The FLRW cosmological model revisited: relation of the local time with the local curvature and consequences on the Heisenberg uncertainty principle

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Abstract

By using the FLRW cosmological model, we calculated the relation between the local time and the local curvature in the case of a vacuum dominated universe. We showed that except for special values of the different constants which enter this equation, the time cannot be equal to zero. By using this assumption, we showed also that the demonstration of the uncertainty principle of Heisenberg is only an approximation.

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One of the most studied cosmological model is the FLRW (Friedmann-Lemaitre-Robertson-Walker) cosmological model. In this model, the metric may be written [1]:

$$ds^2 = c^2dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi)] \tag{1}$$

where c is the speed of light, t the time, k the local curvature and r, θ and ϕ define an appropriate system of coordinates. Therefore, the quantity $a(t)$ represents the cosmic expansion factor and gives the rate at which two points of fixed comoving coordinates (r_1, θ_1, ϕ_1) and (r_2, θ_2, ϕ_2) increase their mutual distance as $a(t)$ increases[1].

By solving Einstein's equations for the FLRW metric given by equation (1), one may find out the time dependence of $a(t)$. Indeed, if the matter content of the Universe can be described by a perfect fluid, such equations reduce to the system of two equations [1]:

$$(\frac{\frac{da}{dt}}{a})^2 = H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2} \tag{2}$$

$$-(\frac{\frac{d^2a}{dt^2}}{a}) = \frac{4\pi G}{3}(\rho + 3p) \tag{3}$$

where G is the gravitational constant, ρ is the density of the fluid, Λ is the cosmological constant, H the Hubble parameter and p is the impulsion, in the case of a vacuum dominated universe.

The aim of this study is to relate the local curvature to the local time. This may be obtained using equation (2):

$$dt = \frac{da}{Ha} \tag{4}$$

with

$$a^2(t) = -k(H^2 - \frac{8\pi G}{3}\rho - \frac{\Lambda}{3})^{-1} \tag{5}$$

therefore the relation of the local time with the local curvature may be written:

$$t = \frac{\ln a}{H} = \frac{1}{2H}\ln(-k(H^2 - \frac{8\pi G}{3}\rho - \frac{\Lambda}{3})^{-1}) \tag{6}$$

Therefore, the behavior of the local time t as a function of the local curvature k depends on the respective values of H, G and Λ.

If we analyze equation (6), we see that the local time t cannot be equal to zero except for the special case where $H^2 - \frac{8\pi G}{3}\rho - \frac{\Lambda}{3} = 1$ and $k = -1$. In this last case, the local time is always equal to zero. This is unphysical although it may happen in the FLRW model for the critical density $\rho_c = 3H^2/8\pi G$ and in the absence of a cosmological constant term.

More explicitly, if we use equation (4) by integrating it on given domain it leads to :

$$\int_{t=0}^{t=t'} dt = \int_{a(t=0)}^{a(t=t')} \frac{da}{Ha} da \tag{7}$$

which gives:

$$t' = \frac{1}{2H}\ln(-k(H^2 - \frac{8\pi G}{3}\rho(t') - \frac{\Lambda}{3})^{-1}) \tag{8}$$
$$-\frac{1}{2H}\ln(-k(H^2 - \frac{8\pi G}{3}\rho(t=0) - \frac{\Lambda}{3})^{-1})$$

Here again the same analysis as for equation (6) applies. Moreover, the difference between the two parts of the right hand side of equation (8) cannot be equal to zero except for $\rho(t') = \rho(t = 0)$ and $k(t') = k(t = 0)$ which is once again a particular case. Therefore, in the general case, t' cannot be equal to zero.

We can interpret the fact that the coordinates t, r, θ and ϕ which define a local referential may be replaced by the coordinates k, r, θ and ϕ which are also define a local referential. Physically, it means that the three dimensional curved space represented by the three coordinates r, θ, ϕ is embedded in a four dimensional space which mean curvature is not known but which fourth dimension is represented by the coordinate k. As k is the local curvature it may be related to the local radius of curvature. This means that because of the relation between time t and k given by equation (6), the irreversibility of time is no more intrinsic to it but only due to the expansion of the universe.

Now let us make the hypothesis that the definition of local time in the FLRW cosmological model is the same as the definition of time in quantum mechanics. The previous paragraph states that time t cannot be equal to zero except for a critical density.

If we place ourselves in the general case where time cannot be equal to zero and depends on the local curvature, let us begin the demonstration which usually leads to the uncertainty principle of Heisenberg [2]:

$$\psi(\mathbf{r}, t) = \frac{1}{(2\pi)^3/2} \int g(\mathbf{k}) e^{i[\mathbf{k_w}.\mathbf{r} - \omega(k_w)t]} d^3k_w \tag{9}$$

which may be simplified in one dimension into:

$$\psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(k_w) e^{i[k_w x - (\hbar k_w^2/2m)t]} dk_w \tag{10}$$

where k_w is the wavenumber, $\psi(\mathbf{r}, t)$ is the wavefunction of the particle of mass m, $\hbar$ is Planck's constant, The demonstration made in order to obtain the Heisenberg uncertainty principle makes the hypothesis that $t = 0$. This allows one to simplify this last equation into a simple Fourier transform. But, as obtained in equation (6), t has a non zero value and cannot be equal to zero except in very special cases. Therefore, the simplification which leads to a simple Fourier transform in the demonstration of the uncertainty principle of Heisenberg [2] reduces to an approximation. Indeed, the demonstration remains valid with the approximation that:

$$(\hbar k^2/2m)t << kx \tag{11}$$

The numerical value of $\hbar$ is equal to $1.054589.10^{-34} J.s.$ So the approximation is almost always valid. This is in good agreement with all the experiments which have been made in quantum mechanics.

As a conclusion, we can say that the local time can be related to the local curvature in the FLRW cosmological model, and therefore that the local time cannot be equal to zero except in very special cases of the values of the constants which enter the FLRW model. This means also that our curved universe may be seen as embedded in a four dimensional space where the fourth local coordinate is the local radius of curvature. Using the fact that local time cannot be equal to zero, we showed also that the demonstration of the uncertainty principle of Heisenberg is only a very good approximation. Indeed, the demonstration of the uncertainty principle of Heisenberg is not as simple as a Fourier transform.

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