H^2}{2}\delta = S_{\mathsf{NL}}[\delta] + S_{\mathsf{Rel}}[\delta] \equiv S(t). \quad (6)$$

The solution reads (setting the decaying mode to 0):

$$\delta(\boldsymbol{k}, t) = c_1(\boldsymbol{k})a(t) + \int_0^t dt' G(t, t')S(t'). \quad (7)$$

Newtonian case

One sets: $c_1(\boldsymbol{k}) = \frac{2k^2}{3H_0^2}\phi$ (linear level).

In principle IC should be set at the non-linear level but $F_2 \propto a^2 \propto \frac{k^4}{a^4 H^4}$.
$\rightarrow$ non-linear IC are suppressed by ϵ^4.

Relativistic case

However $F_2^R \propto a^4 H^2 \propto \frac{k^2}{a^2 H^2} = \mathcal{O}(\epsilon^2)$ should not be ignored!
At higher order in perturbation theory, IC's can be ignored.
Fitzpatrick 0902.2814
NB: Second order results agree with Matarrese 9707278
NB: To my knowledge, is not included in `gevolution` or `GRAMSES` (Barrera 1905.08890).

Renormalization of the background

In LSS, split between large scale *background* (expanding universe, **well defined mean density**) and intermediate scale *perturbations* (density differs little from background).

$$\langle \delta(\boldsymbol{k}) \rangle = (2\pi)^3 \delta_D(\boldsymbol{k}) a^2 \int_{\boldsymbol{q}} F_2(\boldsymbol{q}, -\boldsymbol{q}) P_L(q) \,. \tag{8}$$

While in the Newtonian case: $F_2(\boldsymbol{q}_1, \boldsymbol{q}_2) \xrightarrow[|\boldsymbol{q}_1+\boldsymbol{q}_2| \to 0]{} (\boldsymbol{q}_1 + \boldsymbol{q}_2)^2$,

in the relativistic case: $F_2^R(\boldsymbol{q}_1, \boldsymbol{q}_2) \xrightarrow[|\boldsymbol{q}_1+\boldsymbol{q}_2| \to 0]{} a^2 H^2$.

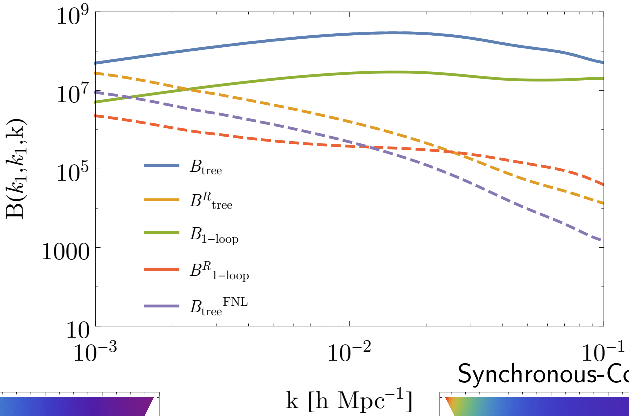
A well defined perturbation theory requires $\langle \delta(\boldsymbol{k}) \rangle = 0$.
Idea: Reabsorb the non-zero part by changing slightly the background (Baumann 1004.2488). We define:

$$\bar{\rho} \to \bar{\rho}\big[1 + \langle \delta(\boldsymbol{k}) \rangle\big]. \tag{9}$$

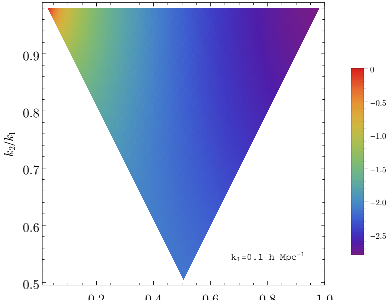
A pressure is also required to ensure $\langle \phi(\boldsymbol{k}) \rangle = 0$. They induce a variation (through Friedmann equation) of the Hubble constant. Explicitly $P \sim \frac{\Delta H}{H} = \mathcal{O}(10^{-5})$.

Results

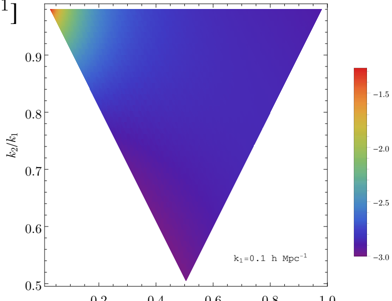
Synchronous-Comoving gauge



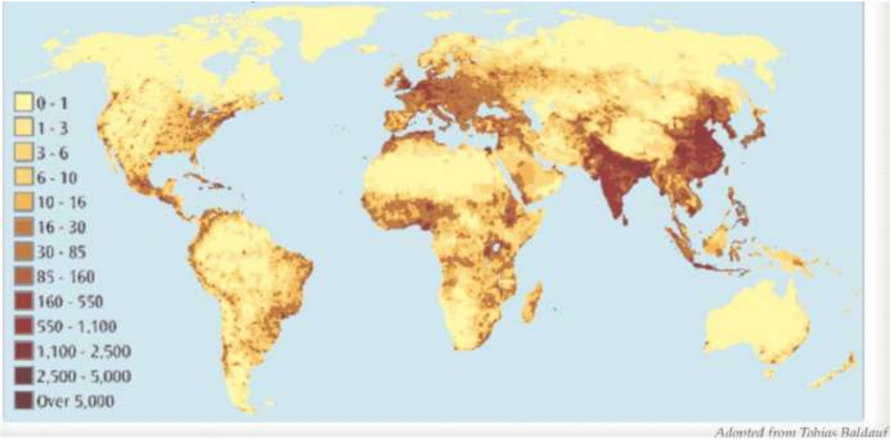
Poisson gauge



Synchronous-Comoving gauge



Human population density



At night

Adopted from Tobias Baldauf

On which geometric quantities the formation of galaxies depends?

Working frame

- Approach à la Effective field Theory: smaller scales are smoothed out and the astrophysical processed are encoded in a handful of *bias* coefficients $b_{\mathcal{O}}$ to be determined (Desjacques 1611.09787).
- Frame of reference of an observer moving with the halo's center of mass ($\rightarrow$ Synchronous-comoving gauge).
- Velocity of dark matter = velocity of halos/galaxies.
- No creation of galaxies.

On which geometric quantities the formation of galaxies depends?

Build on Umeh 1901.07460, generalized to 4th order:

$$\delta_g^{(n)} = a^n \left(F_n^T + \sum b_{\mathcal{O}}^{\mathcal{L}} M_n^{\mathcal{O}} \right) \delta_\ell^n \,, \tag{10}$$

where $F_n^T \equiv F_n + a^2(t)H^2(t)F_n^R$ and $M_n^{\mathcal{O}} \equiv M_n^{\mathcal{O}} + a^2(t)H^2(t)M_n^{\mathcal{O},R}$

at second order ($n = 2$), we find $\mathcal{O} = \{\delta; \delta^2; s^2\}$ such that: (**Calles** 1912.13034)

$$M_2^{\delta}(\boldsymbol{k}_1, \boldsymbol{k}_2) = F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + \frac{4}{21} - \frac{2}{7} s^2(\boldsymbol{k}_1, \boldsymbol{k}_2) \,, \tag{11}$$

$$M_2^{\delta^2}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{1}{2} \,, M_2^{s^2}(\boldsymbol{k}_1, \boldsymbol{k}_2) = s^2(\boldsymbol{k}_1, \boldsymbol{k}_2) \tag{12}$$

$$M_2{}^{\delta,R}(\boldsymbol{k}_1, \boldsymbol{k}_2) = F_2^R(\boldsymbol{k}_1, \boldsymbol{k}_2) \,, \tag{13}$$

with $s^2 \equiv \frac{(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2)^2}{k_1^2 k_2^2} - \frac{1}{3}$. More explicitely:

$$\delta_g^{(2)} = a^2 \left[\left(1 + \frac{b_1}{a} \right) F_2^T + \frac{1}{2} \left(\frac{b_2}{a^2} - \frac{4}{21} \frac{b_1}{a} \right) + \left(\frac{b_{s^2}}{a^2} - \frac{2}{7} \frac{b_1}{a} \right) s^2 \right] \delta_\ell^2 \,, \tag{14}$$

Hassani 1910.01104 : Relativistic corrections including $k-$essence term (eg. Barreira 1411.5965) in the framwork of the EFT of LSS:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\mathsf{Pl}}^2}{2}R - \Lambda(t) - c(t)g^{00} + \frac{M_2^4(t)}{2}\left(\delta g^{00}\right)^2 + \dots\right] \tag{15}$$

Cosmological perturbations studied with the Stückelberg trick: dark energy perturbations are studied with the scalar field $\pi(\vec{x},t)$.
This gives contribution to the stress energy tensor:

$$T_0^0 = -\rho + \frac{\rho+p}{c_s^2}\Big(3c_s^2\mathcal{H}\pi - \zeta\Big), \tag{16}$$

$$T_i^0 = -(\rho+p)\partial_i\pi, \tag{17}$$

$$T_j^i = p\delta_j^i - (\rho+p)\Big(3c_a^2\mathcal{H}\pi - \zeta\Big)\delta_j^i. \tag{18}$$

with $\zeta = \pi' + \mathcal{H}\pi - \Psi$

Main results

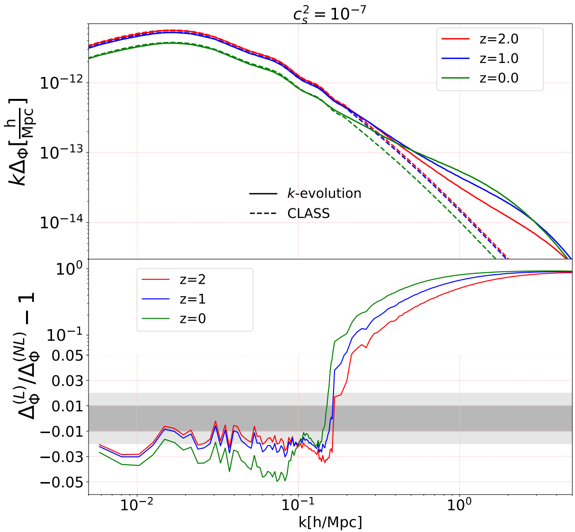
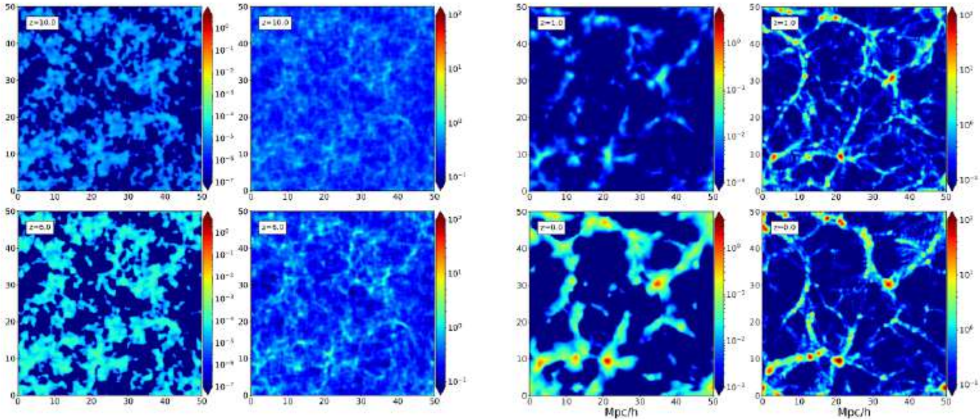


Figure: Compare the gravitational potential ϕ computed with their code `k-evolution` and with `CLASS`

Main results: Observe clustering of dark energy



Conclusions and next steps

- As the bispectrum couples scales, we calculated the one-loop bispectrum within GR in the weak field approximation.
- The bispectrum in the squeezed limit is protected from astrophysics and allow to probe early universe physics (primordial non-gaussianities)
- The relativistic contributions are of the same order than a primordial non-gaussianity of the local type.

This calculation is not finished!

- We calculated the dark matter field and showed the procedure to extend our results to biased tracers (galaxies). No new bias parameters were required.
- Short scales physics (EFT) has to be taken into account.
- The light emitted y galaxies travels in a clumpy universe, one also needs to correct for that ('redshift space distortions')

- The Einstein-de Sitter approximation (full dark matter) is usually a good one (subpercent accuracy, Takahashi 0806.1437) or 1.5% in modified gravity (Fasiello 1604.04612). It possible to check the impact of Λ on our setup following for instance Amendola 0612180.


THANK YOU!