Strong light-matter coupling in a quantum metasurface

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ABSTRACT

Metasurfaces have been used for light manipulation and control, from the point of view of planar optics or polarization control, or non-linear light extraction. The properties of metasurfaces is rooted in the presence of resonant basic elements which are responsible for strongly confined electromagnetic surface modes. We propose to use these modes to enhance light-matter interaction in the vicinity of the surface by considering hybrid structure made of a thin layer, such as a 2D quantum material or a thin film with embedded quantum dots. A semi-classical theory will be proposed along with some functionalities attainable with the device.

1. INTRODUCTION

Metasurfaces have been used for light manipulation and control, from the point of view of planar optics or polarization control, or non-linear light extraction. The properties of metasurfaces is rooted in the presence of resonant basic elements which are responsible for strongly confined electromagnetic surface modes. We propose to use these modes to enhance light-matter interaction in the vicinity of the surface by considering hybrid structure made of a thin layer, such as a 2D quantum material or a thin film with embedded quantum dots.

Quantum metamaterials[1–4] should serve to control the electromagnetic field at the quantum level.[6,13] We do not address this problem here, the field is considered classical and the quantum oscillators A semi-classical theory will be proposed along with some functionalities attainable with the device. The metasurface is described using a non-local impedance operator.[8] This allows efficient numerical treatment as well as a theoretical approach based on a modal expansion. The layer is treated either as a thin slab of material where the excitons are described within the effective mass approximation (single effective particle picture), or as a collection of quantum oscillators (many-body approach). An effective field description is given that allows for the study of topological excitations.

2. MODELING OF THE RESONATORS AND OF THE METAMATERIAL

The structure under study is depicted in fig. 1. The passive structure can be studied in the frequency domain.

The resonators are characterized by their scattering matrix $S(\omega)$.[9–12] When the structure is illuminated by an incident field, it gives rise to a scattered field that can be written:

$$U^s(M) = \sum_{p,n} s_n^p \varphi_n(M - M_p). \tag{1}$$

where

$$\varphi_n(r,\theta) = H_n^{(1)}(k_0|r|)e^{in\theta},$$

and $H_n^{(1)}$ is the nth Hankel function of order 1. From multiple scattering theory,[13–15] it can be shown that:

$$s(\omega, k) = \left[1 - S(\omega)\Sigma(\omega, k)\right]^{-1} S(\omega), \tag{2}$$

where $\Sigma(\omega, k)$ is a lattice sum.[16–18]

The band diagram of the bare structure,[19–21] that is the dielectric metamaterial without the inclusions is given in fig. (2).

We use the band in the vicinity of the resonant transition frequency ω_{12} of the quantum oscillators.

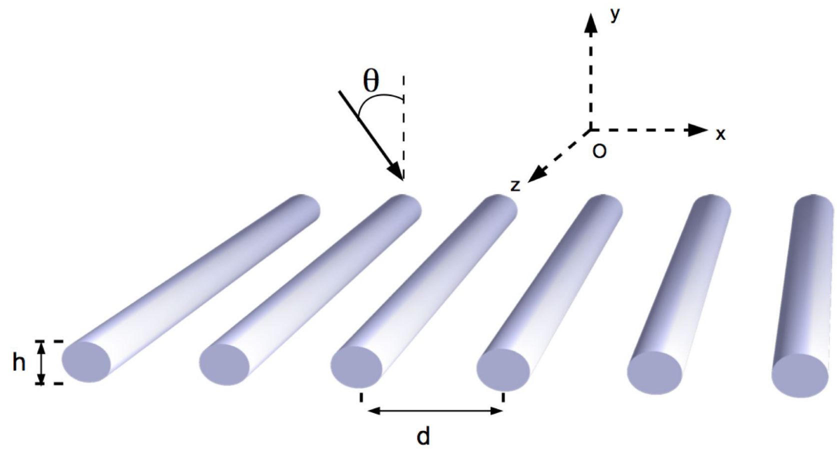

Figure 1. Scheme of the structure under study.

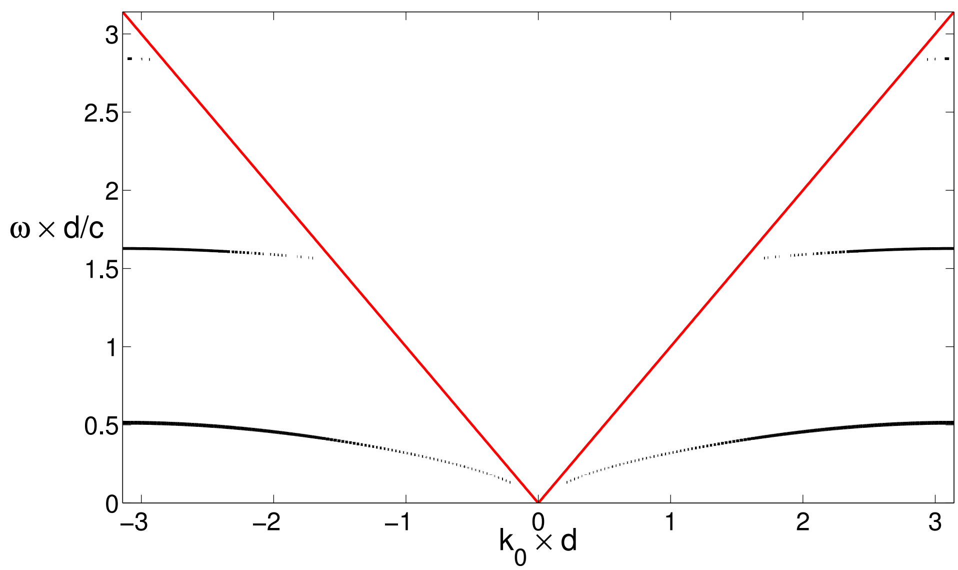

Figure 2. The dispersion curve for the Bloch modes in the presence of resonances. The light cone is depicted in red.

The propagation of a beam is ruled by the Maxwell-Bloch system

$$i\frac{\partial E}{\partial z}+\frac{1}{2\beta_0}\Delta_\perp E-\frac{D_0}{2}\frac{\partial^2 E}{\partial t^2}+\frac{i}{\nu_0}\frac{\partial E}{\partial t}=\frac{i\omega}{2\varepsilon_0 c}\Lambda \tag{3}$$

$$\frac{\partial \Lambda}{\partial t}+\left(\gamma_{12}+i(\omega_{12}-\omega)\right)\Lambda=\frac{ip^2}{\hbar}AN \tag{4}$$

$$\frac{\partial N}{\partial t}+\gamma_{11}(N-N_0)=\frac{2i}{\hbar}\left(A^*\Lambda-A\Lambda^*\right). \tag{5}$$

In the time-independent approximation, the collection of oscillators gives rise to an effective polarization field eta_xi.jpg

Figure 3. Phase portrait of the system (13). The critical point of the abscissa axis corresponds to a solitary wave.

with susceptibility given by

$$\chi(\omega,|E|^2)=\frac{p^2}{\varepsilon_0 h}N_0\frac{(\omega_{12}-\omega)+i\gamma_{12}}{\gamma_{12}^2+(\omega_{12}-\omega)^2+\frac{4p^2A^2}{h^2}\frac{\gamma_{12}}{\gamma_{11}}|F|^2} \tag{6}$$

We assume that the field propagates along the meta-surface below the light-cone, in such a way that it is legit to perform a paraxial approximation. In doing so, and considering that the field propagates along a mode of the chain of wires, one obtains the following equation for the enveloppe of the electric field

$$i\frac{\partial E}{\partial z}=-\frac{1}{2\beta_0}\Delta_\perp E+\frac{D_0}{2}\frac{\partial^2 E}{\partial t^2}-\frac{i}{\nu_0}\frac{\partial E}{\partial t}+i\chi(\omega,|E|^2)E \tag{7}$$

In the low-intensity approximation, this equation can now be put in the form of Landau-Ginzburg equation

$$i\frac{\partial E}{\partial z}=-\frac{1}{2\beta_0}\Delta_\perp E+\frac{D_0}{2}\frac{\partial^2 E}{\partial t^2}-\frac{i}{\nu_0}\frac{\partial E}{\partial t}+i(\alpha E-\mu|E|^2E). \tag{8}$$

This equation cannot be solved by inverse scattering techniques but the existence of solitary waves can be investigated by plugging the generic form of a soliton for the non-linear schrödinger equation. Before doing so, let us simplify the equation further by eliminating the transverse derivatives $\Delta_\perp E$. This retains the main physics of the problem since the modes considered are below the light cone. Finally, since $D_0 \ll \nu_0^{-1}$, we are led to the final equation to be solved

$$\frac{\partial E}{\partial z}+\frac{1}{\nu_0}\frac{\partial E}{\partial t}=\alpha E-\mu|E|^2E. \tag{9}$$

From this, conservation laws can be deduced. The continuity relation reads as

$$\frac{1}{2}\frac{d}{dt}\int(E\partial_z E^*-E^*\partial_z E)\,dx=\int(\alpha E^*\partial_z E-\alpha^*E\partial_z E^*)dx-\int(\beta E^*\partial_z E-\beta^*E\partial_z E^*)|E|^2dx\,, \tag{10}$$

and the conservation of energy leads to

$$\frac{1}{\nu_0}\frac{d}{dt}\int|E|^2dx=\Re(\alpha)\int|E|^2dx-\Re(\beta)\int|E|^4dx. \tag{11}$$

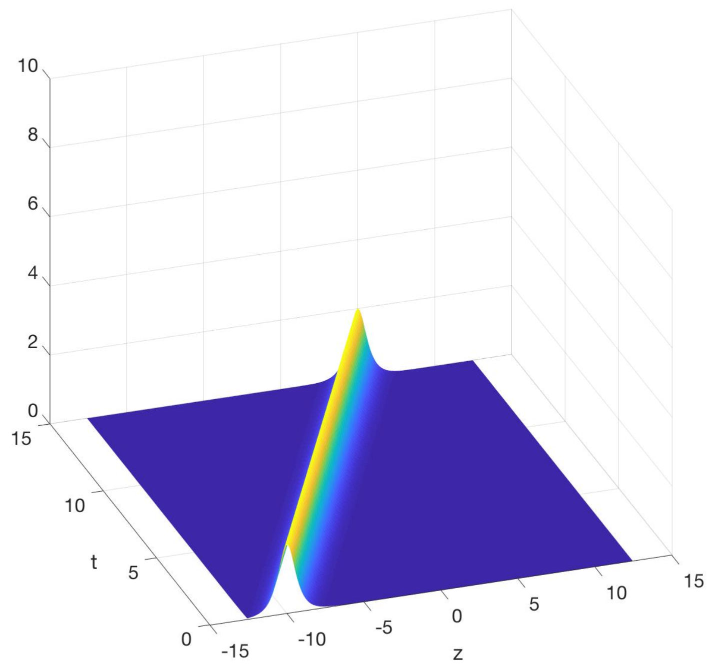

Figure 4. Propagation of a solitary wave along the meta-surface in the inversion regime.

The existence of solitary waves can be probed by injecting the form of a soliton inside the conservation laws. Consider a soliton in the form

$$q(x,t) = 2\eta\,\mathrm{sech}[2\eta(x-\bar{x})]e^{-2i\xi x-2i\sigma}. \tag{12}$$

When it is inserted in the conservation laws, this gives

$$\frac{d\eta}{dt} = \nu_0\alpha\eta - \frac{16}{3}\nu_0\beta\eta^3 \tag{13}$$

$$\frac{d\xi}{dt} = (2-\nu_0)\alpha\xi + \frac{8}{3}(2\nu_0+1)\eta^2\xi. \tag{14}$$

A phase portrait of this dynamical system is given in fig. 3. There a non trivial critical point is seen for

$$\xi_c = 0,\ \eta_c = \frac{\sqrt{3}\alpha}{4\beta}. \tag{15}$$

An example of such a solitary wave propagating along the metasurface is given in fig. 4, when the oscillators are in the inversion regime, i.e. there is gain in the system.

It is interesting to note, that when the parameters of the solitary wave are away from the critical point, the solitary wave tends to spread when propagating along the surface, as seen in fig.5.

In conclusion, we have demonstrated the existence of confined Bloch modes with small group velocity propagating along the metasurface below the light-cone and we have derived a mean-field equation describing the interaction of these modes with a collection of two-level systems situated in the metasurface. This interaction results in the existence of solitary wave that can propagate along the metasurface.

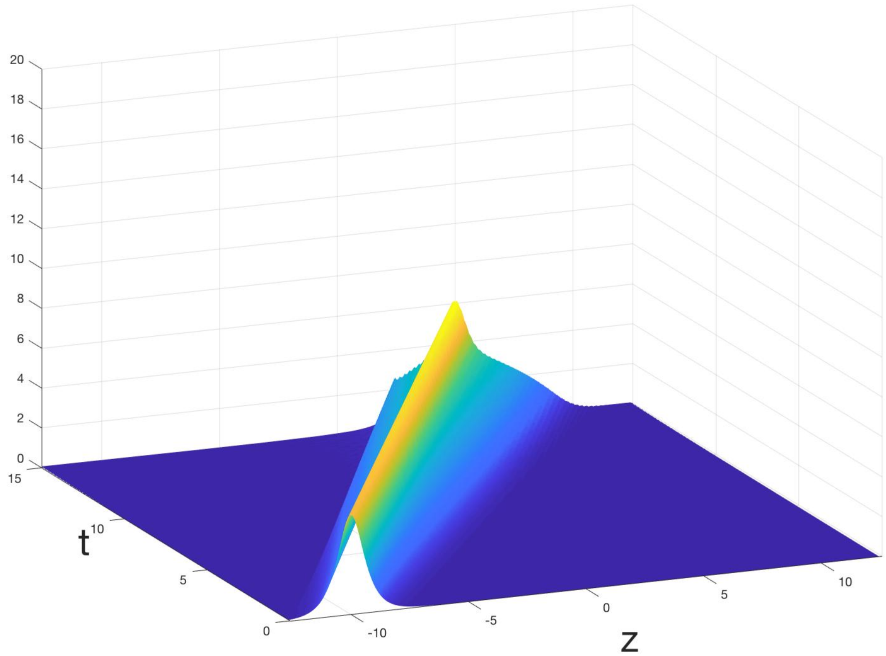

Figure 5. Spreading of a solitary wave along the meta-surface in the inversion regime.

REFERENCES

[1] D. Felbacq and G. Bouchitté, Metamaterials modeling and design, PanStanford Publisher, New York (2017).

[2] B. Guizal, D. Barchiesi, D. Felbacq,"Electromagnetic beam diffraction by a finite lamellar structure: an aperiodic coupled-wave method," J. Opt. Soc. Am. A **20**, 2274-2280 (2003).

[3] D. Felbacq, B. Guizal, F. Zolla,"Ultra-refraction phenomena in Bragg mirrors," J. Opt. A: Pure and Appl. Opt. **2**, L30-L32 (2000)

[4] B. Guizal, H. Yala, D. Felbacq,"Reformulation of the eigenvalue problem in the Fourier modal method with spatial adaptive resolution," Opt. Lett. **34**, 2790-2792 (2009).

[5] A. Rakhmanov, A. Zagoskin, S. Savel'ev and F. Nori, "Quantum metamaterials: Electromagnetic waves in a Josephson qubit line", Phys. Rev. B. **77**, 144507 (2008).

[6] A. Zagoskin, E. Rousseau and D. Felbacq, "Quantum metamaterials in microwave and optical range," EPJ Quantum Technologies (2016).

[7] B. Bellomo, R. Messina, D. Felbacq et al.,"Quantum systems in a stationary environment out of thermal equilibrium," Phys. Rev. A **87**, 012101 (2013).

[8] D. Felbacq, "Impedance operator description of a metasurface," Mathematical Problems in Engineering **2015**, 473079 (2015).

[9] D. Felbacq, G. Tayeb, D. Maystre, "Scattering by a random set of parallel cylinders," J. Opt. Soc. Am. A **11**, 2526-2538 (1997).

[10] D. Felbacq, B. Guizal and F. Zolla, "Wave propagation in one-dimensional photonic crystals," Opt. Comm. **152**, 119-126 (1998).

[11] D. Felbacq, G. Bouchitté,"Theory of mesoscopic magnetism in photonic crystals," Phys. Rev. Lett. **94**, 183902 (2005).

[12] F. Zolla, D. Felbacq, B. Guizal,"A remarkable diffractive property of photonic quasi-crystals," Opt. Comm. **148**, 6-10 (1998).

[13] B. Bellomo, R. Messina, D. Felbacq et al.,"Quantum systems in a stationary environment out of thermal equilibrium," Phys. Rev. A **87**, 012101 (2013).
[14] E. Centeno, D. Felbacq,"Characterization of defect modes in finite bidimensional photonic crystals," J. Opt. Soc. Am. A **16**, 2705-2712 (1999).
[15] G. Bouchitté, C. Bourel, D. Felbacq," Homogenization of the 3D Maxwell system near resonances and artificial magnetism," Comptes Rendus Mathématiques **347**, 571-576 (2009).
[16] C. M. Linton, "Lattice sums for the Helmholtz equation," SIAM Review **52**, 630-674 (2010).
[17] R. Smaali, D. Felbacq, G. Granet, "Bloch waves and non-propagating modes in photonic crystals," Physica E **18**, 443-451 (2003).
[18] D. Felbacq,"Anomalous homogeneous behaviour of metallic photonic crystals," J. Phys. A **33**, 815-821 (2000).
[19] F. Zolla, D. Felbacq and G. Bouchitté," Bloch vector dependence of the plasma frequency in metallic photonic crystals," Phys. Rev. E **74**,056612 (2006).
[20] E. Centeno, D. Felbacq and D. Cassagne,"All-angle phase matching condition and backward second- harmonic localization in nonlinear photonic crystals," Phys. Rev. Lett. **98**, 263903 (2007).
[21] E. Centeno, D. Felbacq,"Second-harmonic emission in two-dimensional photonic crystals," J. Opt. Soc. Am. B **23**, 2257-2264 (2006).